Seminar Slides

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From Big Data to Big Control: Closing Feedback Loops around Large-scale Infrastructure Data

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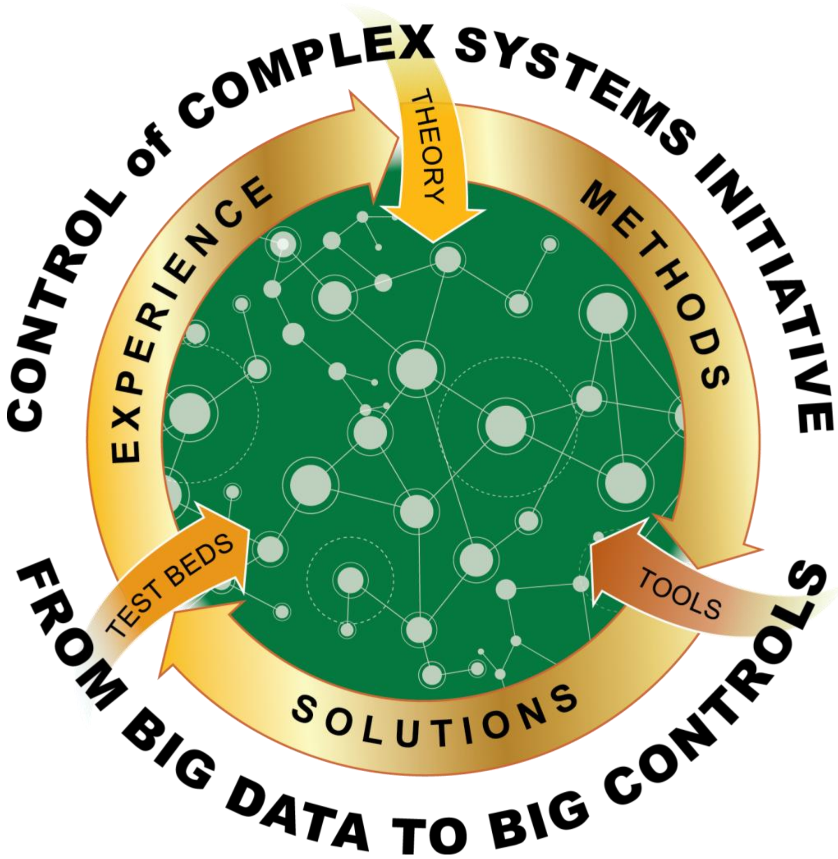
PNNL-SA-107627

Control of Complex Systems Initiative: From Big Data to Big Controls

CCSI: A five year, multi-million dollar internal research investment to build and demonstrate development and delivery of best of class solutions for problems in the control of complex systems.

Challenges for Big Controls:

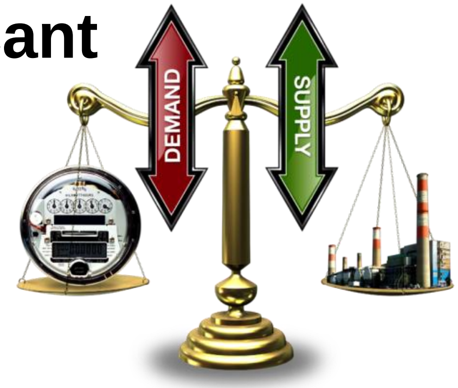
- Large numbers of sensing and/or control end points
- Multiple scales of operation usually with multiple time scales
- Node heterogeneity
- Pervasive computing/autonomous nodes



Control solutions will be:
Scalable, *deployable*, *robust*, *resilient*, and *adoptable*.

Significant Challenges Facing the Grid

The challenges facing the grid are significant and in tension with each other



- Maintain and increase reliability
- Integrate renewables & low-carbon sources
- Potential electrification of vehicle transportation
  (& other end uses as electricity becomes the preferred “fuel”)
- Increase asset utilization, reduce capacity for peak loads
- ***While keeping costs & revenues as low as possible***

Smart grid is the most promising approach to addressing these challenges simultaneously

- Much of smart grid’s promise lies in distributed assets: Demand response, distributed storage & generation, electric vehicles, smart inverters

Future Control Architecture of the Grid

Designing a novel control architecture for the power grid needs a significant number of considerations, e.g.:

- Laws of electro-physics must be observed
- Current/future stakeholder boundaries must be respected
- Architecture must be deployable in a modular, incremental fashion
- For reasons of robustness, resilience & flexibility, the control architecture must be layered
- Considering the huge number of assets, lowest layer must be a *distributed control architecture*

Transactive Controls *is a very promising approach for such a distributed control architecture*

Transactive Controls / Transactive Energy

Refers to *techniques for managing the generation, consumption or flow of electricity* within a power system, *using economic or market-based constructs*, while *respecting grid reliability constraints*.

The term "*transactive*" comes from considering that *decisions are made based on a value*. These decisions may be analogous to, or literally, economic transactions.

Transactive Energy Workshop Proceedings 2012, prepared by the GridWise® Architecture Council, March 2012, PNNL-SA-90082 (http://www.gridwiseac.org/historical/tew2012/tew2012.aspx)

What Problems or Issues is Transactive Control and Coordination Designed to Address?

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Principal Challenges Addressed by TC2

Principal Challenge	Approach
▶ Centralized optimization is unworkable ■ *for such large numbers of controllable assets, e.g. ~10^9 for full demand response participation*	▶ Distributed approach with self-organizing, self-optimizing properties of market-like constructs
▶ Interoperability	▶ Simple information protocol, common between all nodes at all levels of system: *quantity, price or value, & time*
▶ Privacy & security ■ *due to sensitivity of the data required by centralized techniques*	▶ Minimizes risks & sensitivities by limiting content of data exchange to simple transactions
▶ Scalability	▶ Self-similar at all scales in the grid ▶ Common paradigm for control & communication among nodes of all types ▶ Ratio of parent to child nodes limited to ~10^3

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Principal Challenges Addressed by TC2 (cont.)

Principal Challenge	Approach
▶ Level playing field for all assets of all types: ▪ *existing infrastructure & new distributed assets*	▶ Market-like construct provides equal opportunity for all assets ▶ Selects lowest cost, most willing assets to "get the job done"
▶ Maintain customer autonomy ▪ *"Act locally but think globally …"*	▶ Incentive-based construct maintains free will ▪ *customers & 3rd-parties fully control their assets* ▪ *yet collaborate (<u>and get paid for it</u>)*
▶ Achieving multiple objectives with assets essential for them to be cost effective	▶ Allows (but does not require) distribution utility to act as natural aggregator ▪ *address local constraints while representing the resource to the bulk grid*
▶ Stability & controllability	▶ Feedback provides predictable, smooth, stable response from distributed assets ▶ Creates what is effectively closed loop control needed by grid operators

PNNL Transactive Energy Approach: *Transactive Control & Coordination (TC2)*

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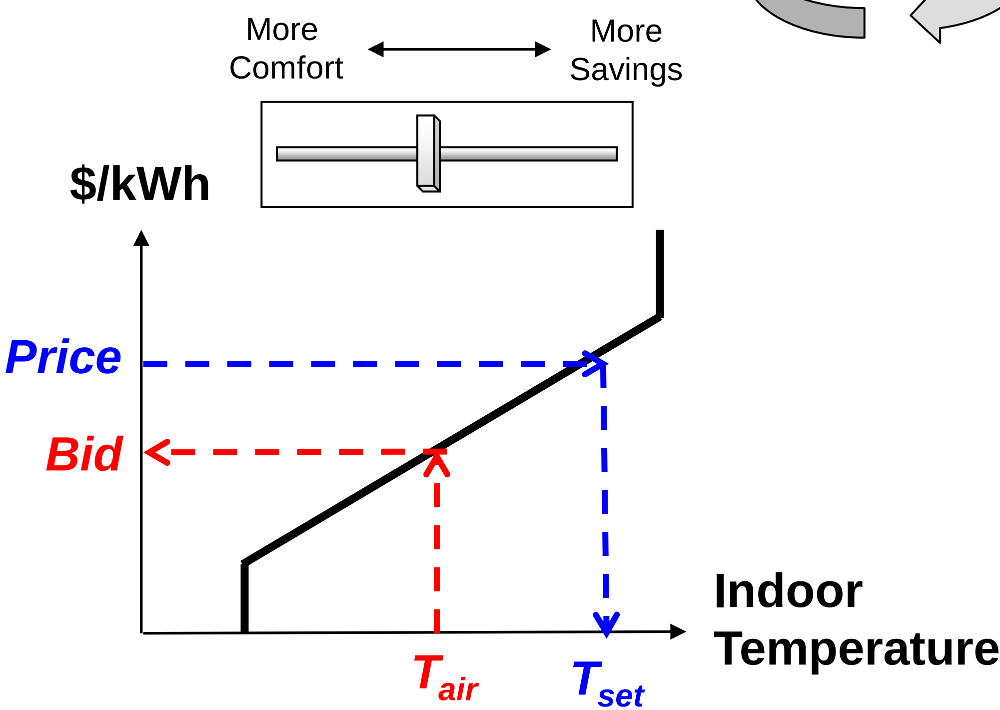
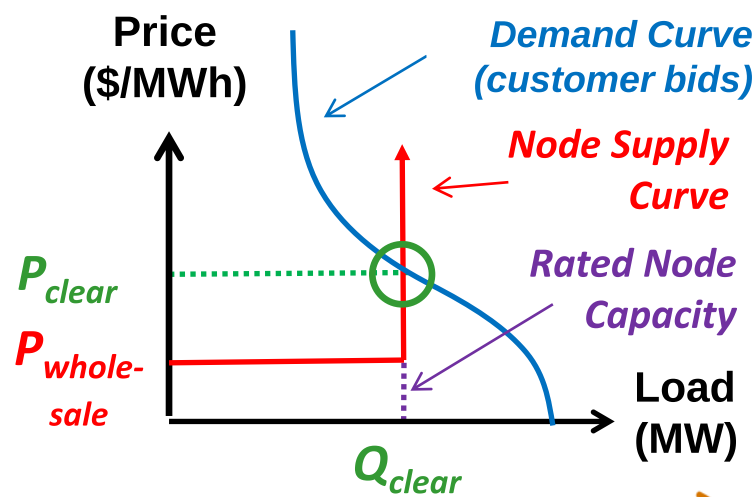
Transactive Control from Interaction of Price Discovery & Customer Bidding Algorithms

Precise, stable control of congested grid nodes derived from customer price-responsive bidding algorithm interacting with price discovery mechanism (e.g., a market)

Transactive Cooling Thermostat



Real-time Market Clears Customer Bids



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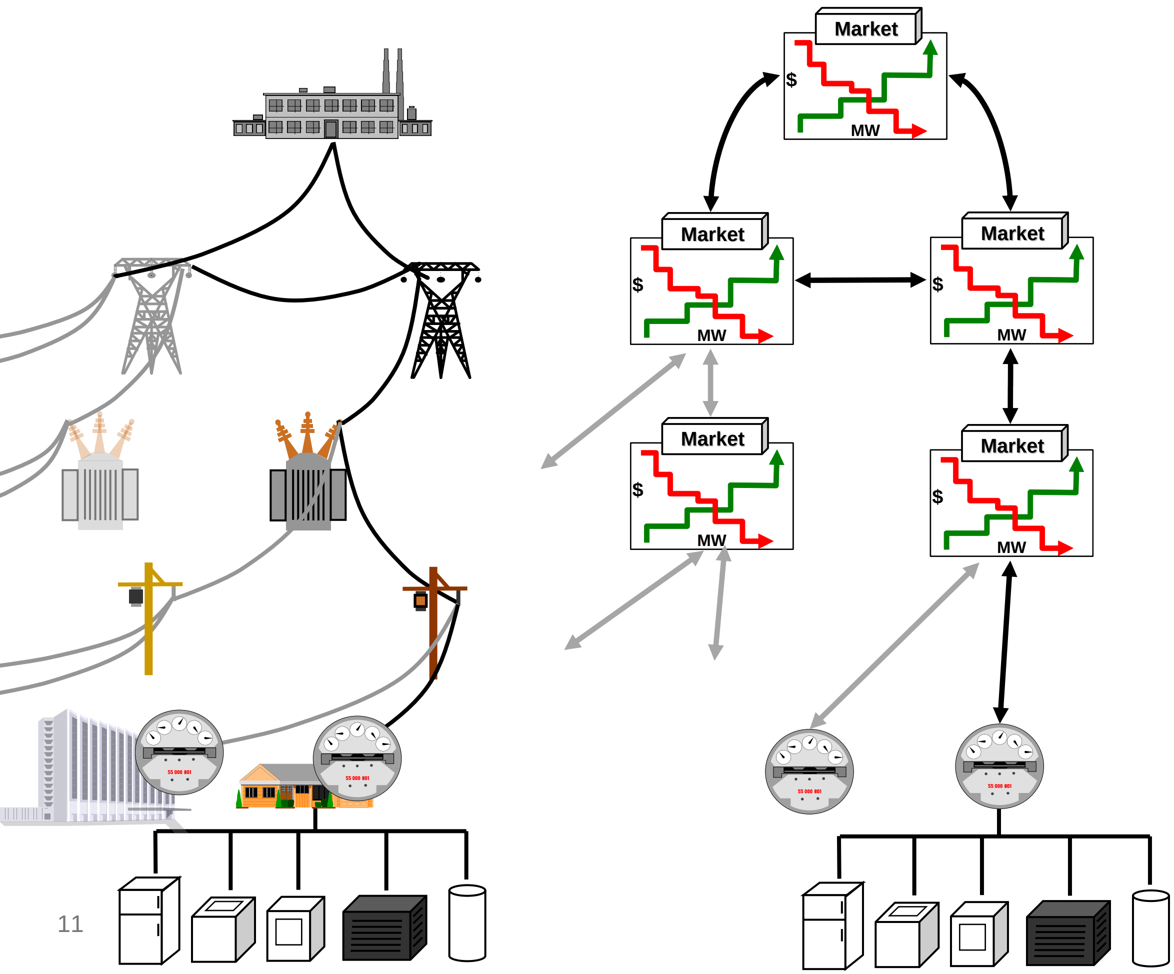
Hierarchical Network of Transactive Nodes Parallels the Grid Infrastructure

Node: point in the grid where flow of power needs to be managed



Node Functionality:

- "Contract" for power it needs from the nodes supplying it
- "Offer" power to the nodes it supplies
- Resolve price (or cost) & quantity through a price discovery process
    - market clearing, for example
- Implement internal price-responsive controls

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Properties of Transactive Nodes

- Use <u>local conditions</u> & <u>global information</u> to make control decisions for its own operation
- Indicate their response to the network node(s) serving them
  - to an *incentive signal* from the node(s) serving them
  - as a *feedback signal* forecasting their projected net flow of electricity (production, delivery, or consumption)
- Setting incentive signal for nodes serves to obtain precise response from them, based on their feedback signals
- <u>Responsiveness</u> is voluntary (set by the node owner)
- <u>Response</u> is typically automated (and reflected in the feedback signal)

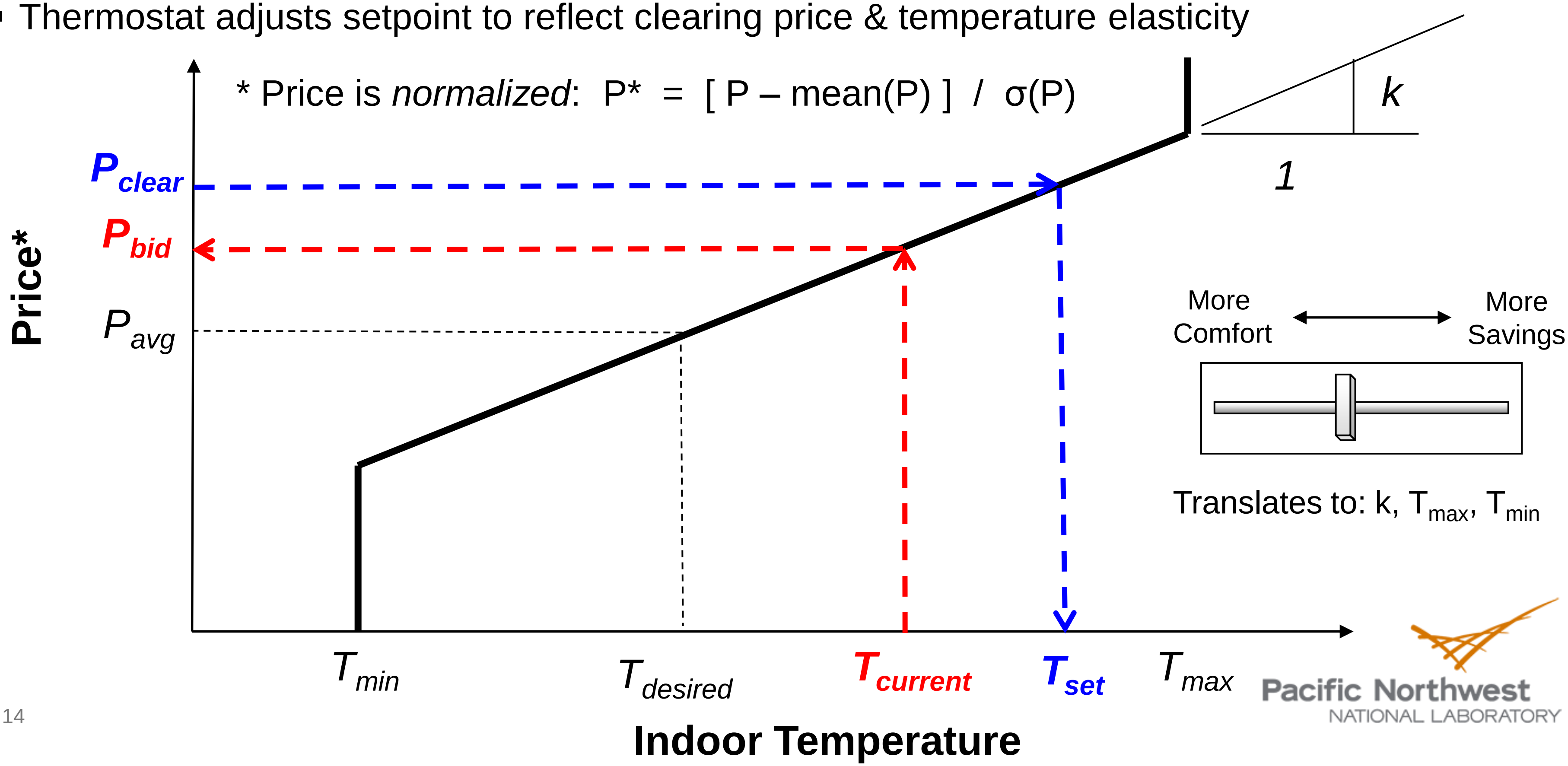
Links All Values/Benefits in Multi-Objective Control

Long-term objective for TC2 is to simultaneously achieve combined benefits

- Reduce peak loads (minimize new capacity, maximize asset utilization) – generation, transmission, <u>& distribution</u>
- Minimize wholesale prices/production costs
- Reduce transmission congestion costs
- Provide stabilizing services on dynamically-constrained transmission lines to free up capacity for renewables
- Provide ancillary services, ramping, & balancing (especially in light of renewables)
- Managing distribution voltages in light of rapid fluctuations in rooftop solar PV system output

Transactive Cooling Thermostat Generates Demand Bid based on Customer Settings

- User's *comfort/savings* setting implies limits around normal setpoint ($T_{desired}$), *temp. elasticity* (*k*)
- Current temperature used to generate bid price at which AC will "run"
- AMI history can be used to estimate bid quantity (AC power)
- Market sorts bids & quantities into demand curve, clears market returns clearing price
- Thermostat adjusts setpoint to reflect clearing price & temperature elasticity



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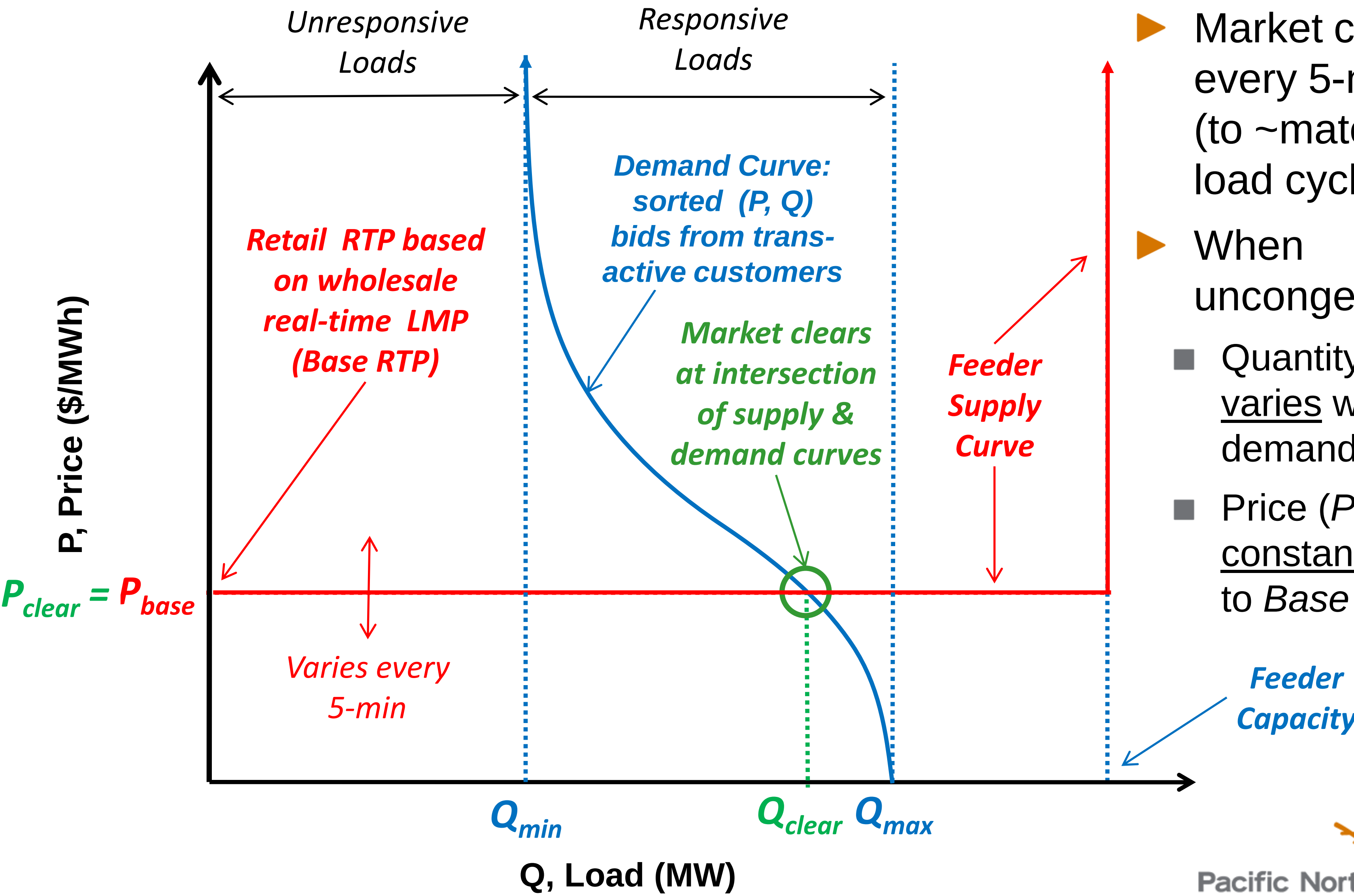
RTP Double Auction Market – *Uncongested*



- Market clears every 5-min (to ~match AC load cycle)
- When uncongested:
  - Quantity (Q_{clear}) varies with demand curve
  - Price (P_{clear}) is constant, equal to *Base RTP*

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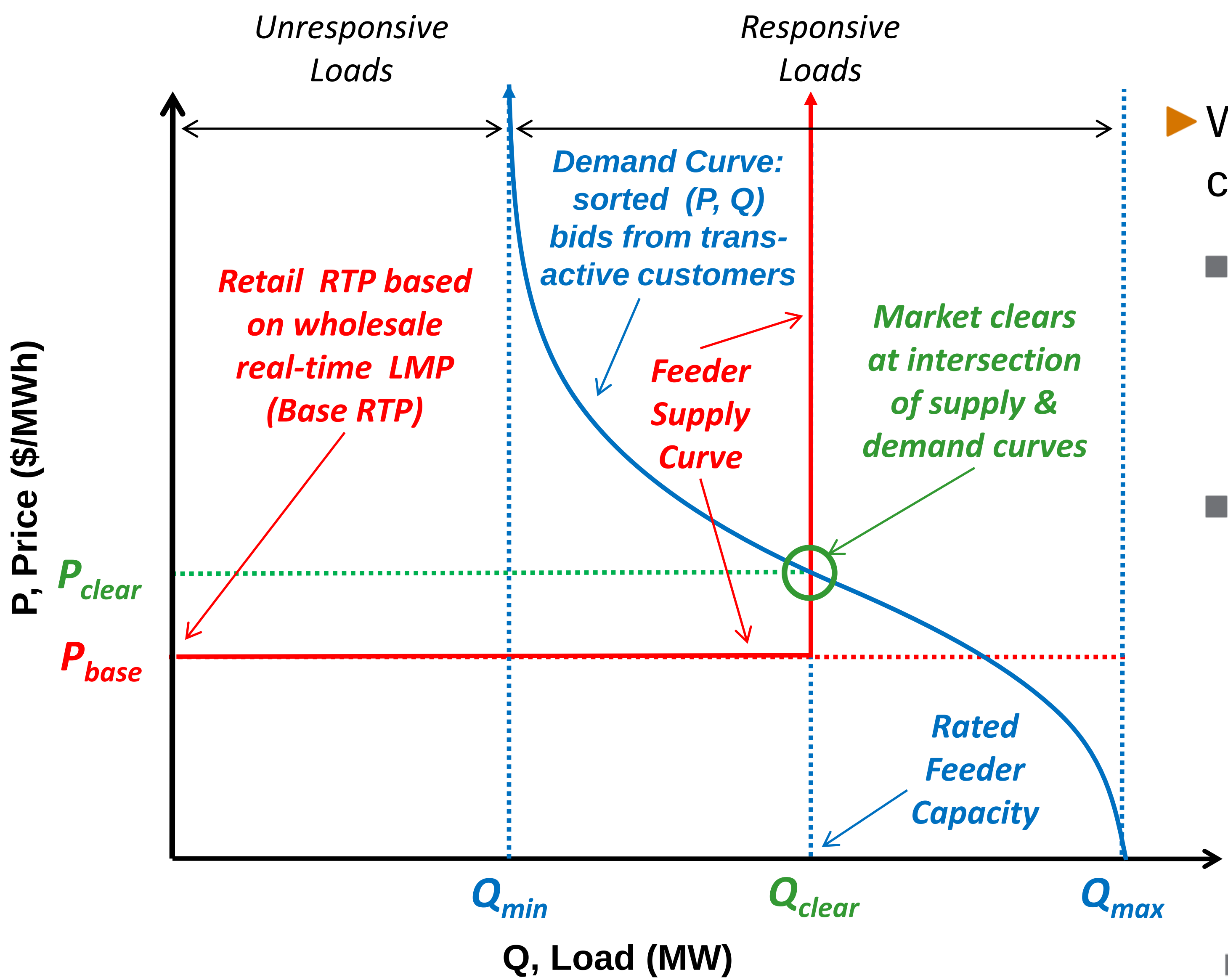
RTP Double Auction Market – *Congested*



- When constrained:
  - Quantity (Q_{clear}) is <u>constant</u> at rated feeder capacity
  - Price (P_{clear}) <u>varies</u> to keep load at rated capacity

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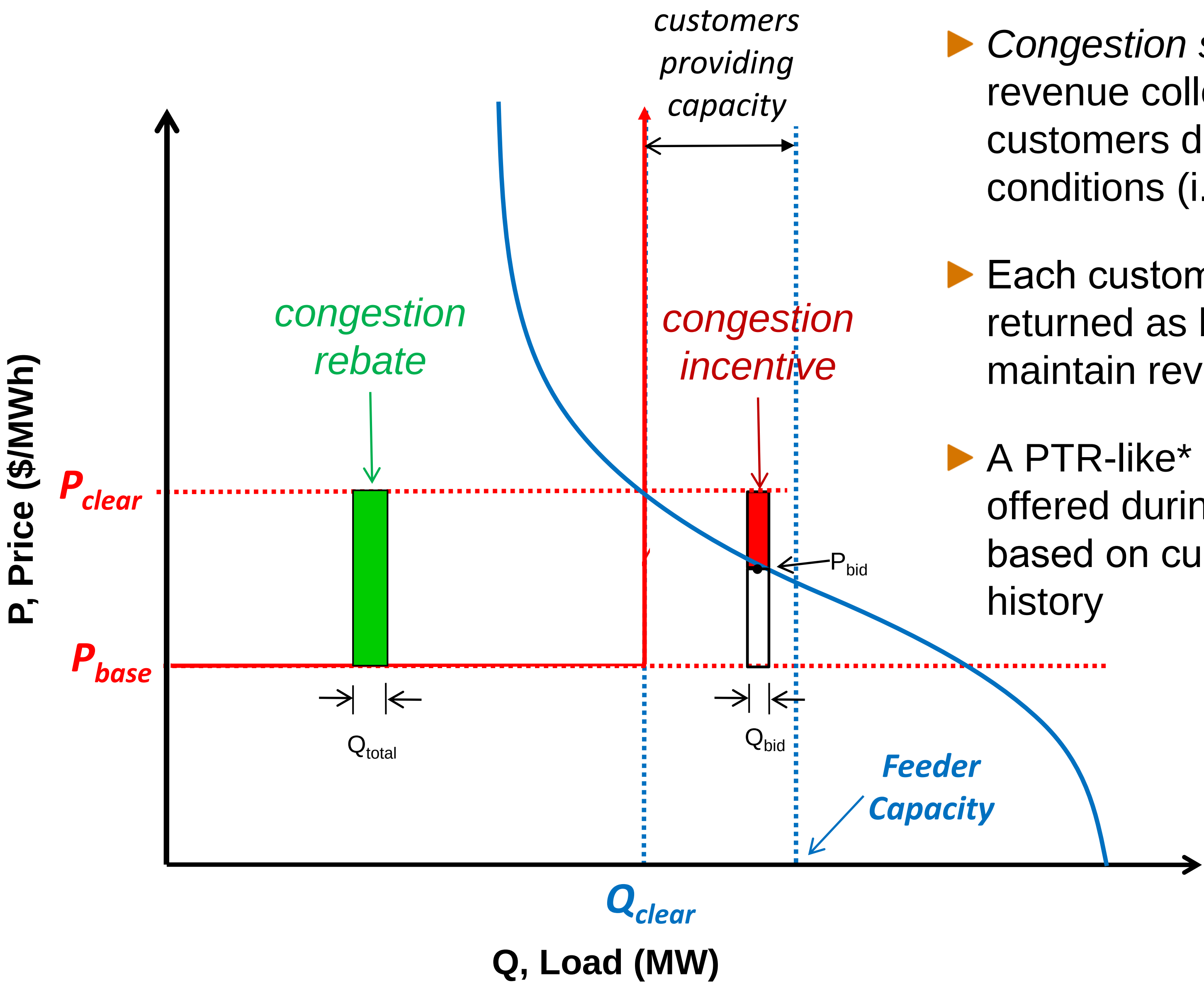
What about the Congestion Surplus?



- *Congestion surplus* is extra revenue collected from customers during constrained conditions (i.e. $P_{clear} > P_{base}$)
- Each customer's surplus returned as billing rebate to maintain revenue neutrality
- A PTR-like* incentive is also offered during congestion, based on customer's bid history

* peak time rebate

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Fully Engaging Demand: What We've Learned from the Olympic Peninsula Demonstration

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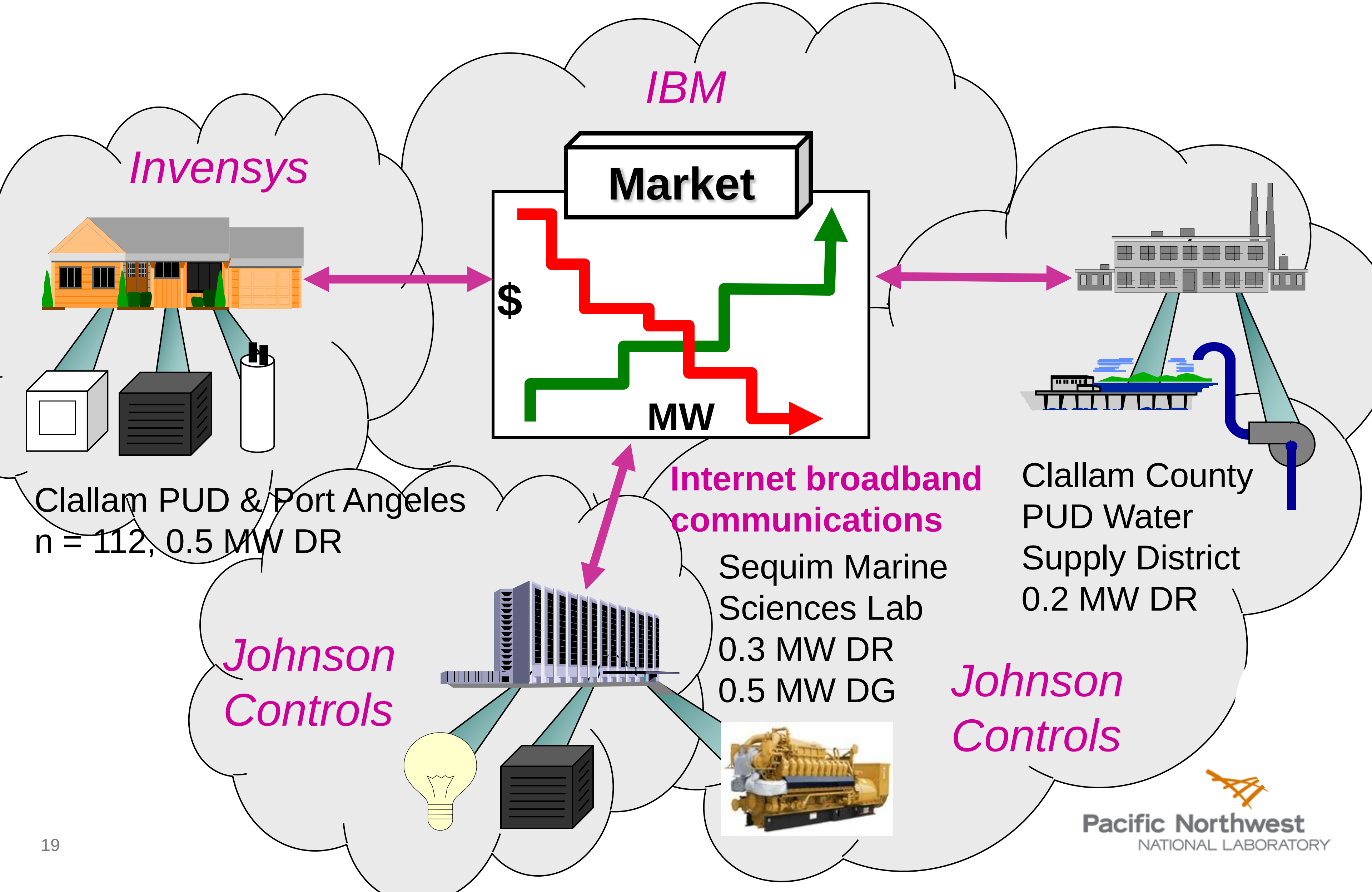
Olympic Peninsula Demonstration



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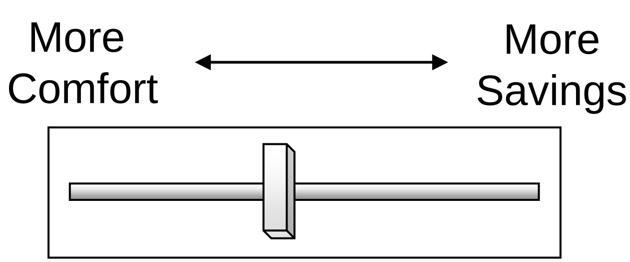
Olympic Peninsula Demo: Key Findings (1)

Customers can be recruited, retained, and will respond to *dynamic pricing* schemes <u>**if they are offered**</u>:

- Opportunity for significant savings (~10% was suggested)
- A "no-lose" proposition compared to a fixed rate
- Control over how much they choose to respond, with which end uses, and a 24-hour override
  - prevents fatigue: reduced participation if called upon too often
- Technology that automates their desired level of response
- A simple, intuitive, semantic interface to automate their response



Translates to control parameters:

K, T_{max}, T_{min} (see *Virtual Thermostat*)

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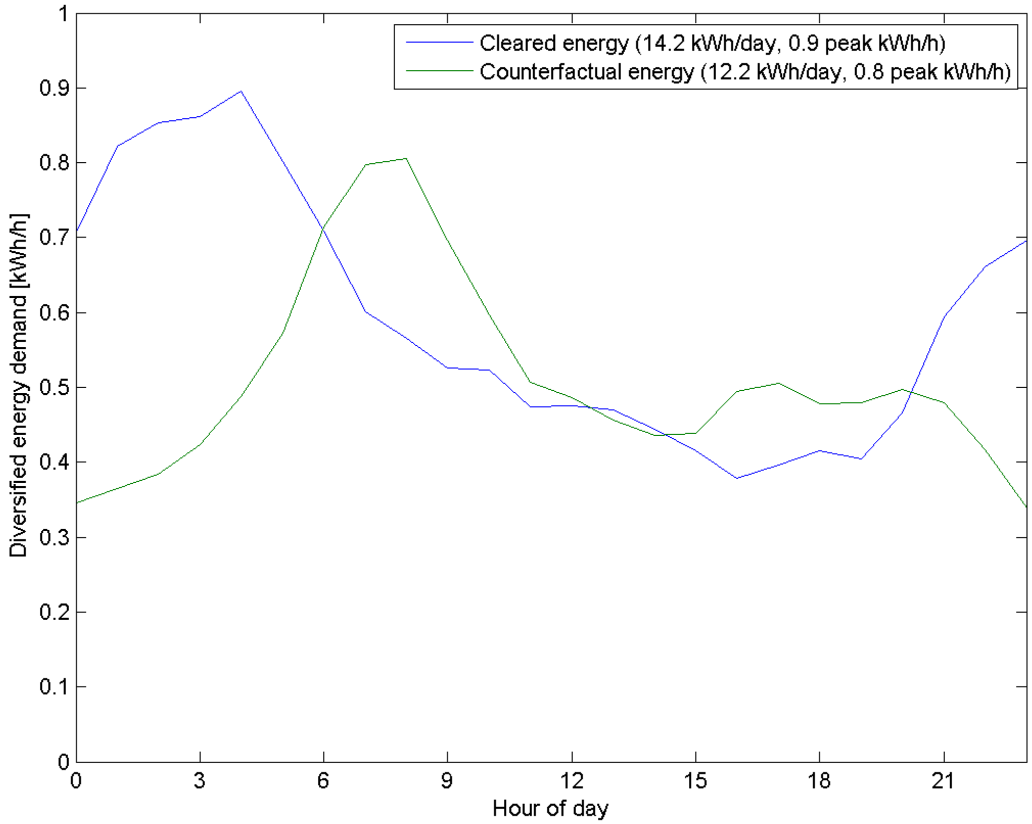
Olympic Peninsula Demo: Key Findings (2)

Significant demand response was obtained:

- 15% reduction of peak load
- Up to 50% reduction in total load for several days in a row during shoulder periods
- Response to wholesale prices + transmission congestion + <u>distribution congestion</u>
- Able to cap net demand at an arbitrary level to manage local distribution constraint
- Short-term response capability <u>could provide regulation, other ancillary services</u> adds significant value at very low impact and low cost)
- Same signals integrated commercial & institutional loads, distributed resources (backup generators)

Load Shifting Results for RTP Customers



- Winter peak load shifted by pre-heating
- Resulting new peak load at 3 AM is non-coincident with system peak at 7 AM
- Illustrates key finding that a portfolio of contract types may be optimal – i.e., we don't want to just create a new peak

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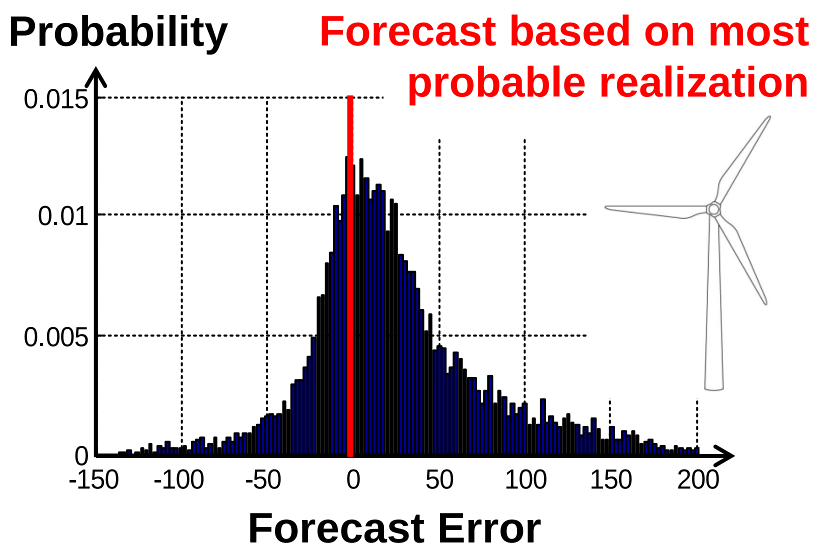
ETH zürich

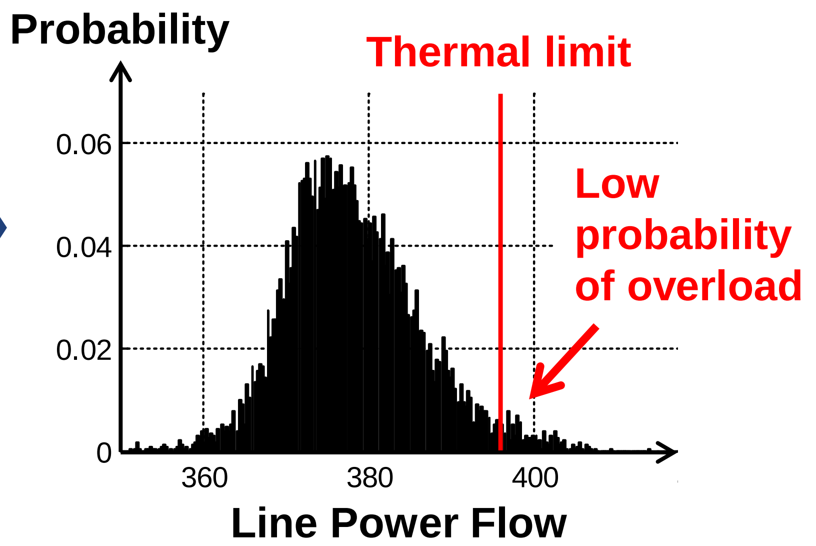
Security Constrained Optimal Power Flow with Distributionally Robust Chance Constraints

Line Roald, Frauke Oldewurtel, Bart Van Parys, Göran Andersson

Santa Fe, 16.01.2015

PROBLEM: Uncertain power injections → uncertain power flows





Uncertainty from:
- Renewables and load
- Intra-day trading

Not always normally distributed!

GOAL: Keep system operation N-1 secure, despite uncertainty!

Chance constrained optimal power flow

- Formulation based on DC power flow
- Chance constraint reflects probability of constraint violation

Post-contingency line flow constraint:

Scheduled power flow + change due to outage

Change due to fluctuations $\boldsymbol{\omega}$

Desired confidence level

$$\mathbb{P}\left(\boldsymbol{A^i_{(l,\cdot)} P^i_{inj}} + \boldsymbol{D^i_{(l,\cdot)}\omega} \leq \boldsymbol{P^{max}_{L(l)}}\right) \geq \boldsymbol{1-\varepsilon}$$

Analytical Reformulation of Chance Constraints

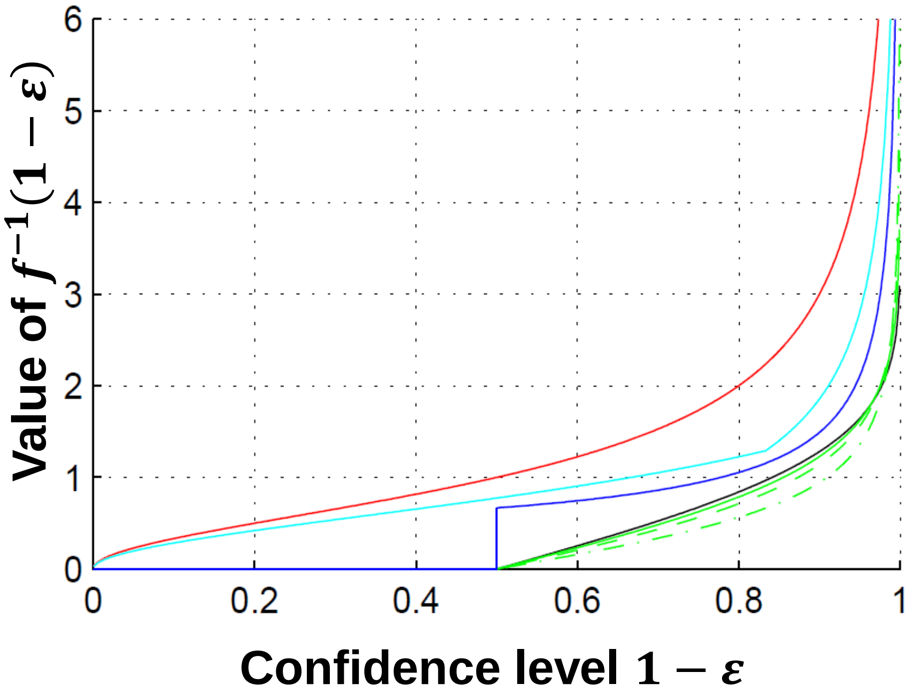
$$\mathbb{P}\left(A^i_{(l,\cdot)}P^i_{inj} + D^i{}_{(l,\cdot)}\omega \leq P^{max}_{L(l)}\right) \geq 1-\varepsilon$$

$$\underbrace{A^i_{(l,\cdot)}P^i_{inj} \leq P^{max}_{L(l)}}_{\text{deterministic constraint}} - \underbrace{f^{-1}(1-\varepsilon)\left\|D^i_{(l,\cdot)}\Sigma^{\frac{1}{2}}\right\|_2 - D^i_{(l,\cdot)}\mu}_{\text{stochastic tightening}}$$

Different (unknown) distributions of ω lead to different expressions for $f^{-1}(1-\varepsilon)$!

- If multivariate normal (or elliptical): Exact reformulation
- If only partially known: Probabilistic inequalties

$$A^{i}_{(l,\cdot)} P^{i}_{inj} \leq P^{max}_{L(l)} - f^{-1}(1-\varepsilon) \left\| D^{i}_{(l,\cdot)} \Sigma^{\frac{1}{2}} \right\|_2 - D^{i}_{(l,\cdot)} \mu$$



Exact reformulation:

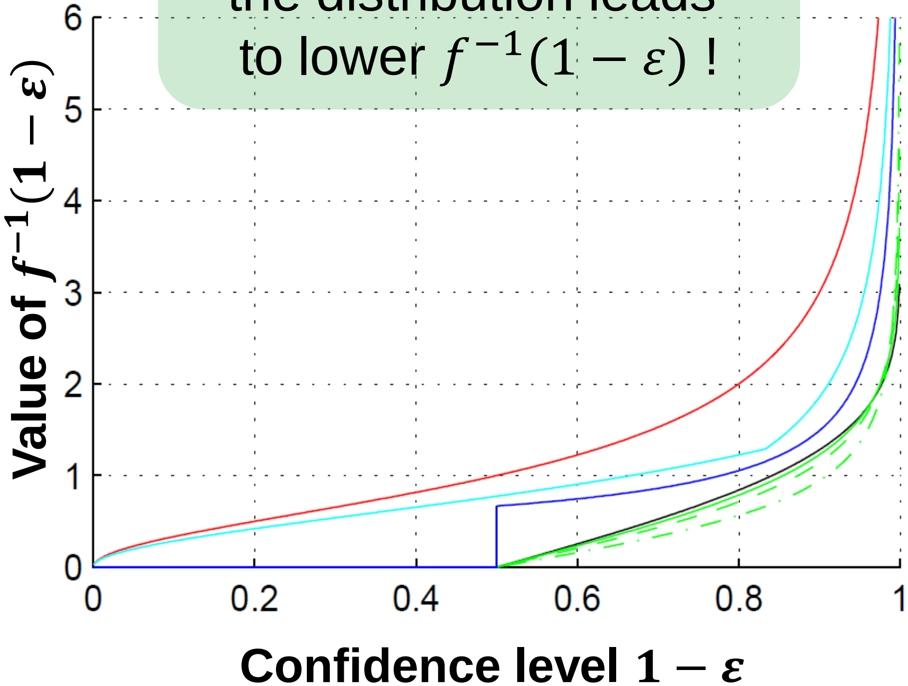
- Normal distribution
- t distribution

Distributionally robust:

- Symmetric, unimodal with known μ & Σ
- Unimodal with known μ & Σ
- Chebyshev (known μ & Σ)

$$A^i_{(l,\cdot)} P^i_{inj} \leq P^{max}_{L(l)} - f^{-1}(1-\varepsilon) \left\| D^i_{(l,\cdot)} \Sigma^{\frac{1}{2}} \right\|_2 - D^i_{(l,\cdot)} \mu$$

More information about the distribution leads to lower $f^{-1}(1-\varepsilon)$!



Exact reformulation:

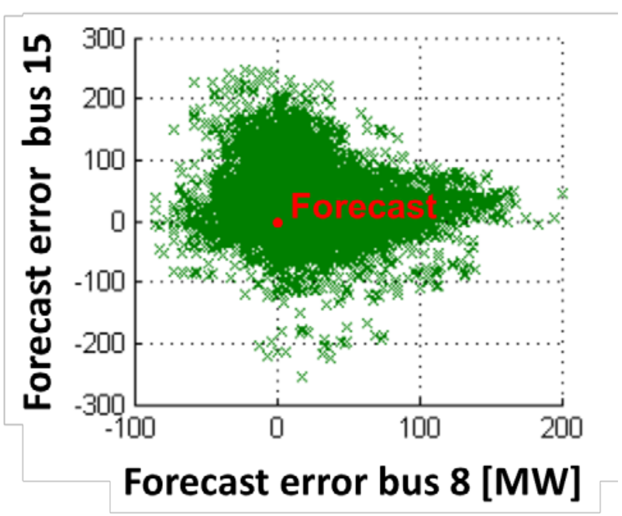
- Normal distribution
- t distribution

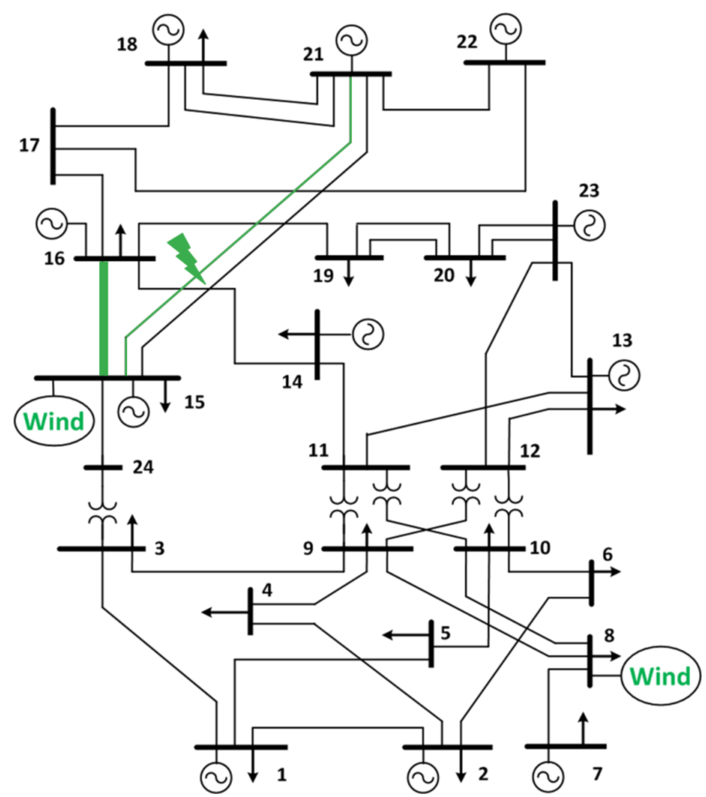
Distributionally robust:

- Symmetric, unimodal with known μ & Σ
- Unimodal with known μ & Σ
- Chebyshev (known μ & Σ)

Case study: IEEE RTS 96 with uncertain in-feeds

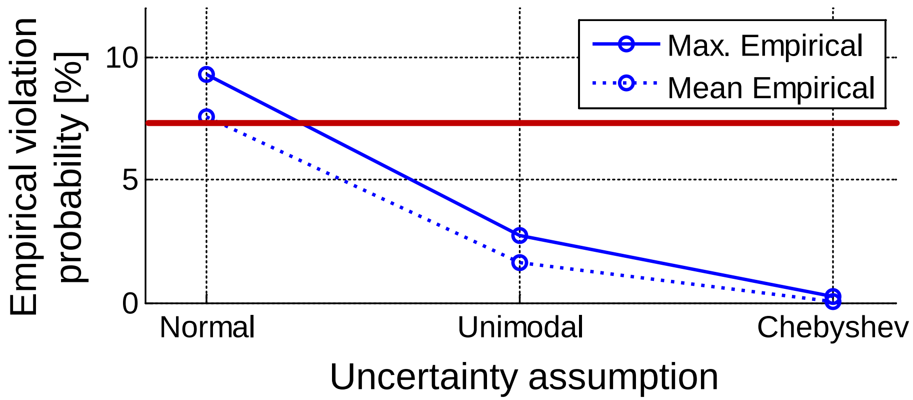
- Two uncertain in-feeds (bus 8, 15)
- μ, Σ based on samples of historical data from APG
- $\varepsilon = 0.075$
- Constant $D^{i}_{(l,\cdot)}$ (LP)
- Different assumptions on ω
- **Not normally distributed!**



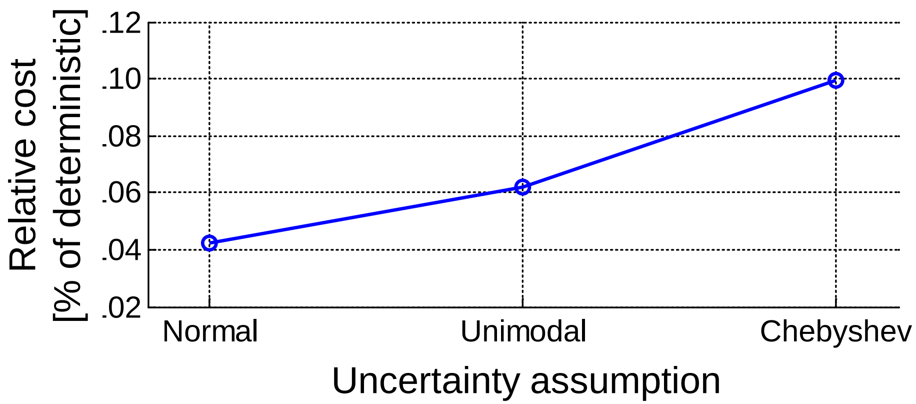


Case study: IEEE RTS 96 with uncertain in-feeds

Empirical violation probability



Relative generation cost



Normal distribution:
«good guess»,
no probabilistic guarantees

Chebyshev:
probabilistic guarantees,
very conservative

Unimodal:
probabilistic guarantees,
less conservative

Summary

- Analytic reformulation for separate chance constraints can be applied to non-normal distributions
- Assuming unimodality might be a good way to provide probabilistic guarantees, without being too conservative
- Next: German network with more uncertainty sources

Thank you!

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Umbrella Project

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Load-side Frequency Control

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Enrique Mallada*
Steven Low
EE, CMS, Caltech

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Elec & Sys Engr
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Lina Li
LIDS/MIT
Harvard

Jan 2015

Outline

Motivation

Network model

Load-side frequency control

Simulations

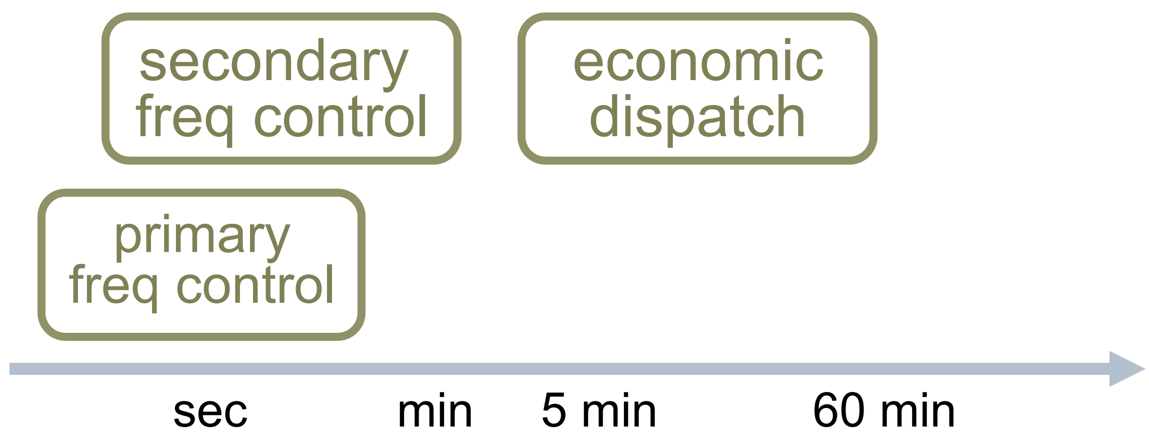
Main references:

Zhao, Topcu, Li, Low, TAC 2014
Mallada, Zhao, Low, Allerton 2014
Zhao, Low, CDC 2014

Why frequency regulation

Control signal to balance supply & demand

Andersson's talk in am



Why frequency regulation

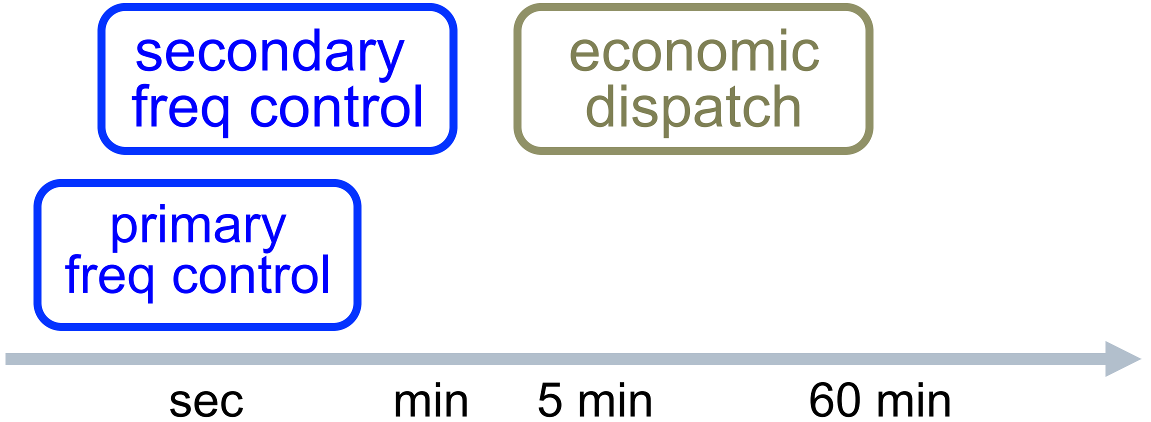
Traditionally done on generator-side

- Frequency control: Lu and Sun (1989), Qu et al (1992), Jiang et al (1997), Wang et al (1998), Guo et al (2000), Siljak et al (2002)
- Stability analysis: Bergen and Hill (1981), Hill and Bergen (1982), Arapostathis et al (1982), Tsolas et al (1985), Tan et al (1995), …
- Recent analysis: Andreasson et al (2013), Zhang and Papachristodoulou (2013), Li et al (2014), Burger et al (2014), You and Chen (2014), Simpson-Porco et al (2013), Dorfler et al (2014), Zhao et al (2014)

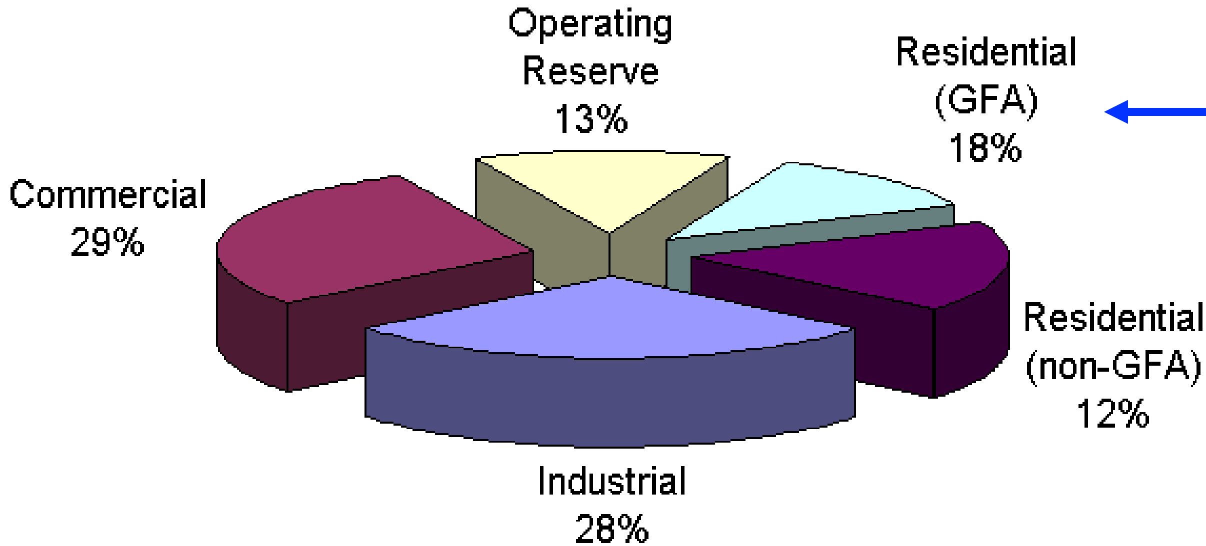
Why load-side participation

Ubiquitous continuous load-side control can supplement generator-side control

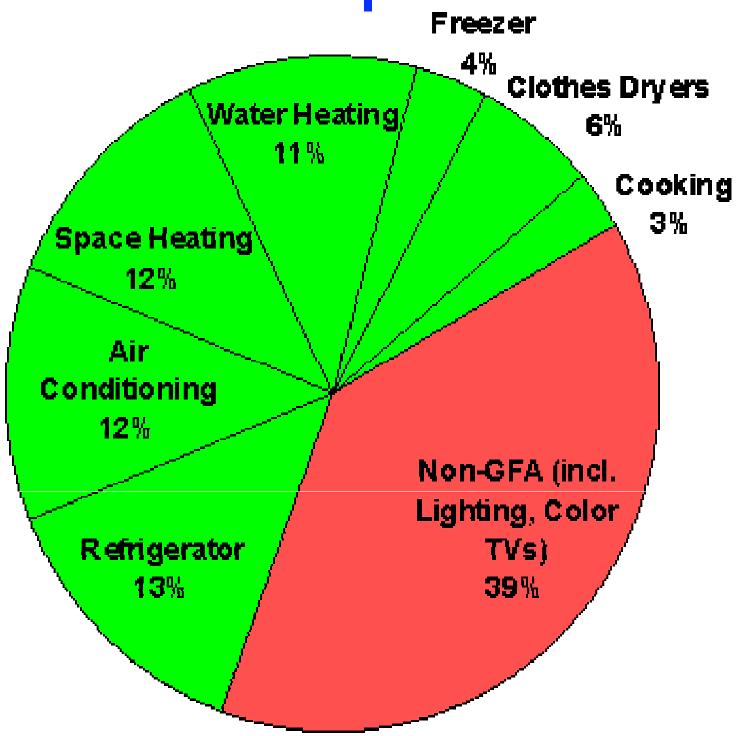
- faster (no/low inertia)
- no extra waste or emission
- more reliable (large #)
- better localize disturbances
- reducing generator-side control capacity



What is the potential

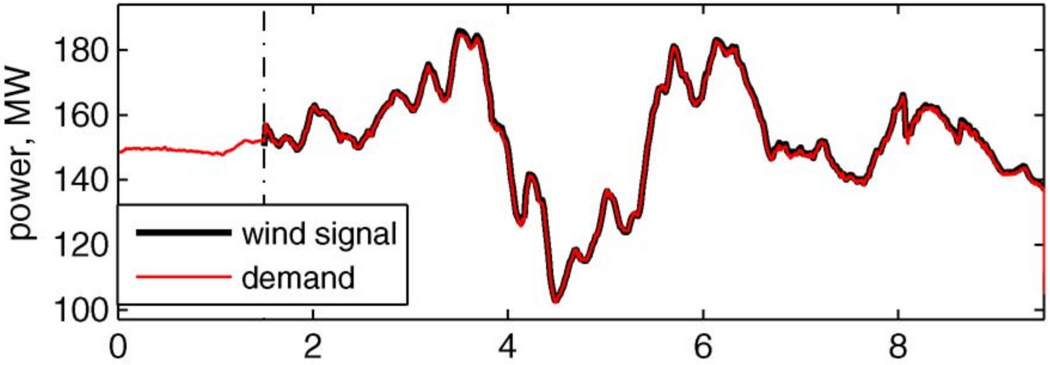


- Residential load accounts for ~1/3 of peak demand
- 61% residential appliances are Grid Friendly



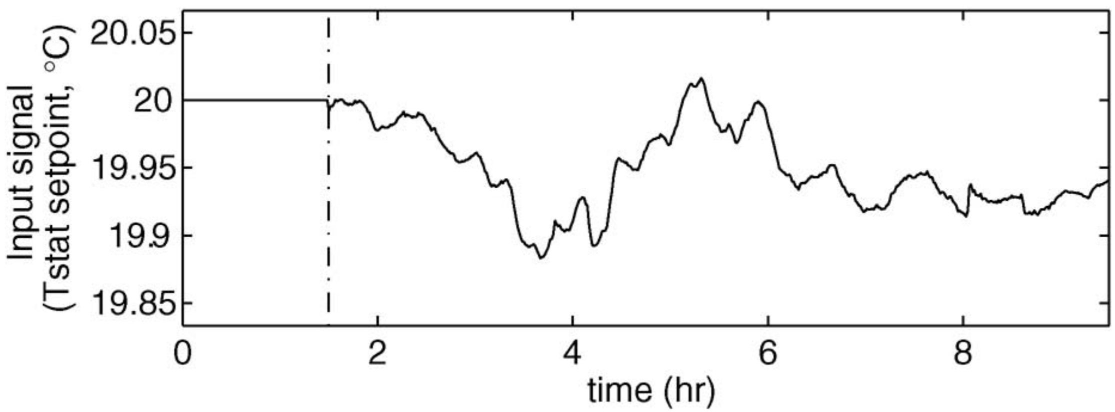
US:
operating reserve: 13% of peak
total GFA capacity: 18%

Lu & Hammerstrom (2006), PNNL



Can household Grid Friendly appliances follow its own PV production?

- 60,000 AC
- avg demand ~ 140 MW
- wind var: +- 40MW
- temp var: 0.15 degC



Dynamically adjust thermostat setpoint

Fig. 7. ***Load control example for balancing variability from intermittent renewable generators, where the end-use function—in this case, thermostat setpoint—is used as the input signal.***

Callaway, Hiskens (2011)
Callaway (2009)

How

How to design load-side frequency control ?

How does it interact with generator-side control ?

Literature: load-side control

Original idea

- Schweppe et al 1979, 1980

Small scale trials around the world

- D.Hammerstrom et al 2007, UK Market Transform Programme 2008

Numerical studies

- Trudnowski et al 2006, Lu and Hammerstrom 2006, Short et al 2007, Donnelly et al 2010, Brooks et al 2010, Callaway and I. A. Hiskens, 2011, Molina-Garcia et al 2011

Analytical work

- Zhao et al (2012/2014), Mallada and Low (2014), Mallada et al (2014)
- Simpson-Porco et al 2013, You and Chen 2014, Zhang and Papachristodoulou (2014), Zhao, et al (2014)

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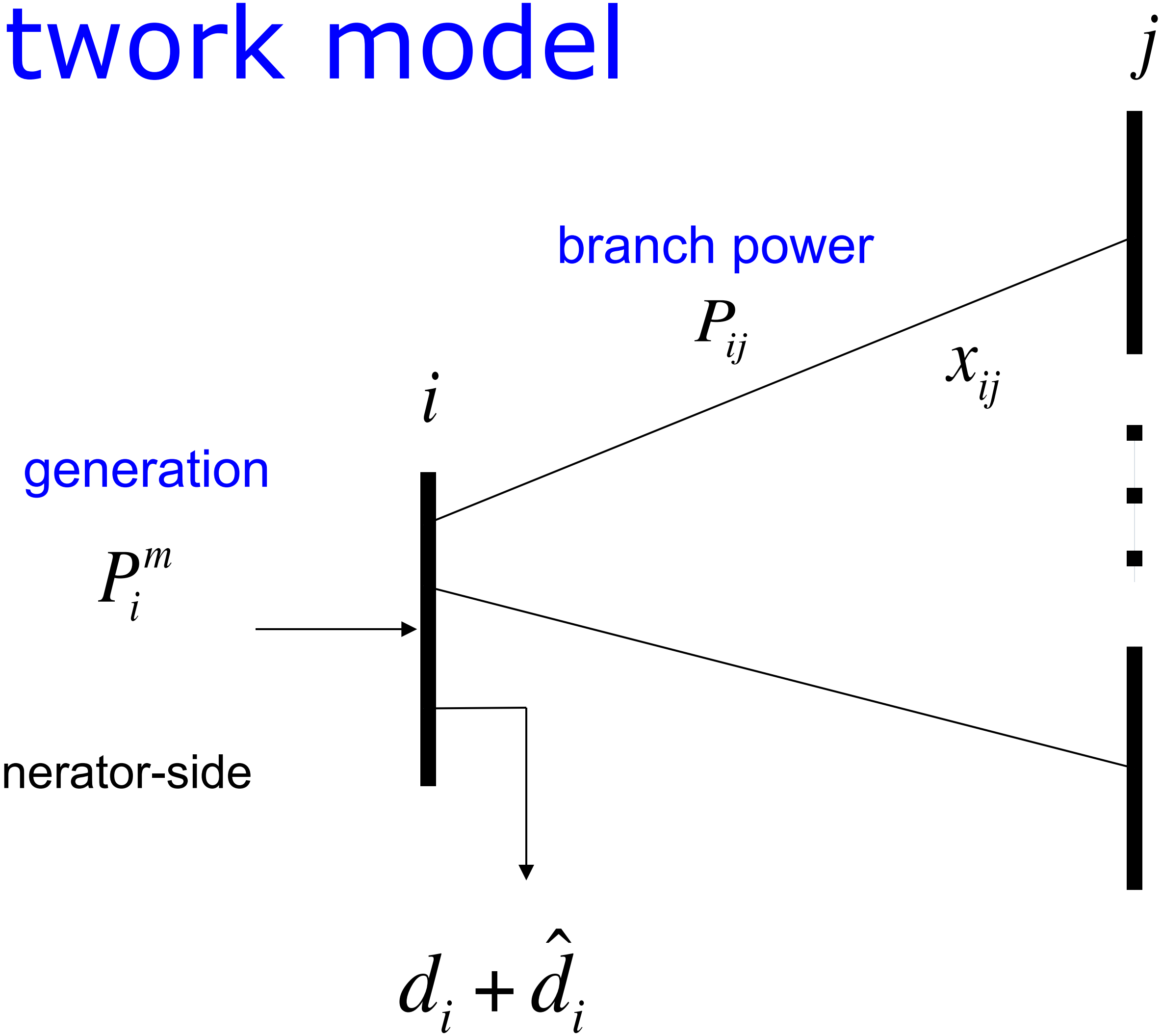
Network model



Will include generator-side control later

loads:
controllable + freq-sensitive

i : region/control area/balancing authority

Network model

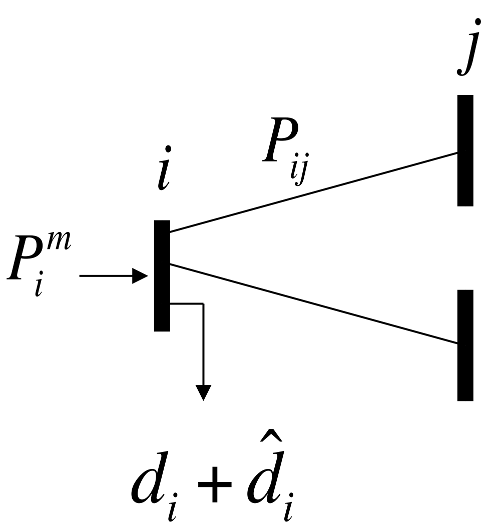
$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

Generator bus: $M_i > 0$

Load bus: $M_i = 0$

Damping/uncontr loads: $\hat{d}_i = D_i \omega_i$

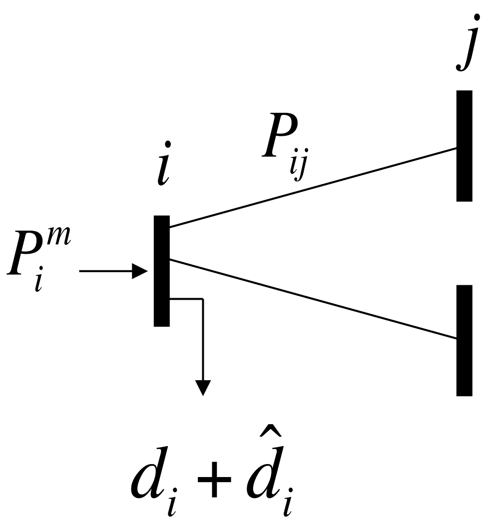
Controllable loads: d_i



Network model

$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij} \left(\omega_i - \omega_j \right) \qquad \forall\ i \to j$$



- swing dynamics
- all variables are deviations from nominal
- nonlinear : Mallada, Zhao, Dorfler

Frequency control

$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i - \omega_j\right) \qquad \forall\, i \to j$$

Suppose the system is in steady state

$$\dot{\omega}_i = 0 \qquad \dot{P}_{ij} = 0 \qquad \omega_i = 0$$

and suddenly …

Frequency control

Given: disturbance in gens/loads

Current: adapt remaining generators P_i^m

- re-balance power
- restore nominal freq and inter-area flows (zero ACE)

Our goal: adapt controllable loads d_i

- re-balance power
- restore nominal freq and inter-area flows
- … while minimizing disutility of load control

Questions

How to design load-side frequency control ?

How does it interact with generator-side control ?

Limitations

- Modeling assumptions
- Preliminary design and analysis

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Frequency control

$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij} \left(\omega_i - \omega_j \right) \qquad \forall\ i \to j$$

current approach

new approach

Load-side controller design

$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij} \left(\omega_i - \omega_j \right) \qquad \forall\ i \rightarrow j$$

How to design feedback control law

$$d_i = F_i \left(\omega(t), P(t) \right)$$

Load-side controller design

$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij} \left(\omega_i - \omega_j \right) \qquad \forall\ i \rightarrow j$$

Control goals

Zhao, Topcu, Li, Low
TAC 2014

Mallada, Zhao, Low
Allerton, 2014

- Rebalance power
- Stabilize frequency
- Restore nominal frequency
- Restore scheduled inter-area flows

Load-side controller design

$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij} \left(\omega_i - \omega_j \right) \qquad \forall\ i \rightarrow j$$

Desirable properties of $d_i = F_i\left(\omega(t), P(t)\right)$

- simple, scalable
- decentralized/distributed

Motivation: reverse engineering

Dj interpreted power flows as solution of an optimization problem

- PF equations = stationarity condition

We interpret swing dynamics as algorithm for an optimization problem

- eq pt of swing equations = optimal sol
- dynamics = primal-dual algorithm

Other examples: Internet congestion control (2000s), …
What are the advantages of this design approach?

Motivation: reverse engineering

$$M_i \dot{\omega}_i = P_i^m \qquad - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij} \left(\omega_i - \omega_j \right) \qquad \forall\ i \to j$$

primal-dual algorithm

Equilibrium point is unique optimal of:

$$\min_{\hat{d},P} \quad \sum_i \frac{\hat{d}_i^2}{2D_i}$$

$$\text{s. t.} \quad P_i^m - \hat{d}_i - \sum_j C_{ij} P_{ij} = 0 \quad \forall i$$

demand = supply

Load-side controller design

$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij} \left(\omega_i - \omega_j \right) \qquad \forall\, i \to j$$

Proposed approach: forward engineering

- formalize control goals into OLC objective
- derive local control as distributed solution

Outline

Motivation

Network model

Load-side frequency control

- Primary control Zhao et al SGC2012, Zhao et al TAC2014
- Secondary control
- Interaction with generator-side control

Simulations

Optimal load control (OLC)

$$\min_{d,\hat{d},P} \quad \sum_i \left(c_i(d_i) + \frac{\hat{d}_i^2}{2D_i} \right)$$

$$\text{s. t.} \quad P_i^m - \left(d_i + \hat{d}_i\right) - \sum_e C_{ie} P_{ie} = 0 \quad \forall i$$

demand = supply

disturbances

controllable loads

Decoupled dual (DOLC)

$$\max_{\nu} \quad \sum_i \Phi_i(\nu_i)$$

$$\text{s.t.} \quad \nu_i = \nu_j \quad \forall\, i \sim j$$

decouples areas/buses i

$$\Phi_i(\nu_i) \quad := \quad \min_{d_i,\,\hat{d}_i} \ \text{Lagrangian}\,(d_i, \hat{d}_i, \nu_i)$$

$$c_i(d_i) + \frac{1}{2D_i}\hat{d}_i^2 - \nu_i\left(d_i + \hat{d}_i - P_i^m\right)$$

primal objective

constraint penalty

Decoupled dual (DOLC)

$$\max_{v} \quad \sum_i \Phi_i\left(v_i\right)$$

$$\text{s.t.} \quad v_i = v_j \quad \forall\, i \sim j$$

Lemma

A unique optimal $v^* := (v^*, \ldots, v^*)$ is attained

There is no duality gap (assuming Slater's condition)

- ➔ solve DOLC and recover optimal solution to primal (OLC)

system dynamics + load control = primal dual alg

swing dynamics

$$\dot{\omega}_i = -\frac{1}{M_i}\left(d_i(t) + D_i\omega_i(t) - P_i^m + \sum_{i\to j} P_{ij}(t) - \sum_{j\to i} P_{ji}(t)\right)$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i(t) - \omega_j(t)\right)$$

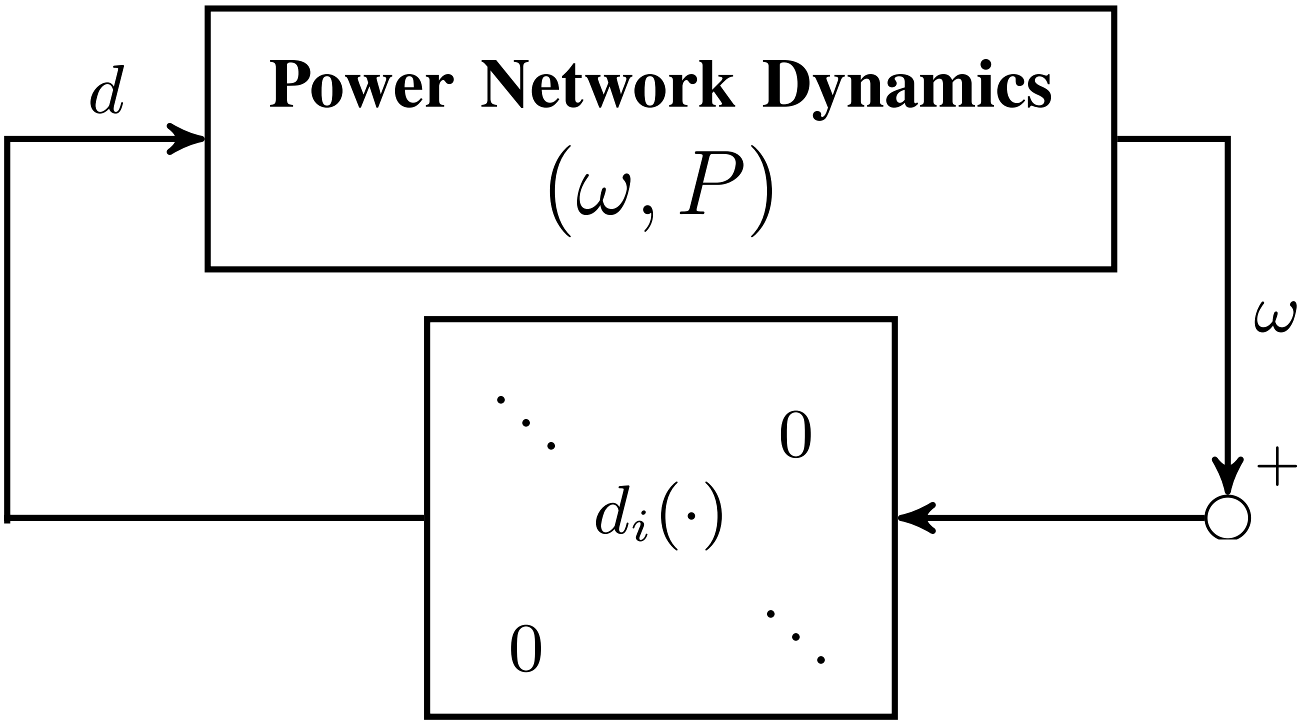
implicit

load control

$$d_i(t) := \left[c_i^{'-1}\left(\omega_i(t)\right)\right]_{\underline{d}_i}^{\overline{d}_i}$$

active control

Control architecture



Load-side primary control works

Theorem

Starting from any $\left(d(0),\ \hat{d}(0),\ \omega(0),\ P(0)\right)$

system trajectory $\left(d(t),\ \hat{d}(t),\ \omega(t),\ P(t)\right)$

converges to $\left(d^*,\ \hat{d}^*,\ \omega^*,\ P^*\right)$ as $t \to \infty$

- $\left(d^*,\ \hat{d}^*\right)$ is unique optimal of OCL
- ω^* is unique optimal for dual

- completely decentralized
- frequency deviations contain right info for local decisions that are globally optimal

Implications

- Freq deviations contains right info on global power imbalance for local decision

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- Decentralized load participation in primary freq control is stable

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- ω^*: Lagrange multiplier of OLC
  info on power imbalance

Implications

- Freq deviations contains right info on global power imbalance for local decision

- Decentralized load participation in primary freq control is stable

- ω^*: Lagrange multiplier of OLC
  info on power imbalance

- P^*: Lagrange multiplier of DOLC
  info on freq asynchronism

Recap: control goals

Yes
- Rebalance power

Yes
- Stabilize frequencies

No
- Restore nominal frequency $\left(\omega^* \neq 0\right)$

No
- Restore scheduled inter-area flows

Proposed approach: forward engineering

- formalize control goals into OLC objective
- derive local control as distributed solution

Outline

Motivation

Network model

Load-side frequency control

- Primary control
- Secondary control
- Interaction with generator-side control

Mallada, Low, IFAC 2014
Mallada et al, Allerton 2014

Simulations

Recall: OLC for primary control

$$\min_{d,\hat{d},P} \quad \sum_i \left(c_i(d_i) + \frac{1}{2D_i}\hat{d}_i^2 \right)$$

$$\text{s. t.} \quad P^m - (d + \hat{d}) = CP$$

demand = supply

OLC for secondary control

$$\min_{d,\hat{d},P,v} \quad \sum_i \left(c_i(d_i) + \frac{1}{2D_i}\hat{d}_i^2 \right)$$

$$\text{s. t.} \quad P^m - (d + \hat{d}) = CP$$

demand = supply

key idea: "virtual flows"

$$BC^T v$$

in steady state: virtual = real flows

$$BC^T v = P$$

OLC for secondary control

$$\min_{d,\hat{d},P,v} \quad \sum_i \left(c_i(d_i) + \frac{1}{2D_i}\hat{d}_i^2 \right)$$

$$\text{s. t.} \quad P^m - (d + \hat{d}) = CP$$

demand = supply

$$P^m - d = CBC^T v$$

restore nominal freq

in steady state: virtual = real flows

$$BC^T v = P$$

OLC for secondary control

$$\min_{d,\hat{d},P,v} \quad \sum_i \left(c_i(d_i) + \frac{1}{2D_i}\hat{d}_i^2 \right)$$

s. t.

$$P^m - (d + \hat{d}) = CP$$ demand = supply

$$P^m - d = CBC^T v$$ restore nominal freq

$$\hat{C}BC^T v = \hat{P}$$ restore inter-area flow

$$\underline{P} \leq BC^T v \leq \overline{P}$$ respect line limit

in steady state: virtual = real flows

$$BC^T v = P$$

Recall: primary control

swing dynamics:

$$\dot{\omega}_i = -\frac{1}{M_i}\left(d_i(t) + D_i\omega_i(t) - P_i^m + \sum_{e\in E} C_{ie}P_e(t) \right)$$

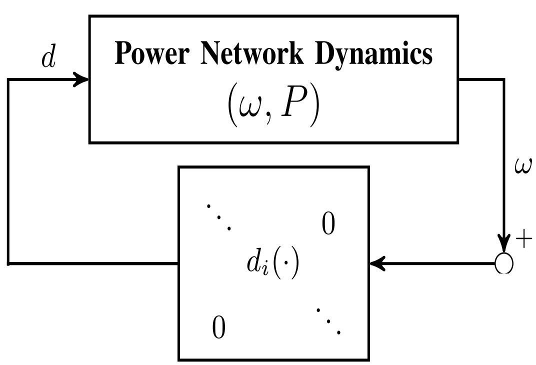
$$\dot{P}_{ij} = b_{ij}\left(\omega_i(t) - \omega_j(t)\right)$$

implicit

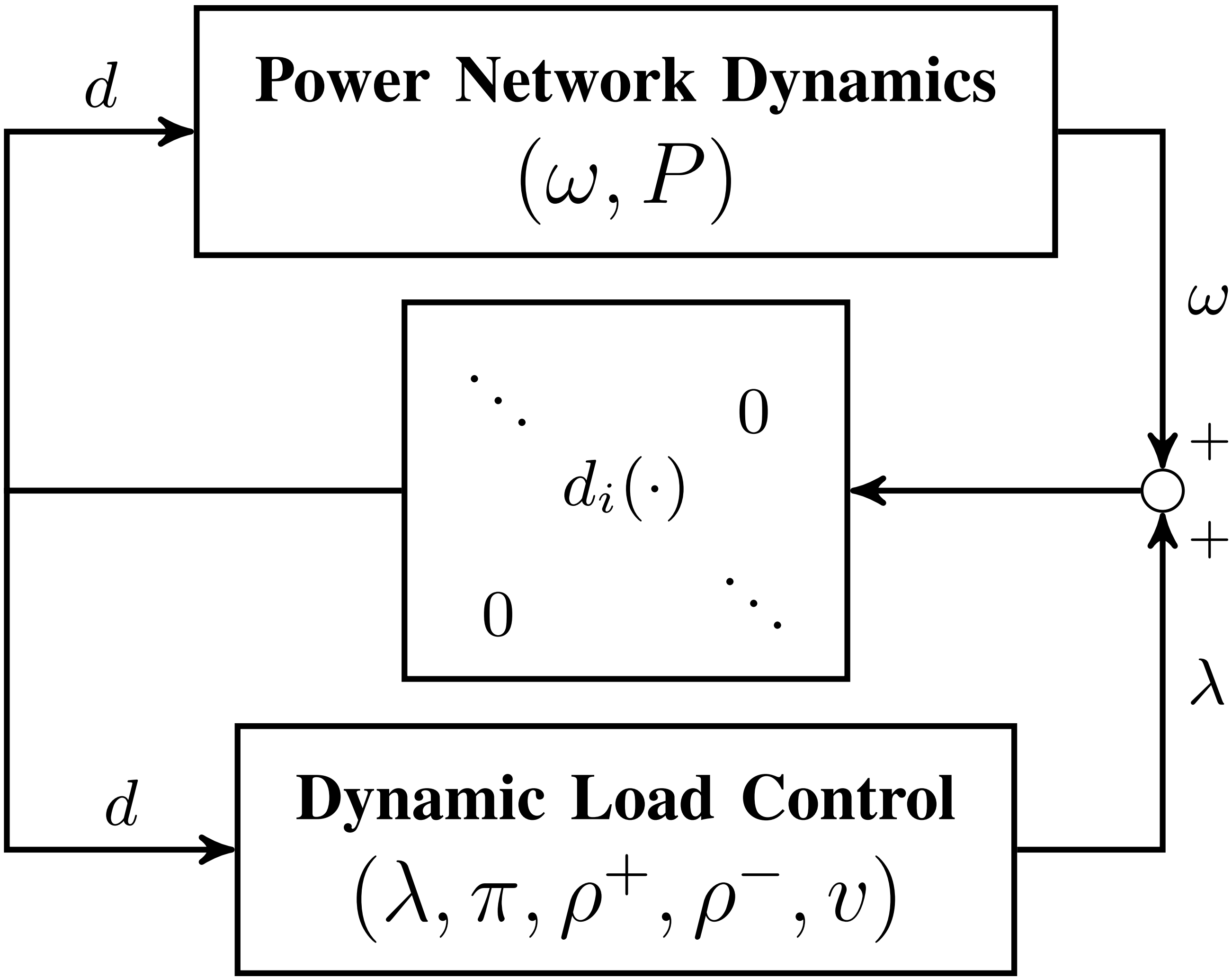
load control:

$$d_i(t) := \left[c_i'^{-1}\left(\omega_i(t)\right) \right]_{\underline{d}_i}^{\bar{d}_i}$$

active control



Control architecture



Secondary frequency control

load control: $$d_i(t) := \left[c_i'^{-1} \left(\omega_i(t) + \lambda_i(t) \right) \right]_{\underline{d}_i}^{\overline{d}_i}$$

computation & communication:

primal var: $$\dot{v} = \chi^v \left(L_B \lambda - C D_B \hat{C}^T \pi - C D_B (\rho^+ - \rho^-) \right)$$

dual vars: $$\dot{\lambda} = \zeta^\lambda \left(P^m - d - L_B v \right)$$

$$\dot{\pi} = \zeta^\pi \left(\hat{C} D_B C^T v - \hat{P} \right)$$

$$\dot{\rho}^+ = \zeta^{\rho^+} \left[D_B C^T v - \bar{P} \right]_{\rho^+}^+$$

$$\dot{\rho}^- = \zeta^{\rho^-} \left[\underline{P} - D_B C^T v \right]_{\rho^-}^+$$

Secondary control works

Theorem

starting from any initial point, system trajectory converges s. t.

- $\left(d^*, \hat{d}^*, P^*, v^*\right)$ is unique optimal of OLC
- nominal frequency is restored $\omega^* = 0$
- inter-area flows are restored $\hat{C}P^* = \hat{P}$
- line limits are respected $\underline{P} \le P^* \le \overline{P}$

Recap: control goals

- Yes ■ Rebalance power
- Yes ■ Resynchronize/stabilize frequency

Zhao, et al TAC2014

- Yes ■ Restore nominal frequency $\left(\omega^{*} \neq 0\right)$
- Yes ■ Restore scheduled inter-area flows

Mallada, et al Allerton2014

Secondary control restores nominal frequency but requires local communication

Outline

Motivation

Network model

Load-side frequency control

- Primary control
- Secondary control
- Interaction with generator-side control

Zhao and Low, CDC2014

Simulations

Generator-side control

New model: nonlinear PF, with generator control

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + \boxed{p_i} - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\ i \rightarrow j$$

Recall model: linearized PF, no generator control

$$M_i \dot{\omega}_i = -D_i \omega_i + \boxed{P_i^m - d_i} - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i - \omega_j\right) \qquad \forall\ i \rightarrow j$$

Generator-side control

New model: nonlinear PF, with generator control

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + p_i - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\ i \rightarrow j$$

generator bus: real power injection
load bus: controllable load

Generator-side control

New model: nonlinear PF, with generator control

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + p_i - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\ i \to j$$

generator buses:

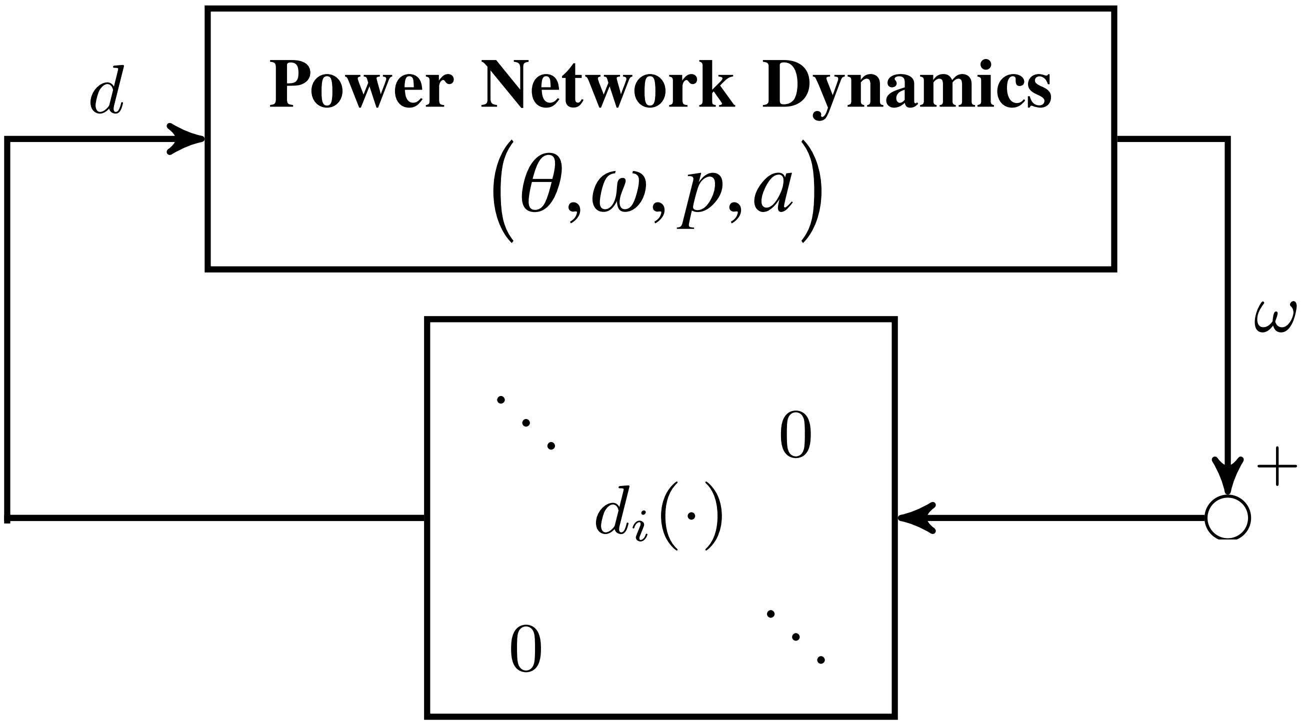
$$\dot{p}_i = -\frac{1}{\tau_{bi}}\left(p_i + a_i\right)$$

primary control $p_i^c(t) = p_i^c\left(\omega_i(t)\right)$

e.g. freq droop $p_i^c\left(\omega_i\right) = -\beta_i \omega_i$

$$\dot{a}_i = -\frac{1}{\tau_{gi}}\left(a_i + p_i^c\right)$$

Load-side (primary) control



load-side control

$$d_i(t) := \left[c_i'^{-1}\left(\omega_i(t)\right) \right]_{\underline{d}_i}^{\overline{d}_i}$$

Load-side primary control works

Theorem

- Every closed-loop equilibrium solves OLC and its dual

Suppose $\left| p_i^c(\omega) - p_i^c(\omega^*) \right| \le L_i \left| \omega - \omega^* \right|$

$$\text{near } \omega^* \text{ for some } L_i < D_i$$

- Any closed-loop equilibrium is (locally) asymptotically stable provided

$$\left| \theta_i^* - \theta_j^* \right| < \frac{\pi}{2}$$

Outline

Motivation

Network model

Load-side frequency control

Simulations

Main references:

Zhao, Topcu, Li, Low, TAC 2014
Mallada, Zhao, Low, Allerton 2014
Zhao, Low, CDC 2014

Simulations

Dynamic simulation of IEEE 39-bus system

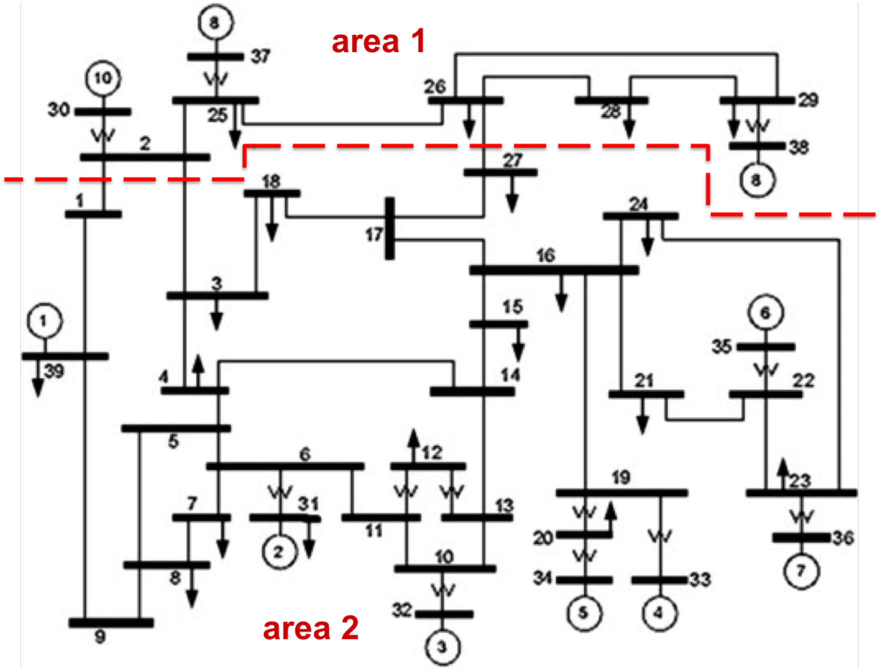
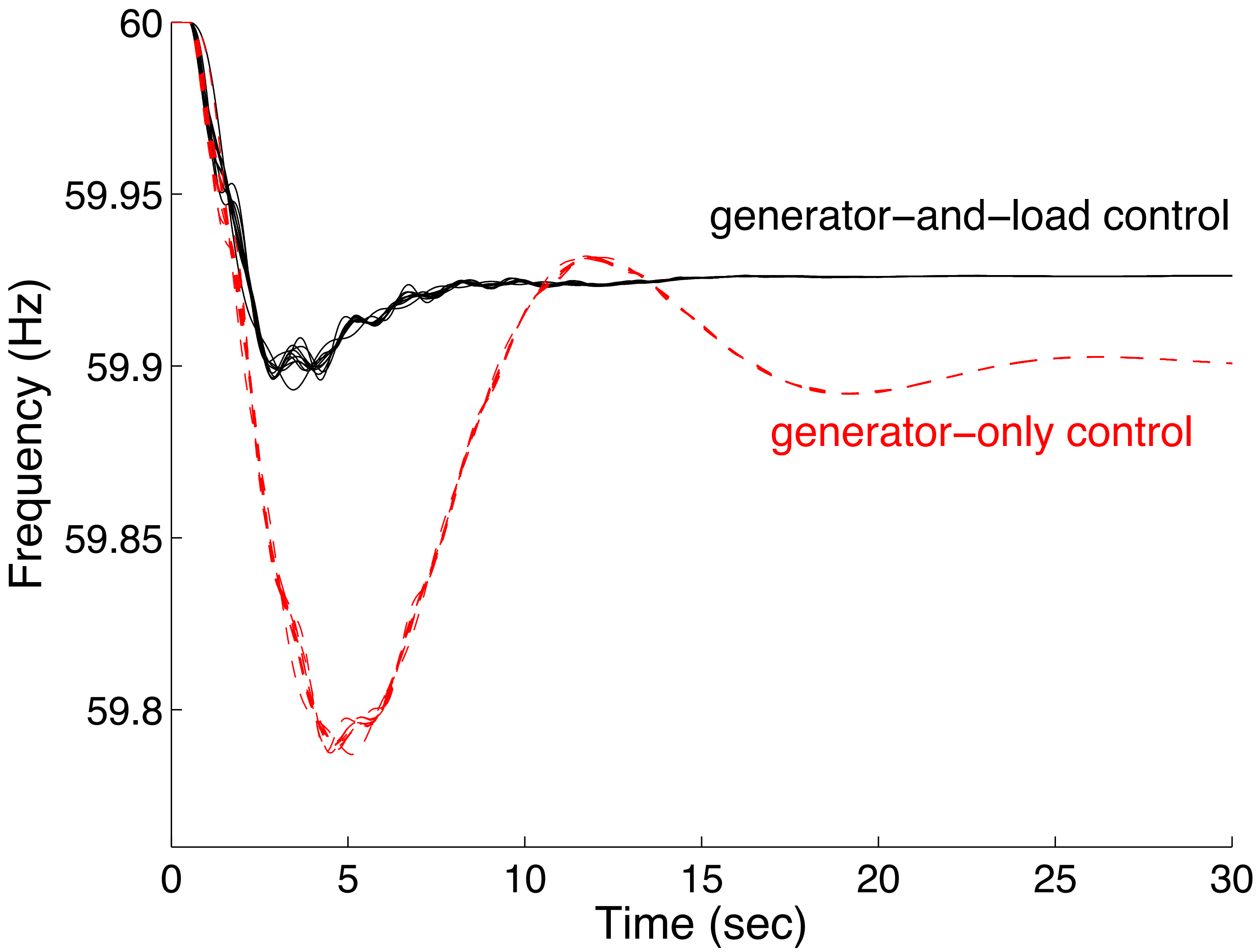


Fig. 2: IEEE 39 bus system: New England

- Power System Toolbox (RPI)
- Detailed generation model
- Exciter model, power system stabilizer model
- Nonzero resistance lines

Primary control



Secondary control

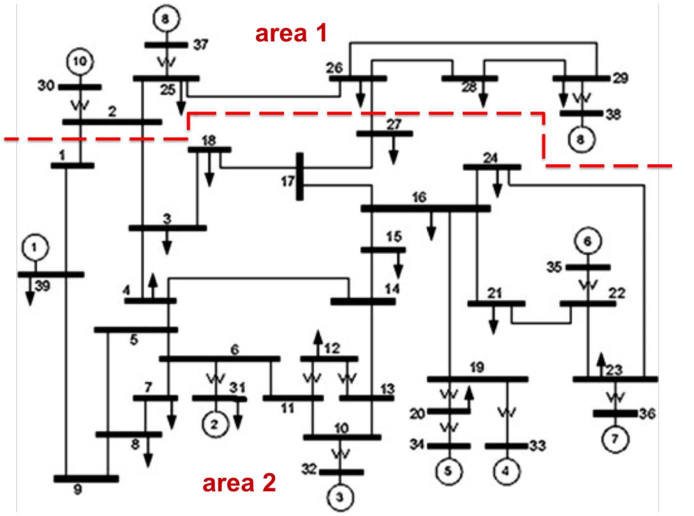
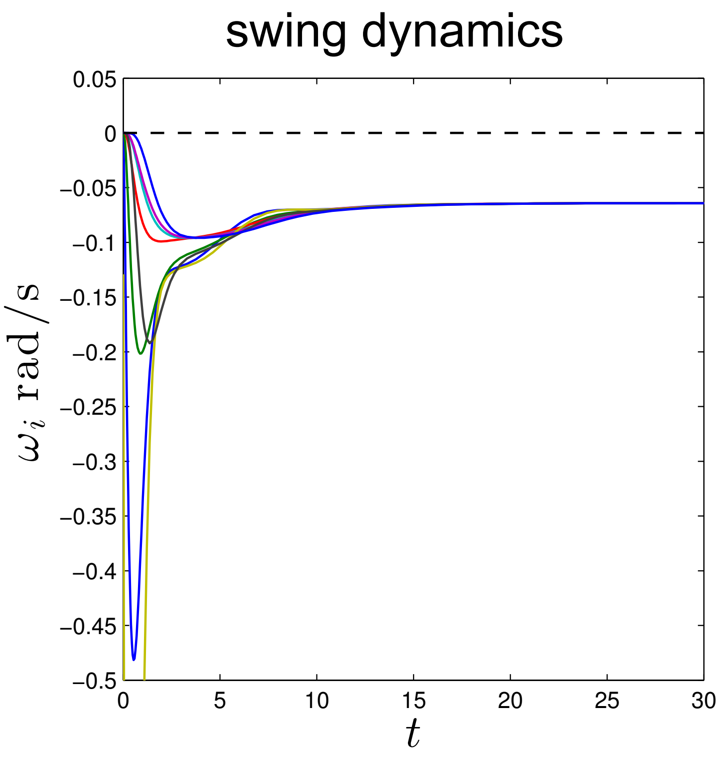
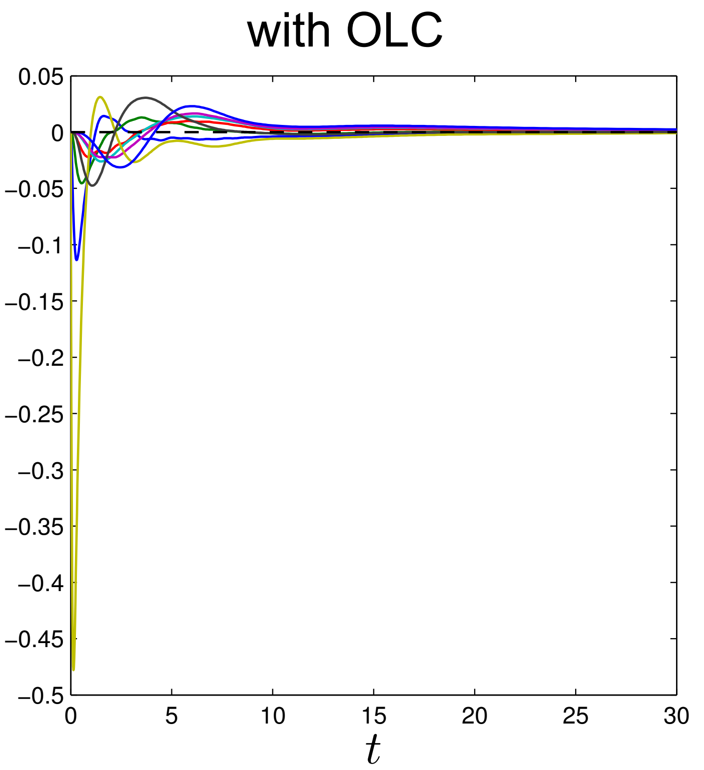


Fig. 2: IEEE 39 bus system: New England





area 1

Conclusion

Forward-engineering design facilitates

- explicit control goals
- distributed algorihtms
- stability analysis

Load-side frequency regulation

- primary & secondary control works
- helps generator-side control

SANTA FE| JANUARY 15-16, 2015

ISO new england

Architecting the Future Grid

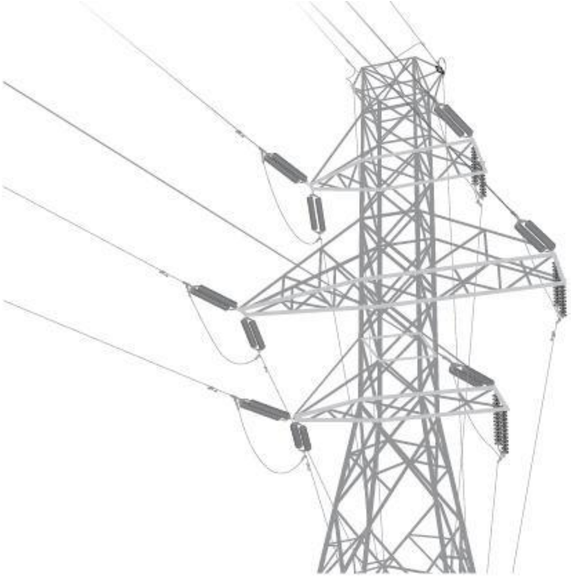
Grid Science Conference

Eugene Litvinov

…complex systems are counterintuitive. That is, they give indications that suggest corrective action which will often be ineffective or even adverse in its results.

Forrester, Jay Wright

Power System: A Traditional View

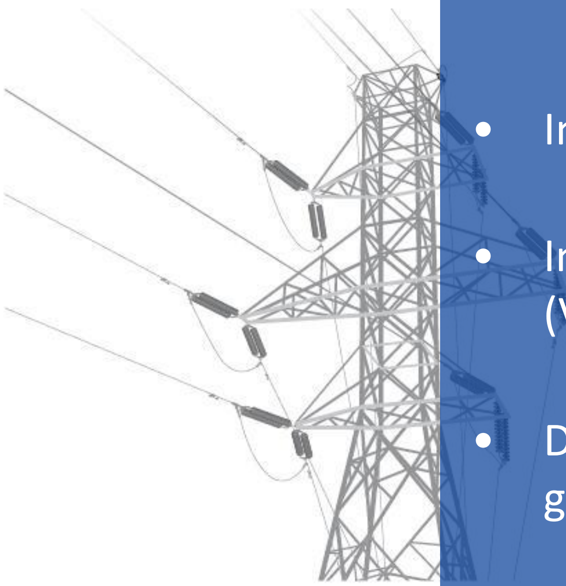


Bulk Power System

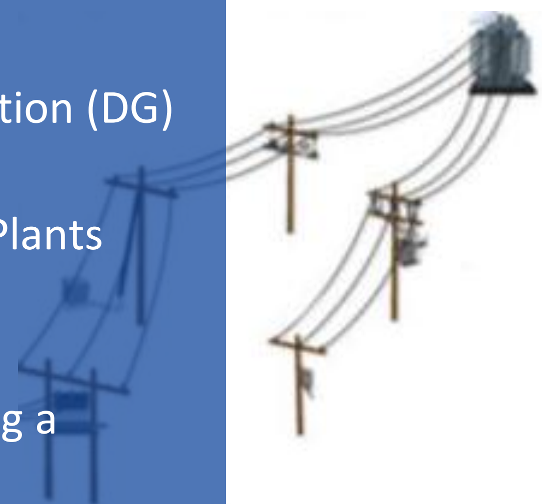
Distribution System

Two separate systems

The Line Between Transmission and Distribution is Blurring



- Increase in Distributed Generation (DG)
- Introduction of Virtual Power Plants (VPP)
- Demand Resources (DR) playing a greater role



Result: traditional power system becomes more "open" and vulnerable to disturbances and attacks

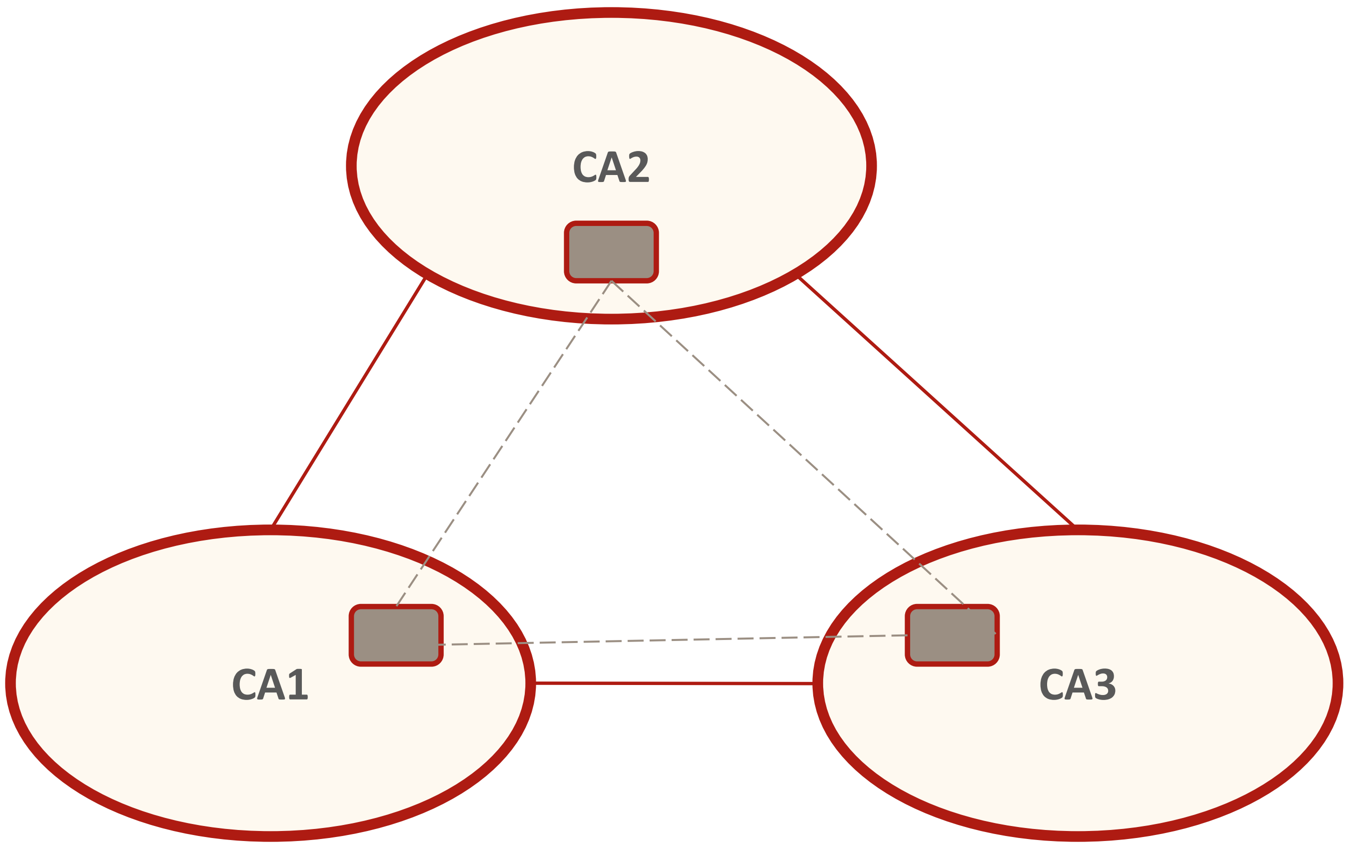
The Smart Grid

Bulk Power System

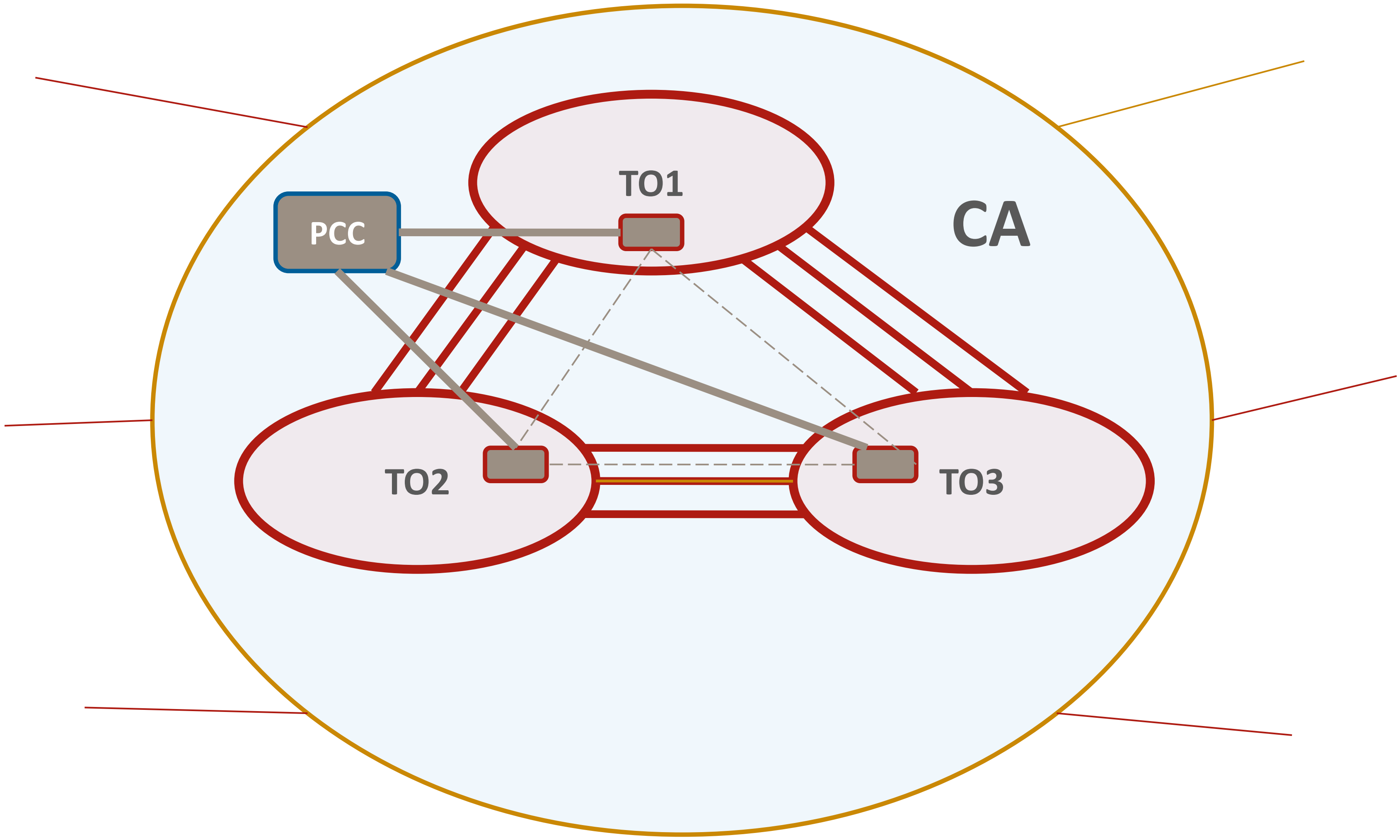
Distribution System

Common policies, reliability and control standards

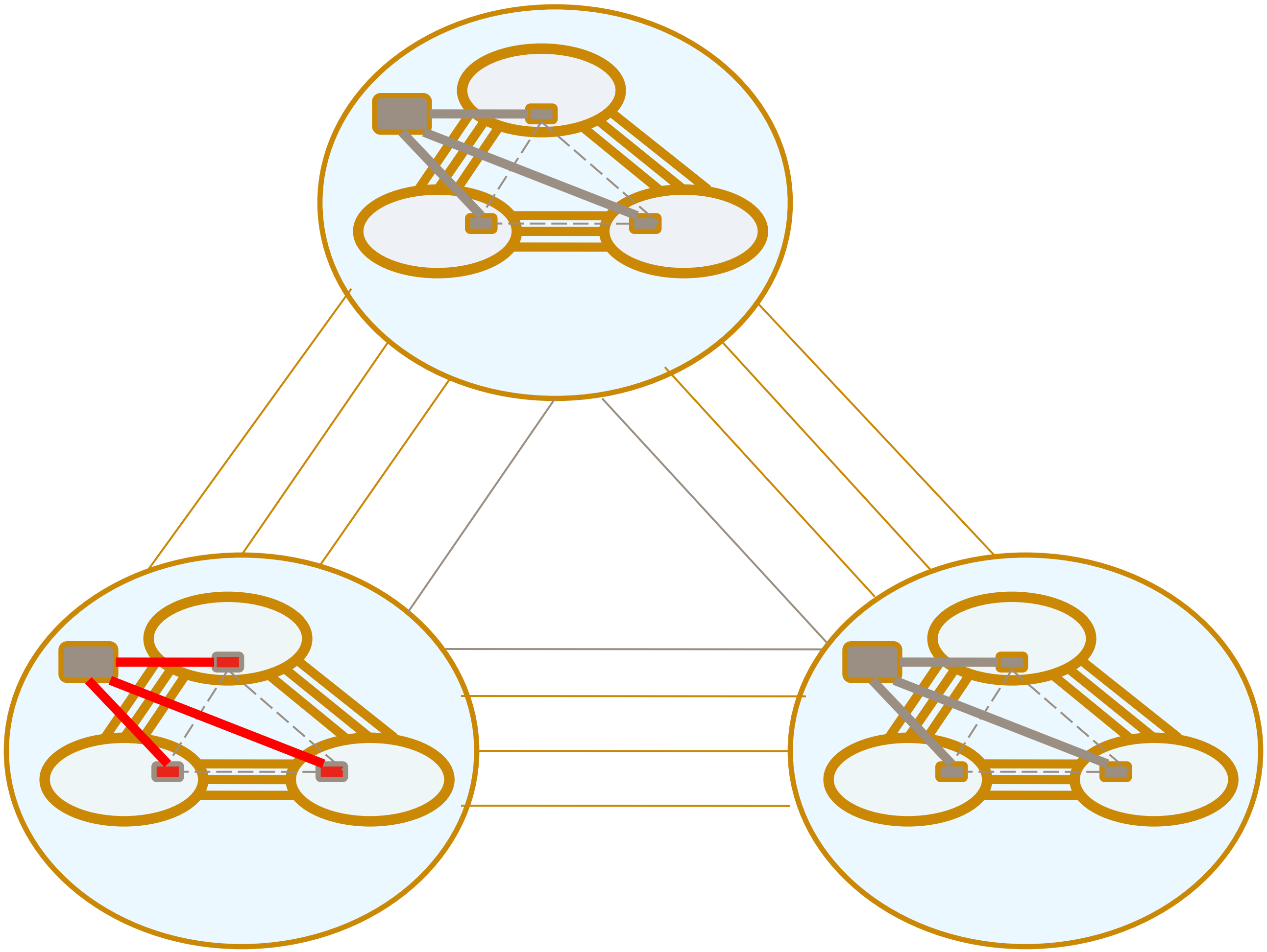
Power System Architecture Evolution (before 1966)



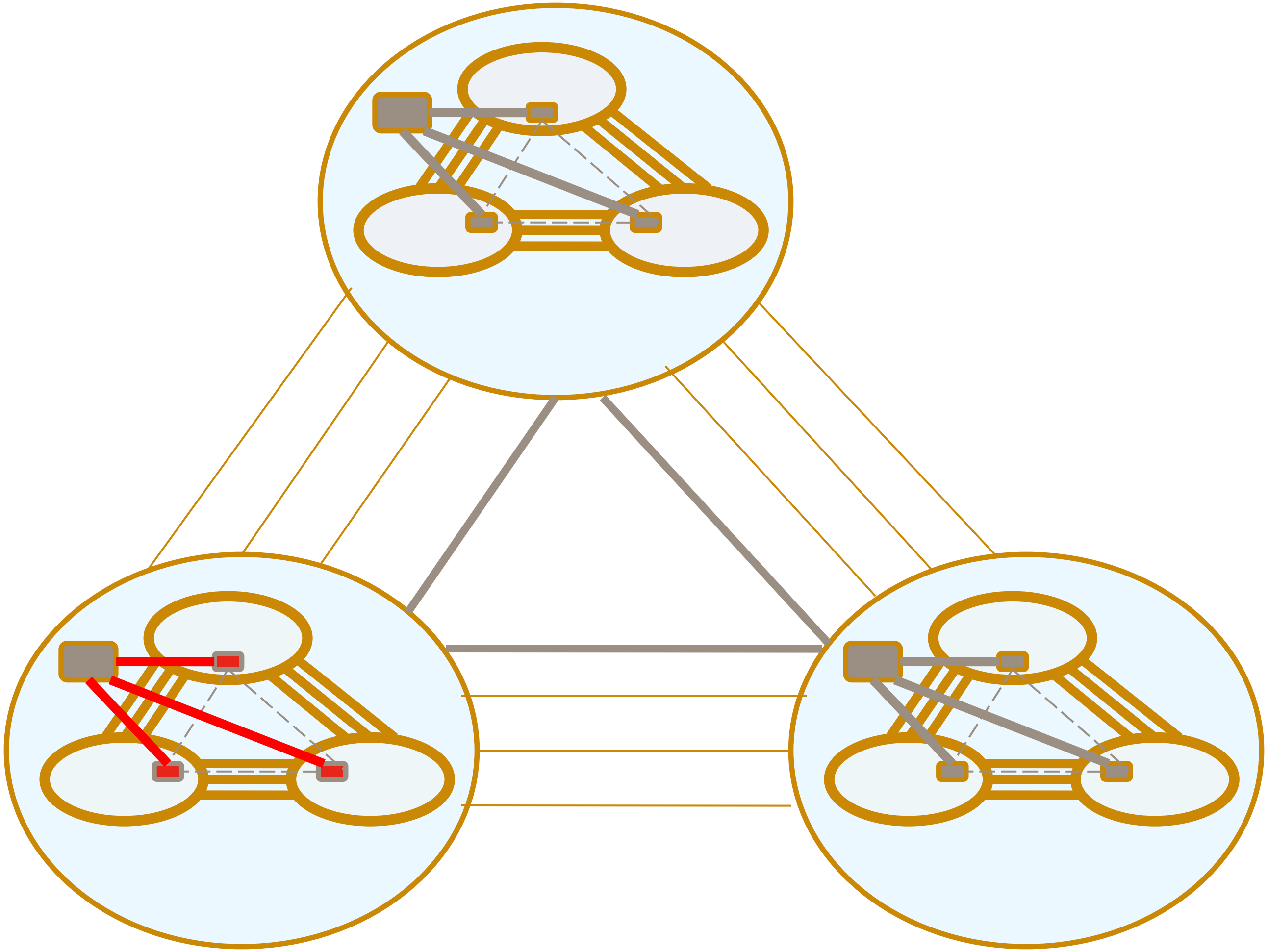
Power System Architecture Evolution (creation of pools)



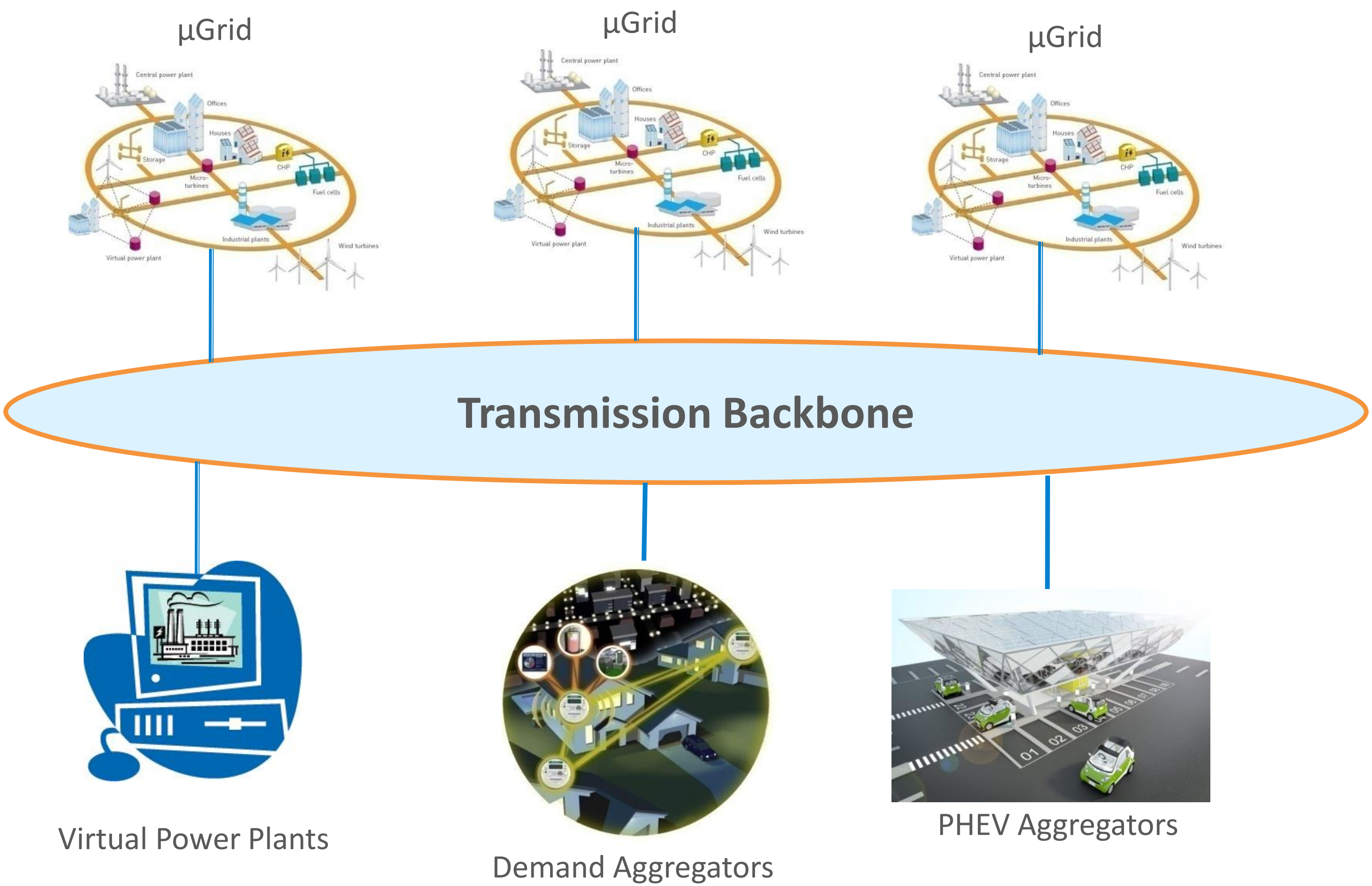
Power System Architecture Evolution (markets)



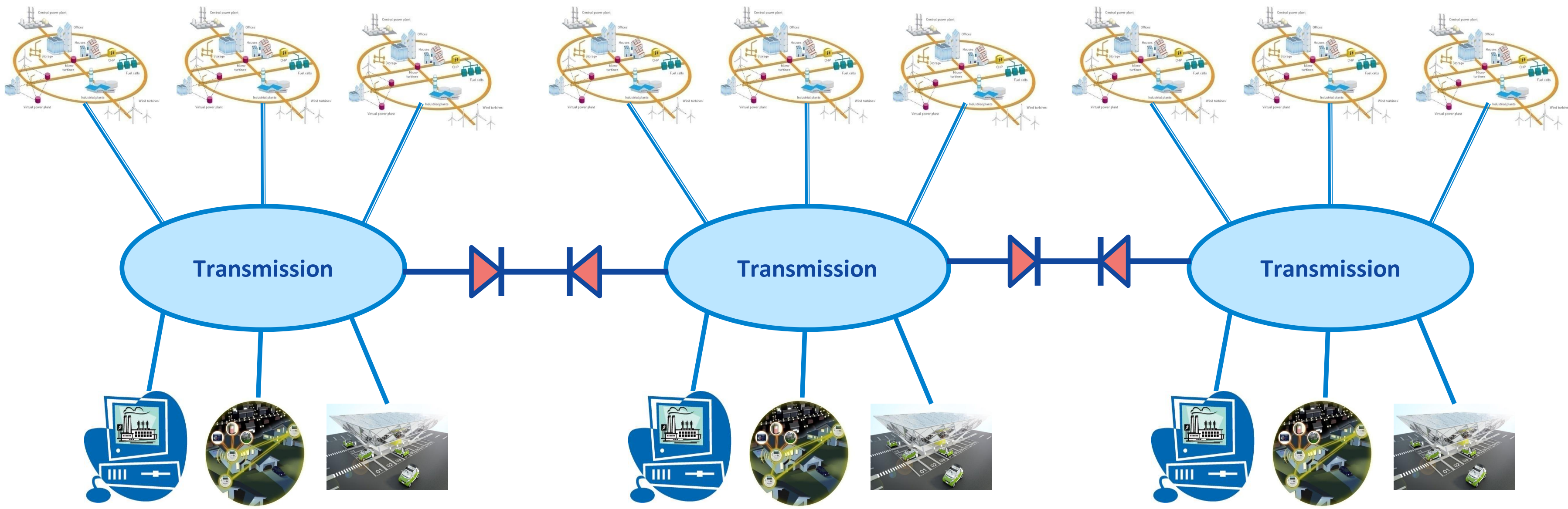
Power System Architecture Evolution (coordinated markets)



Power System Architecture Evolution (what's next?)

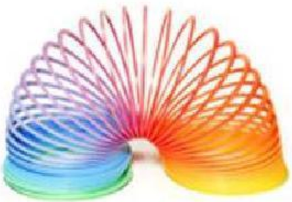


Power System Control Evolution (what's next?)



Maybe this?

The Need for Greater Flexibility



New Planning and Protection Concepts

- Rapid response to different disturbances
- Greater reliance on corrective actions
- System integrity protection
- Power quality standards
- System survivability

New Transmission Technologies

- Power electronics
- Energy storage
- Superconductors
- HVDC and HVDC-lite
- Nanotechnologies

New Operation and Control Strategies

- Risk-based operation
- Wide-area monitoring
- Adaptive islanding
- Transmission switching
- Online constraints calculation
- Dynamic and adaptive line ratings
- Adaptive and distributed control
- New optimization algorithms: robust and stochastic optimization

Reliability

NERC defines reliability as:
Adequacy + Operating Reliability[1]

Challenges to this conventional reliability concept:

- Distributed resources and microgrids
  - System is unbounded – operator cannot completely control perimeter
  - Contingency definition is nontrivial
- Evolving contingency definitions
  - Binary contingency definition → probability distributions
  - Greater effect of computer & communication contingencies
  - Ambiguous definition of "loss-of-load" events with responsive loads
- Non-uniform quality of service and reliability needs

[1] NERC, *Definition of "Adequate Level of Reliability,"* 2007

OE-417 Analysis Overview

- About the data: who reports and what is reported
- Types and frequency of events
- Problems with the data
- Evaluation of historical reliability indices (2002-2011)
- Power law distribution of events

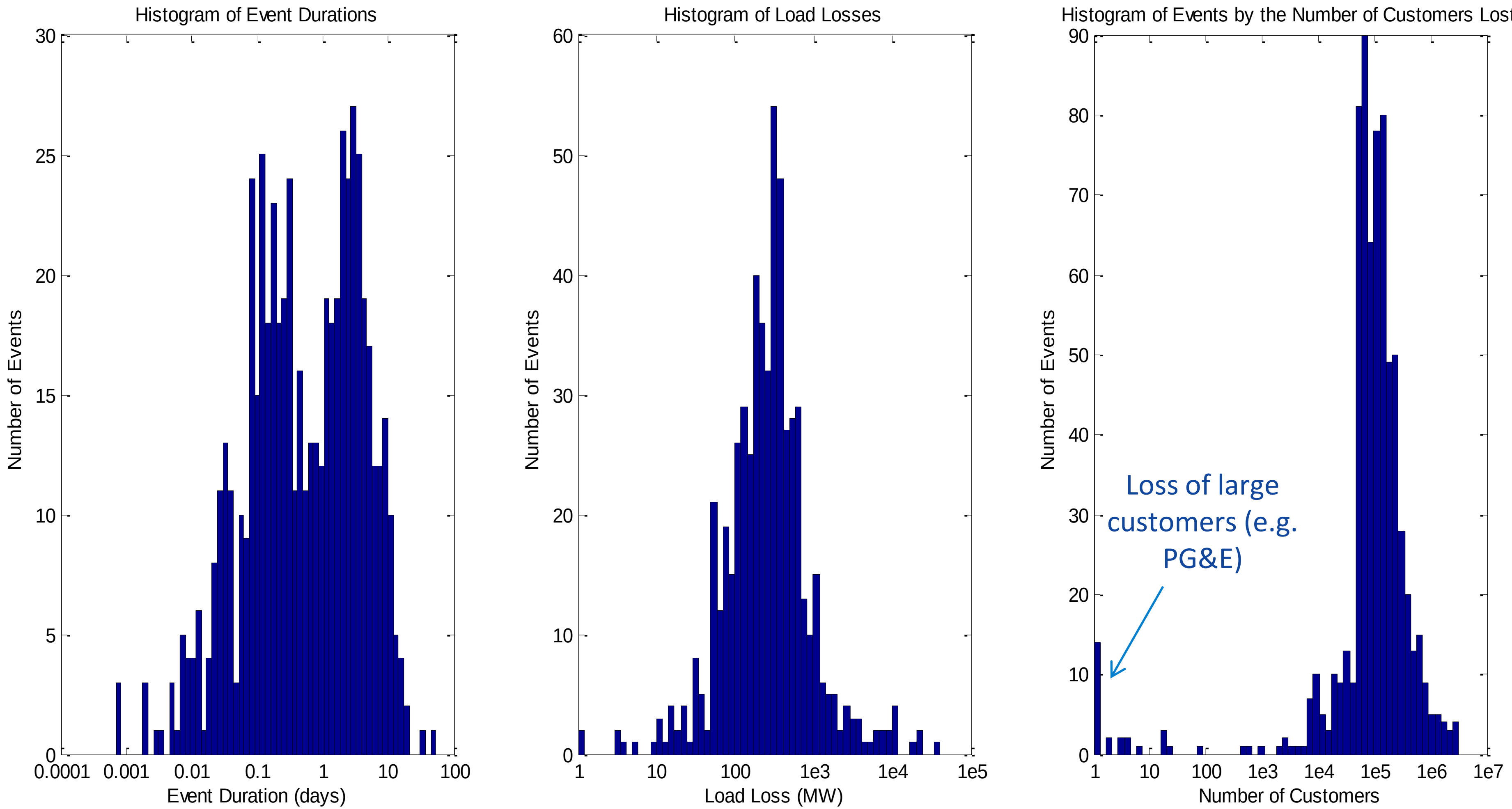
OE-417 Data – Who Reports?

1. Electric Utilities
2. Balancing Authorities
3. Reliability Coordinators
4. Generating entities
5. Local utilities in AK, HI, PR

OE-417 criteria for reporting incidents:

1. Physical, cyber, or communications attack
2. Complete operational failure of transmission and/or distribution
3. Electrical system islanding
4. Uncontrolled loss of 300 MW or more load for 15 or more minutes
5. Load shedding of 100 MW or more
6. System-wide voltage reductions of 3% or more
7. Public appeals to reduce the use of electricity

Event duration and size of losses



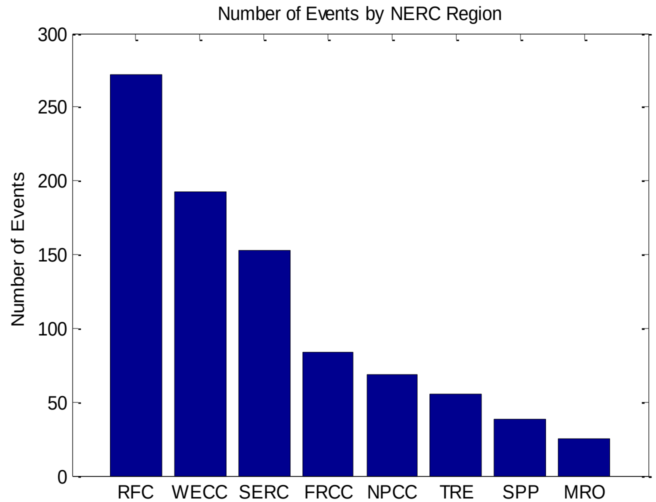
Problems with the data

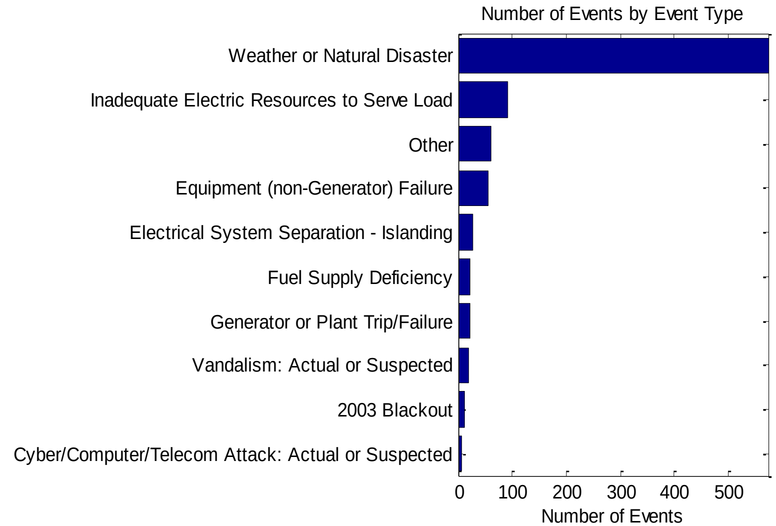
- Event losses are reported either in MW or number of customers, usually not both
  - Limits the useful portion of the data set to about 50%

- Event duration is provided, but the duration of the loss of load is not provided – this inhibits the evaluation of energy-related indices

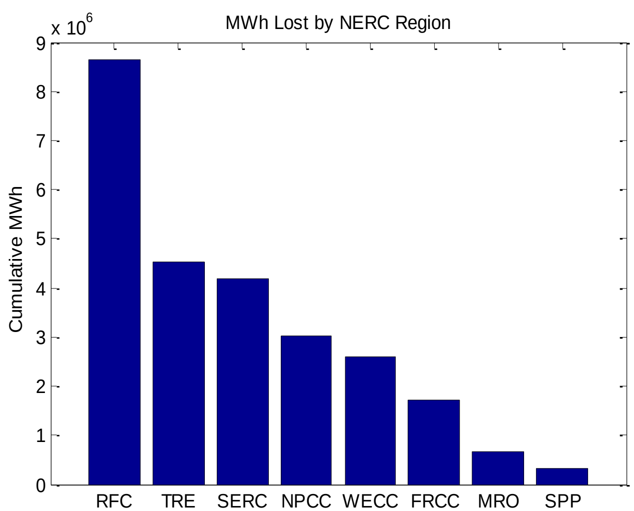
Breakdown of Events by NERC Region and Incident Type

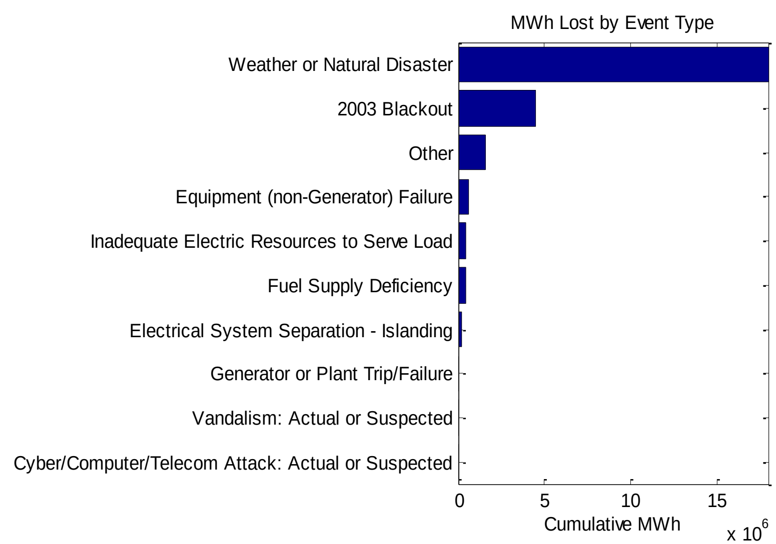
Number of Events





Magnitude of Events (sum of LOL times duration)

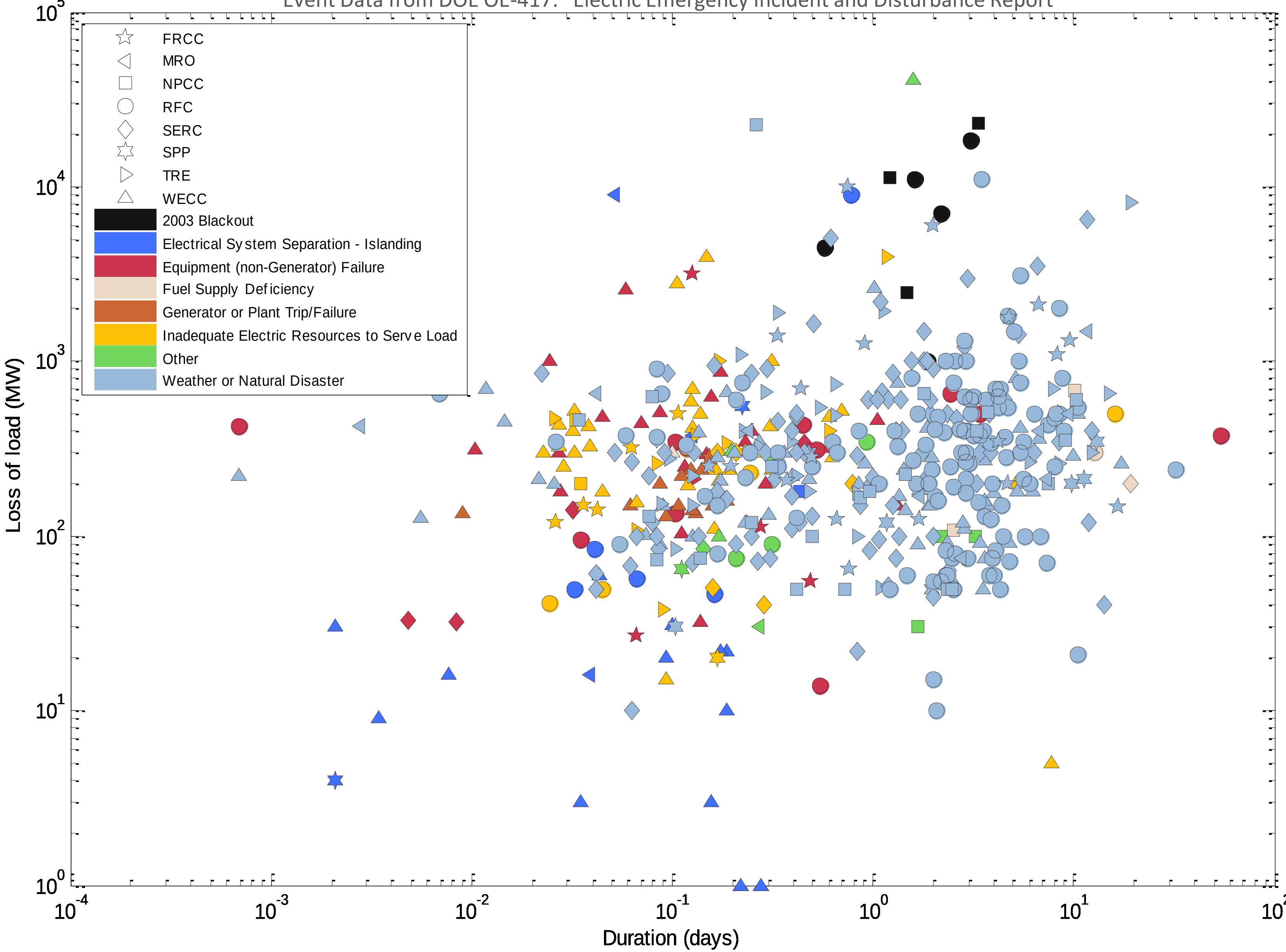




*Note: Since the duration of the event may not correspond to the duration of the loss-of-load, all results regarding unserved energy are inconclusive

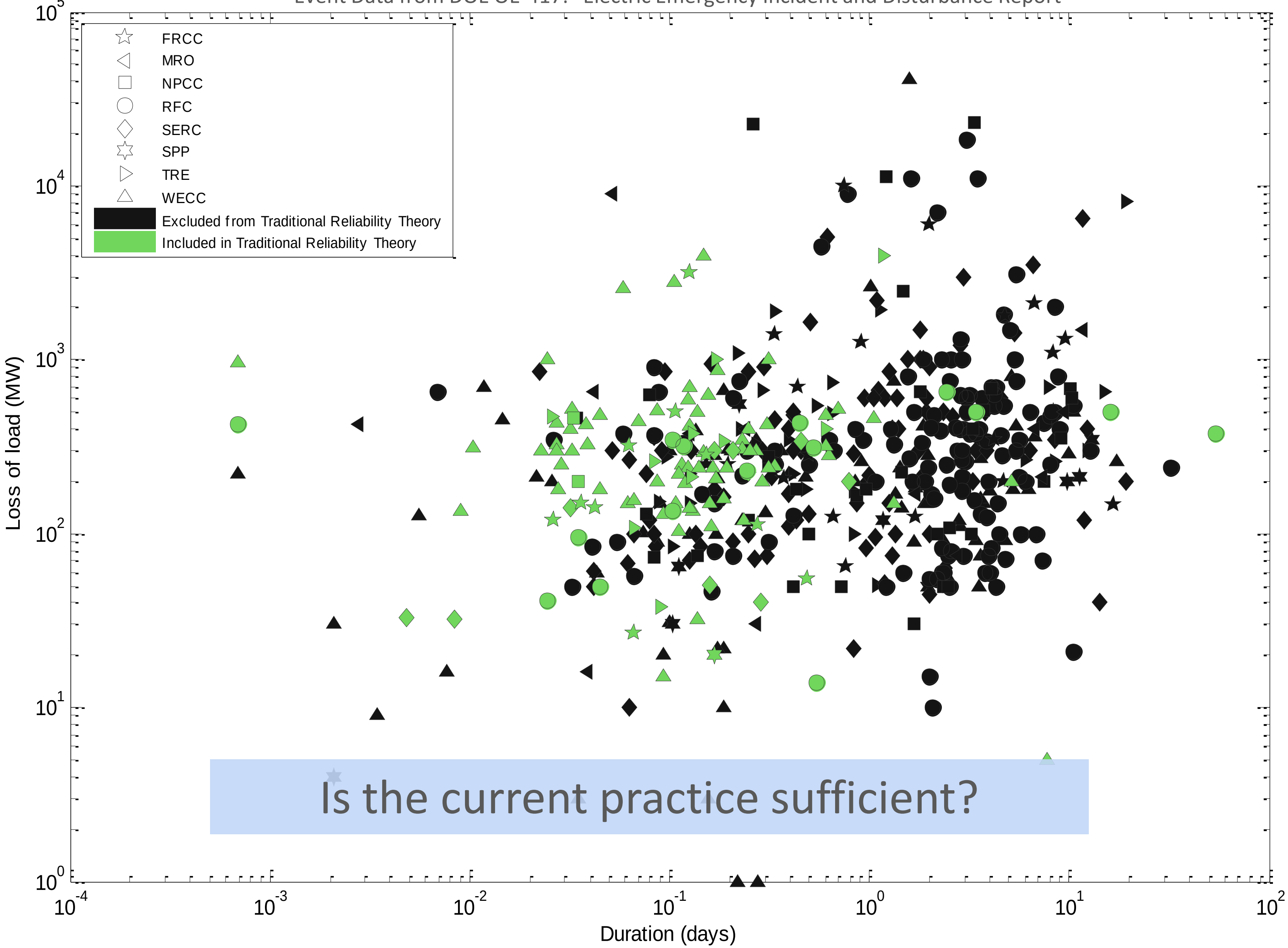
U.S. Power Disturbances Since 2002: By NERC Region and Incident Type

Event Data from DOE OE-417: “Electric Emergency Incident and Disturbance Report”



Many System Disturbances are Not Explicitly Modeled in Traditional Reliability Theory

Event Data from DOE OE-417: “Electric Emergency Incident and Disturbance Report”

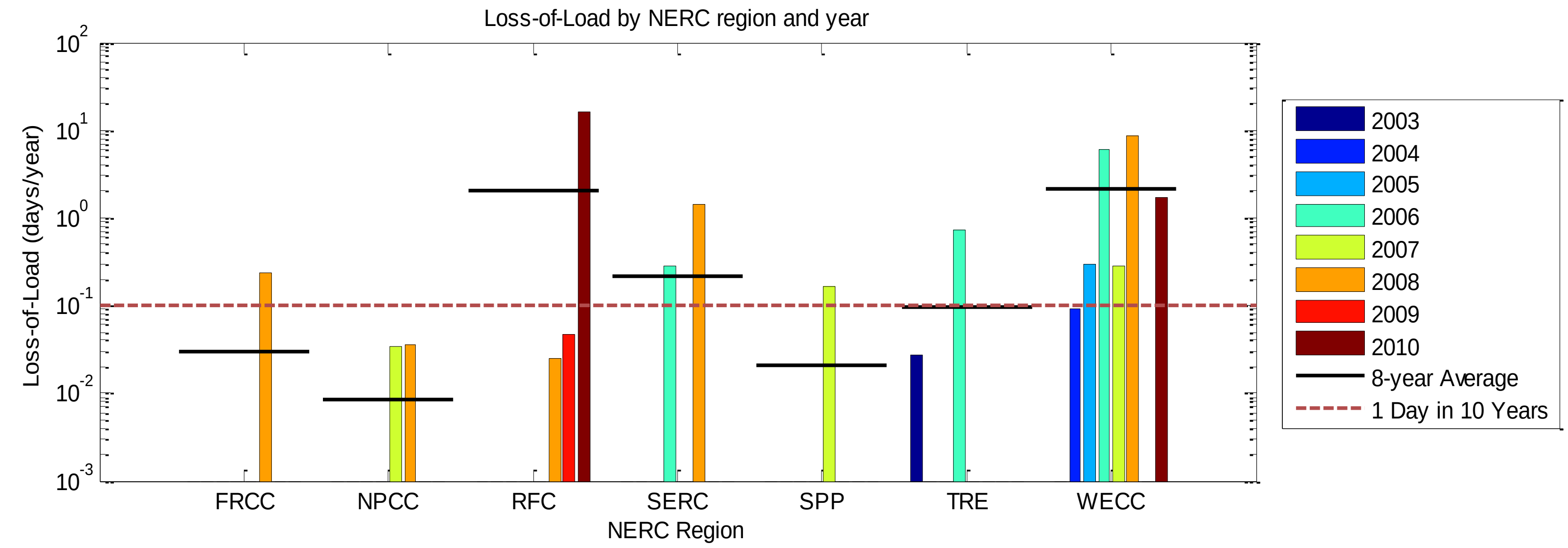


Is the current practice sufficient?

Calculated reliability indices using events categorized as "Inadequate Electric Resources to Serve Load" only.

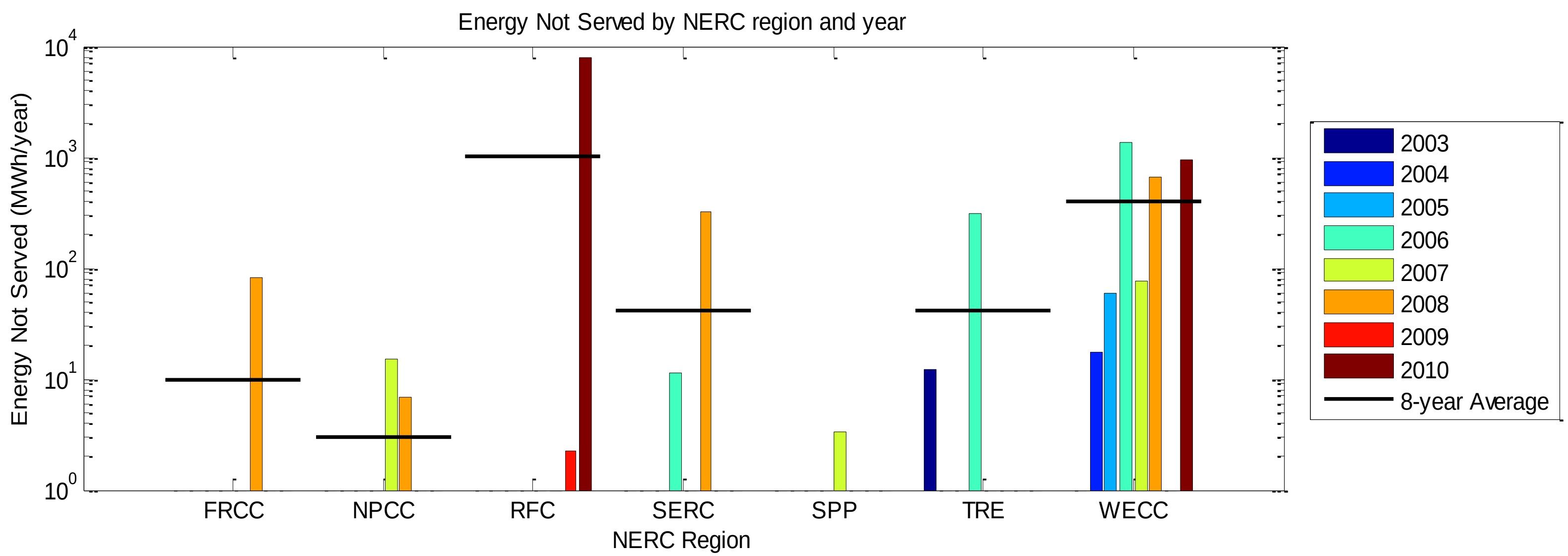
Average Loss-of-Load

1 Day in 10 Years



Average Energy Not Served:

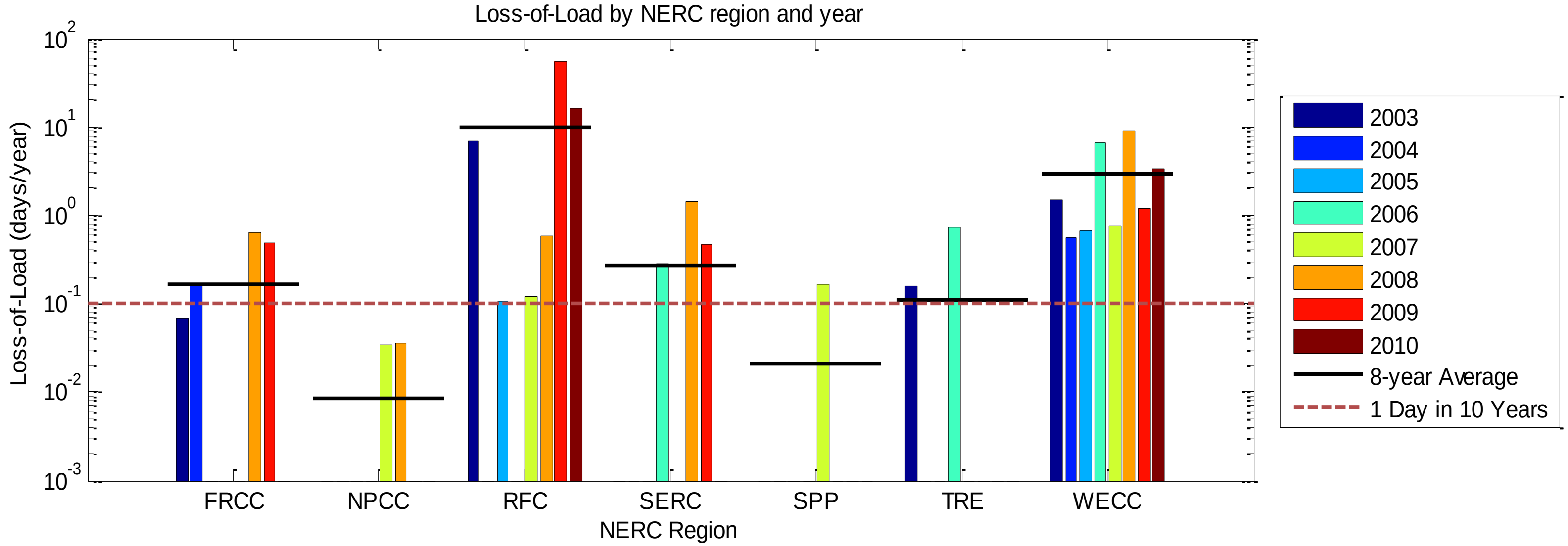
*Note: Since the duration of the event may not correspond to the duration of the loss-of-load, all results regarding unserved energy are inconclusive



Calculated reliability indices using events categorized as "Inadequate Electric Resources to Serve Load," Equipment (non-Generator) Failure," or "Generator or Plant Trip/failure."

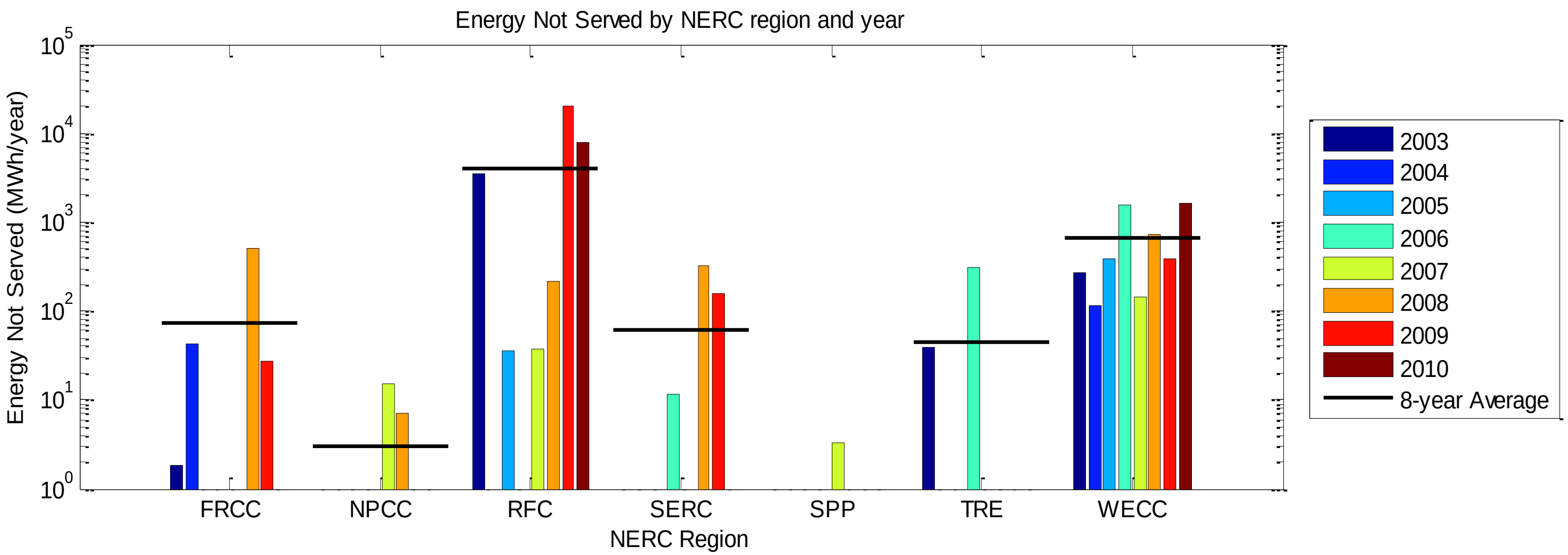
Average Loss-of-Load

1 Day in 10 Years



Average Energy Not Served:

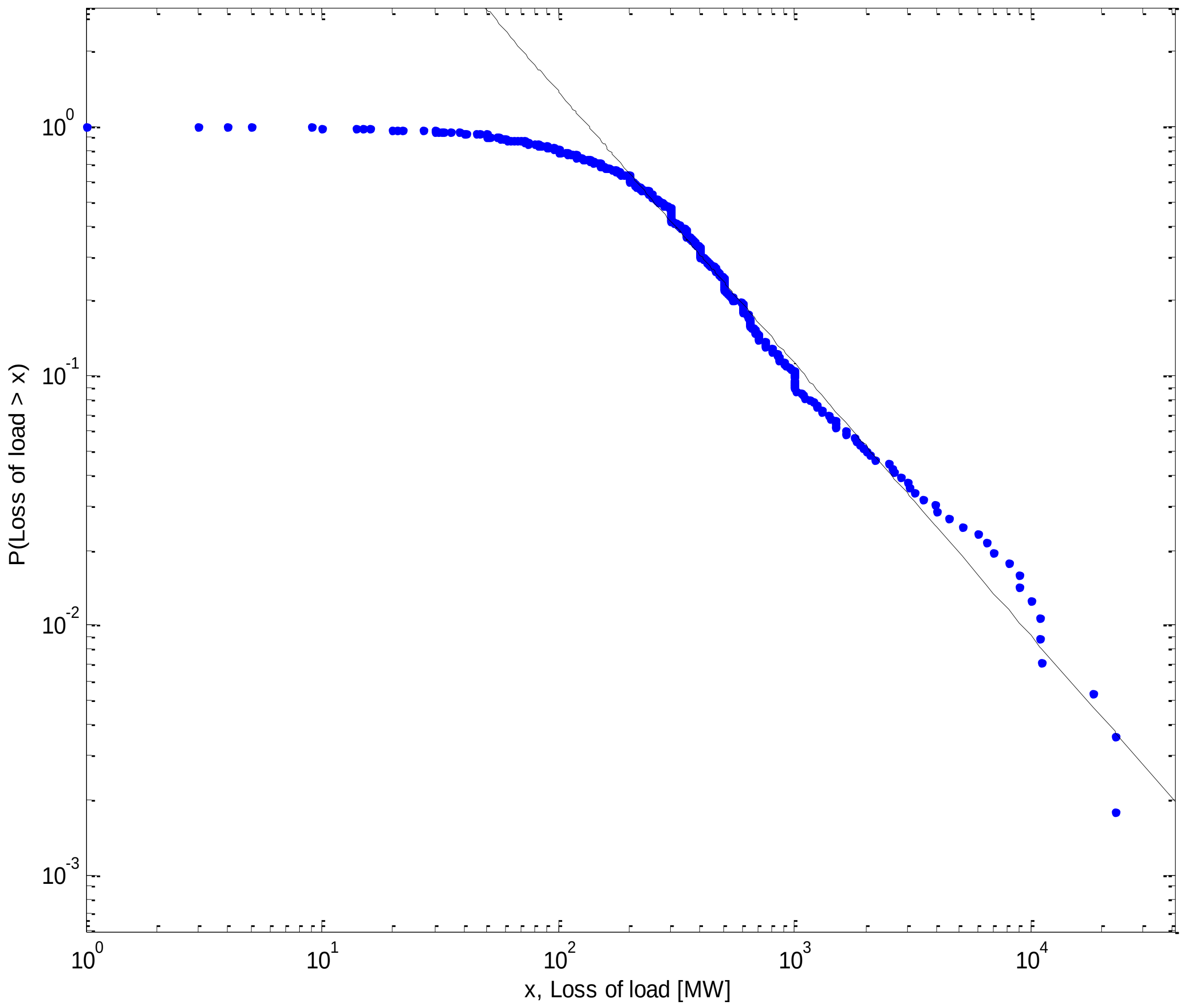
*Note: Since the duration of the event may not correspond to the duration of the loss-of-load, all results regarding unserved energy are inconclusive



Extreme Events appear to follow a power law distribution

- Data: All continental U.S. events with MW losses of load reported from mid-2003 through mid-2011 through OE-417
- The tail appears to follow a power law distribution
- Confirms the findings of a number of studies that there is non-negligible probability in the tails of the distribution. The distribution in heavy-tailed

Probability that loss of load is larger than the indicated value



Conclusions

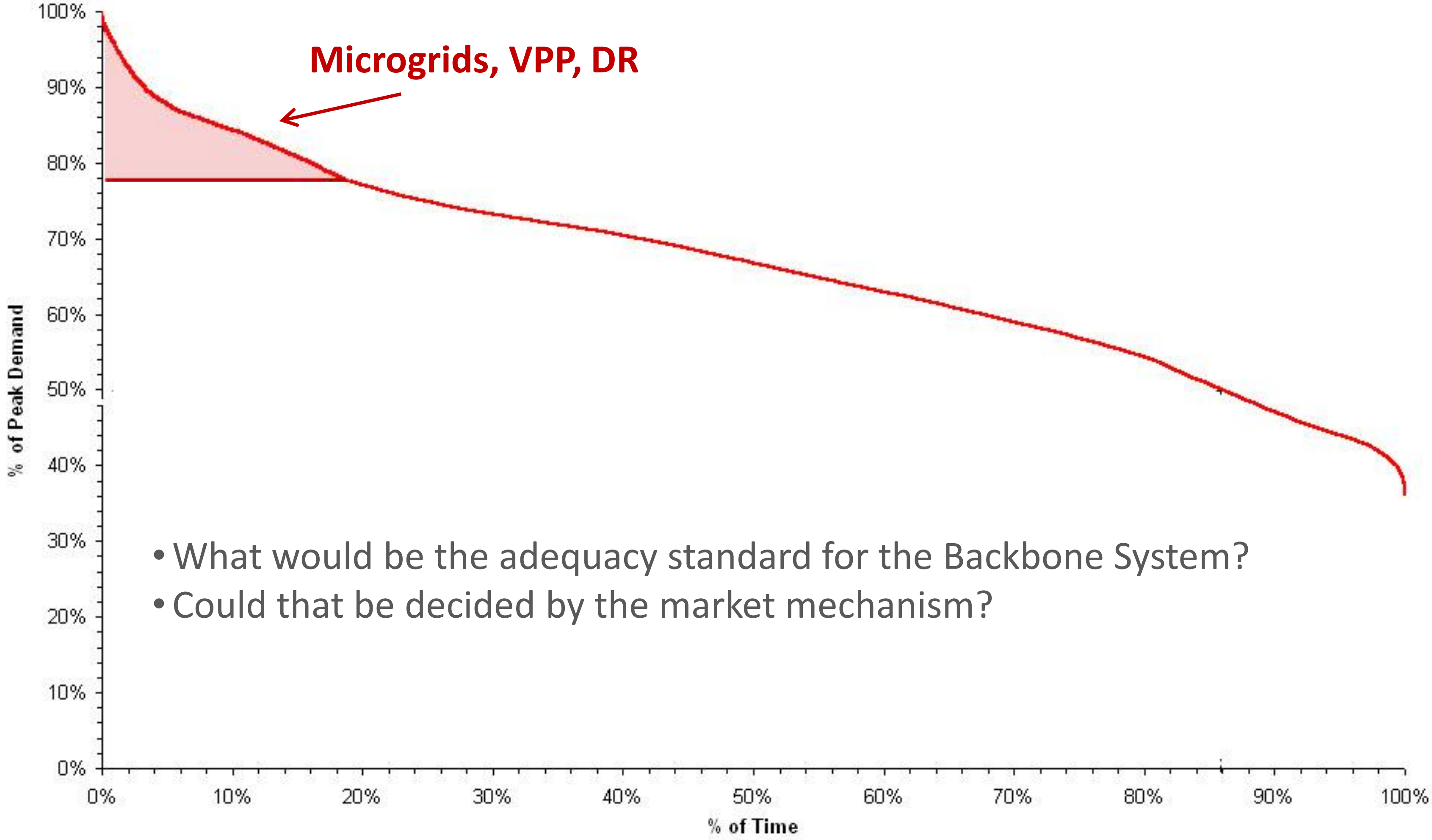
- The available historical data may not be comprehensive enough to accurately evaluate all reliability indices
- Traditional reliability indices cover the effects of a fraction of total events – this may suggest expanding the theory
- Major power system events may follow a power law distribution

Reliability Standards

- Are we compliant?
  - Not enough statistics and evidence to answer
- What do our standards mean?
- What happens if they are relaxed?

New system challenges suggest expanding the framework of traditional reliability theory

Reliability

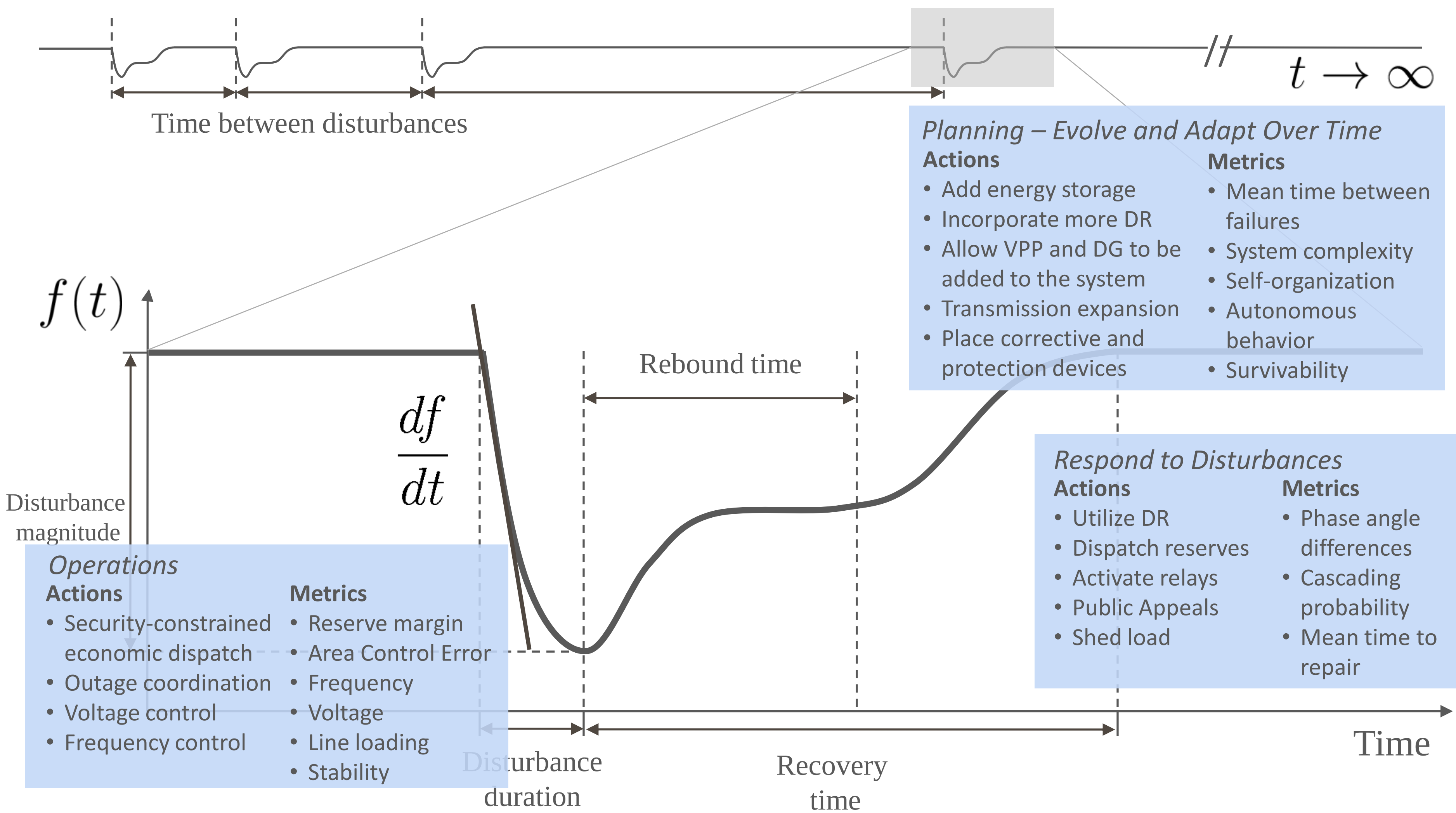


- What would be the adequacy standard for the Backbone System?
- Could that be decided by the market mechanism?

Survivability

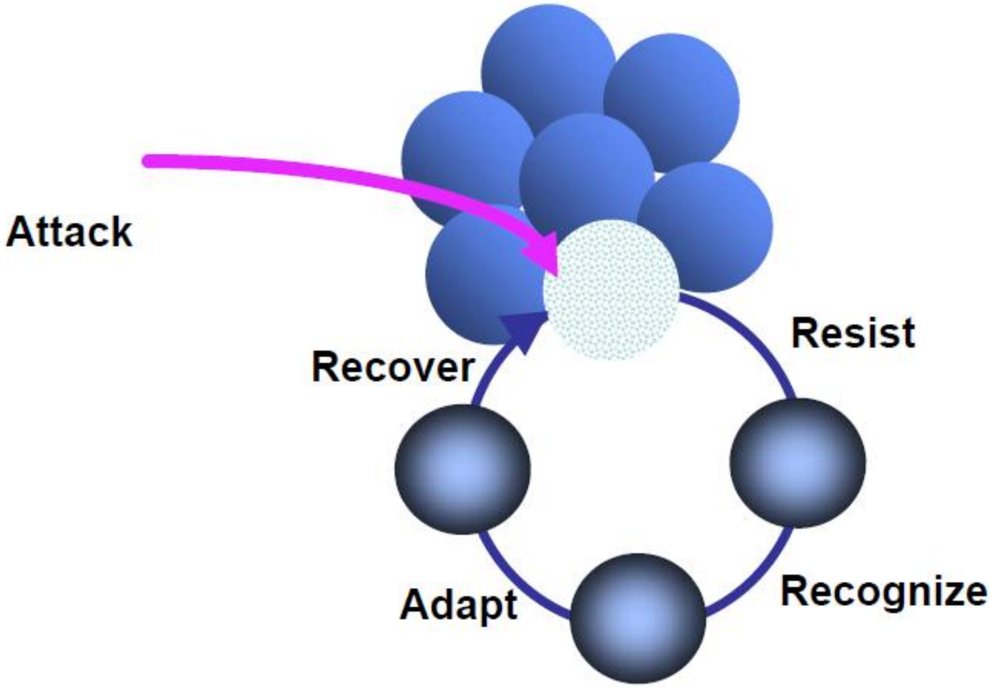
- New technologies will lead to emergent behavior – not necessarily positive
  – Self-Organized Criticality: Blackout cannot be avoided by tightening the current reliability criteria
- Concepts of survivability, resilience and robustness
  – Survivability is an *emergent property* of a system – desired system-wide properties "emerge" from local actions and distributed cooperation
  – The realization of a survivable system will rely on advanced detection, control and coordination techniques
  – How do you effectively model, simulate, and visualize survivability?

Survivability



Survivability

- The ability of the system to continuously provide energy to the customers in the presence of a failure or attack on the system



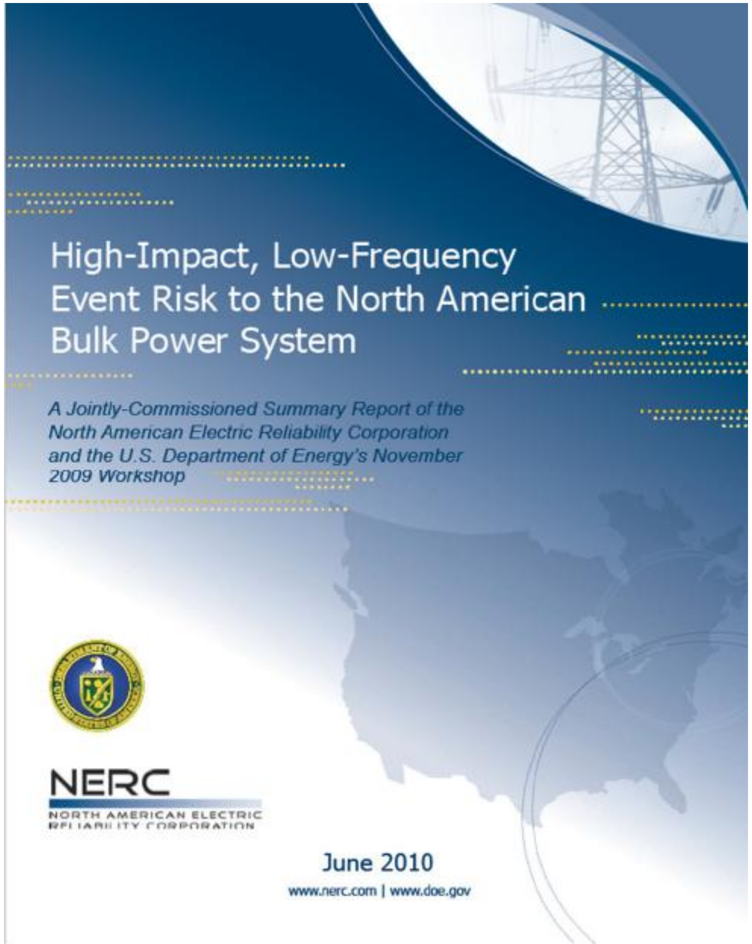
Survivability

- Four properties of survivability:
  - Resistance to attack – system design, short term planning
  - Recognition of intrusion – local and wide-area monitoring
  - Recovery of essential or full service after attack – protection, emergency control, SPS/RAS, WASIP, reconfiguration
  - Adaptation/evolution to reduce effect of future attacks – cognitive systems
- Why is it so difficult to define the metrics for survivability? Rare but high impact events!

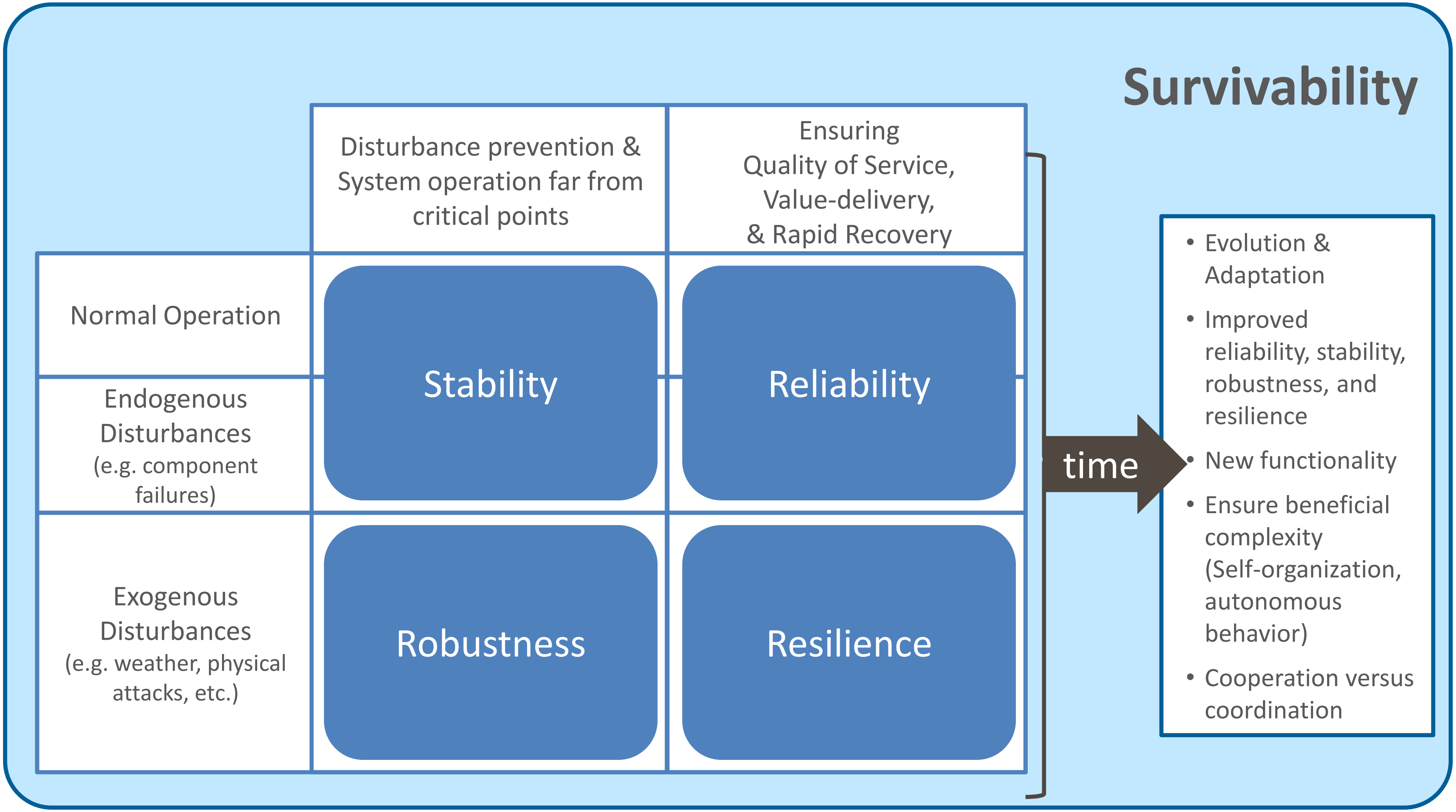
High Impact Low Frequency Report

- NERC/DOE report June 2010
- Based on the results of the HILF workshop

http://www.nerc.com/files/HILF.pdf

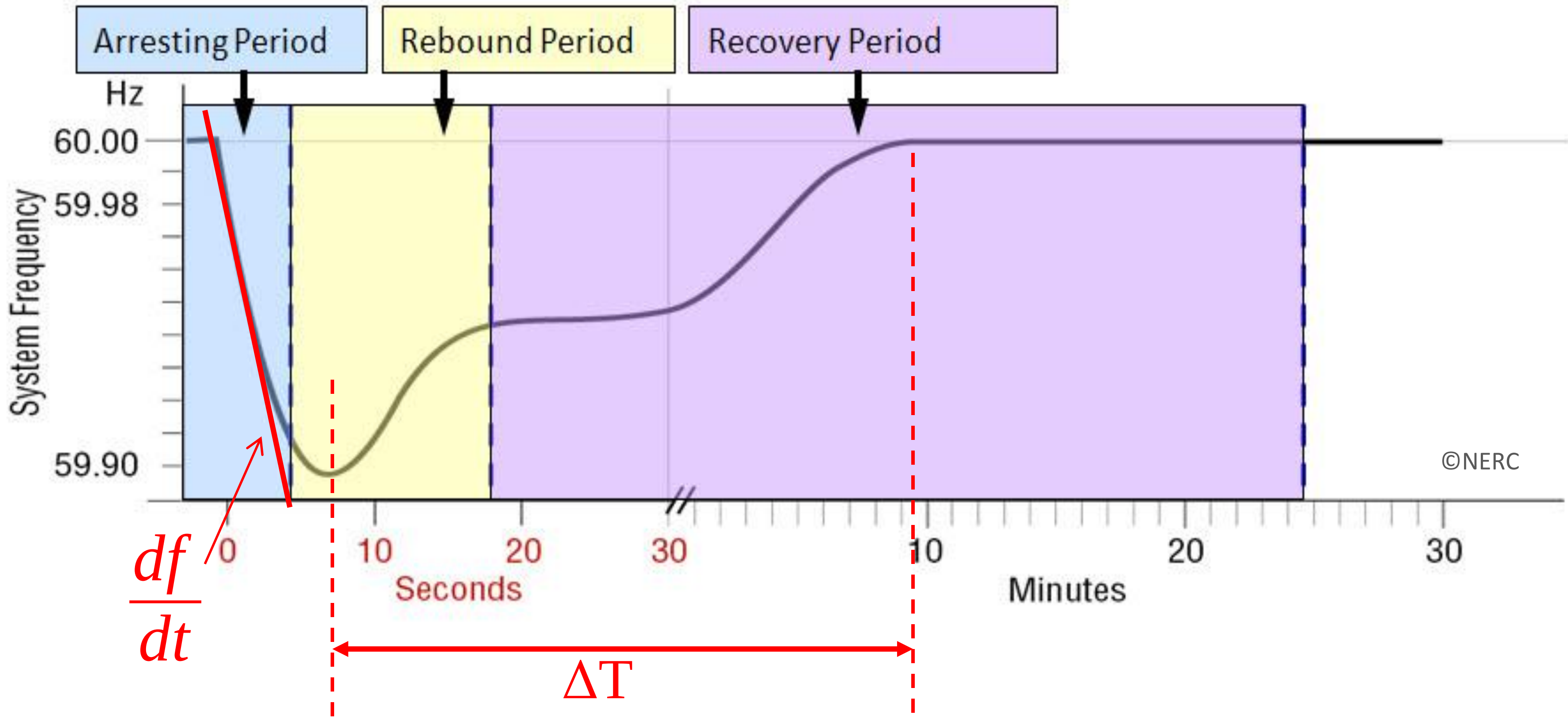


Survivability Characteristics



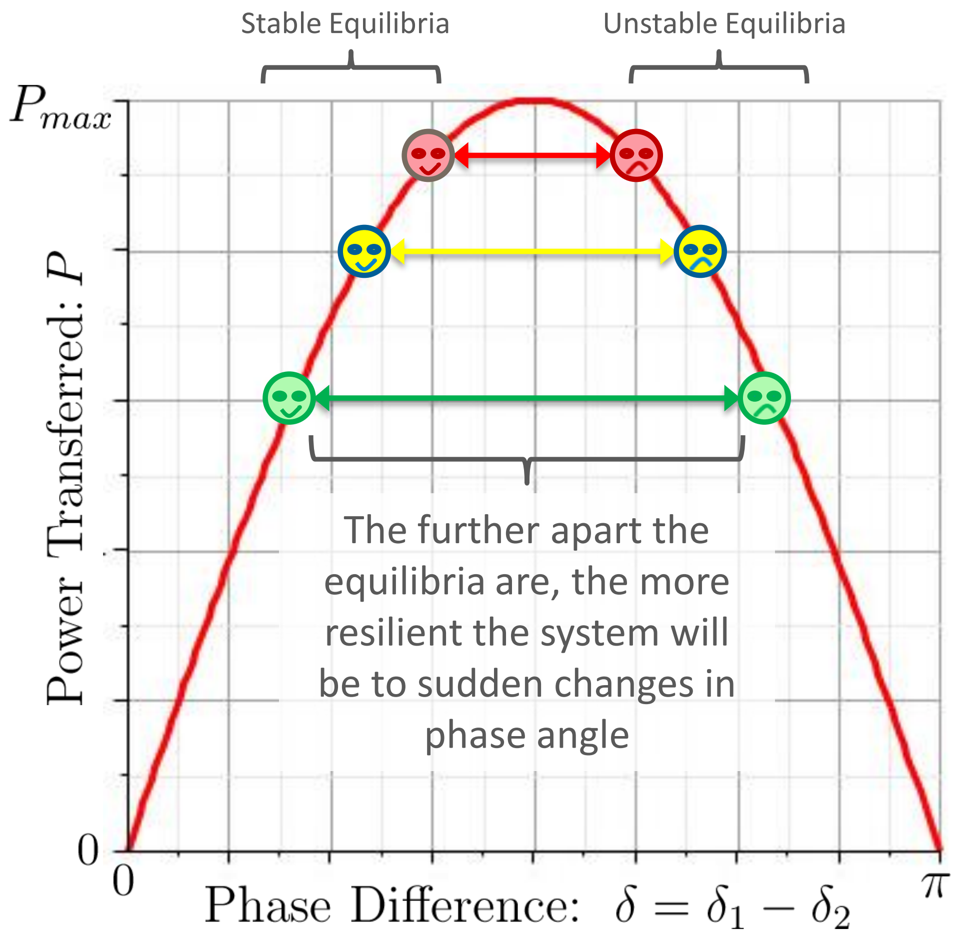
Survivability and Resilience: early detection and fast recovery

Survivability Metrics



During a disturbance, the rate of change of frequency and the time to recover may be used to measure survivability

Survivability Metrics (cont'd)



Flexibility (Motivation)

- The variability of renewable resources requires the system to have the ability to react to a sudden change of system condition and accommodate new state within acceptable time and cost tolerance.
- The importance of flexibility is well recognized, but there is lack of a unified framework for defining and evaluating flexibility.
- A single flexibility framework can
  - Serve as a basis for comparison of different power system designs.
  - Enable the integration of flexibility in the design of power systems

Literature Review

- In finance, flexibility can be reflected by liquidity, i.e. the degree to which assets can be converted to capital.
- In manufacturing system, flexibility represents the capability of manufacturing system to modify manufacturing resources to produce different products efficiently maintaining an acceptable quality. [Sethi et al, 1992]
- In information system, flexibility is the ability of the system to accommodate a certain amount of variation regarding the requirements of the supported business process [Applegate et al, 1999]

Literature Review: Flexibility in Power System

- A flexible plan is the one that enables the utility to quickly and inexpensively change the system's configuration or operation in response to varying market and regulatory conditions. [Hobbs et al, 1994]
- Flexibility is the ability of a system to deploy its resources to respond to changes in the demand not served by variable generation. [Lannoye et al, 2011]
  - They suggest reliability criteria to assess flexibility of a system, similar to the LOLE for capacity adequacy.
- Flexibility is the potential for capacity to be deployed within a certain timeframe. [Bouffard et al, 2011]
  - They associate flexibility with reserves.
- Flexibility is defined as the attitude of the transmission system to adapt, quickly and with limited cost, to every change, from the initial planning conditions. [Capasso et al, 2005]
- A flexibility index is borrowed from the process control literature, and is associated with reserves. [Menemenlis et al, 2011]

Definition of Flexibility

- Flexibility is the ability of a system to respond to a range of uncertain future states by taking an alternative course of actions within acceptable cost threshold and time window.
- Four elements are the determinants of flexibility
  - Response time window (T)
  - Set of corrective actions ($\boldsymbol{a}$)
  - Range of uncertainty ($\boldsymbol{u}$)
  - Response cost threshold ($\overline{C}$)

Target Range of Uncertain State Deviation

- The first step in accounting for flexibility is to define and clarify the target range of uncertain state deviation.
- A system aims to accommodate the uncertainty within the target range.
- For example, while a system is flexible with respect to the N-1 criterion, it may not be flexible with respect to the N-2 criterion.

Response Time Window

- Indicate how fast the system is expected to react to state deviations and restore the system to normal states.
- Short/Long time windows focus on the short-term/long-term flexibility of a system.
- A system may show more flexibility in long term while lacking flexibility in short term.

Set of Corrective Actions

- It represents the corrective actions that can be taken within the response time window under certain operating procedure.

	Control Actions						
Time	AGC	Economic Dispatch	Unit Commitment	Voltage Control	Interchange Scheduling	Short-term Outage Coordination	Long-term Outage Coordination
4 Sec							
5 Min							
1 Hr							
Day							
Month							

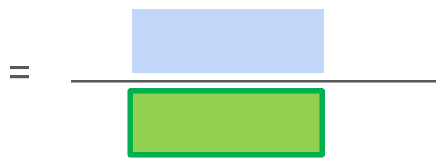
Other Related Complementary Concepts

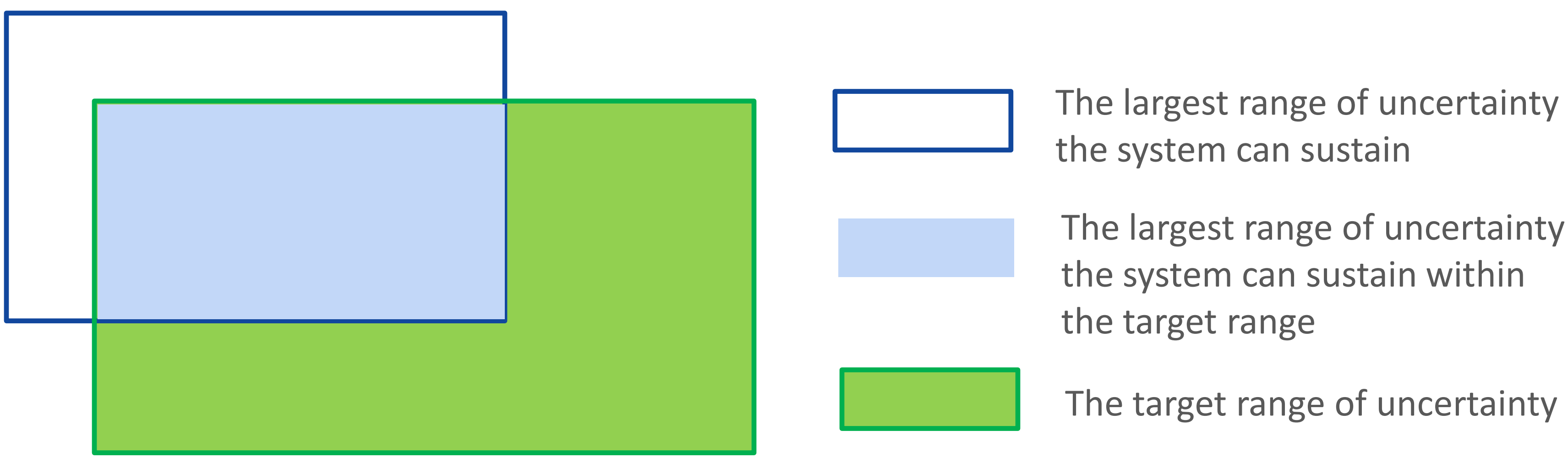
- *Flexibility*: Ability of the system to be modified to do jobs NOT originally included in the requirement.
- *Robustness*: Ability of the system to do its job in unexpected environments.
- *Adaptability*: Ability of the system to be modified to do jobs in expected environments.
- *Reliability*: Probability that the system will do the job it was asked to do.

FLEXIBILITY METRIC

- Flexibility metric is defined as the following

$$\mathit{Fledex} = \frac{\text{The size of the largest range of uncertainty the system can sustain within the target range}}{\text{The size of the target range of uncertainty}}$$





The Range of Uncertainty

- For each time interval t within the response time window T, the range of uncertainty is assumed to be a hypercube

$$\mathcal{U}_t = \left\{ s_t \in \mathbb{R}^n \mid s_t^{LB} \leq s_t \leq s_t^{UB} \right\}$$

- The target range of uncertainty

$$\mathcal{U}_t^{\text{target}} = \left\{ s_t \in \mathbb{R}^n \mid \underline{s}_t^{LB} \leq s_t \leq \overline{s}_t^{UB} \right\}$$

Formulation of the Largest Range of Uncertainty Problem

$$\max_{s^{LB}, s^{UB}, a(\bullet)} \sum_{t=1}^{T} e^T (s_t^{UB} - s_t^{LB})$$ → Find the largest range of uncertainty $\mathcal{U}_t^{\max}$

$$\text{s.t. } A_t a_t(s) + B_t s_t \le b_t, \quad \forall s_t \in [s_t^{LB}, s_t^{UB}], \ \forall t = 1, \ldots, T$$ → Corrective action

$$c_t^T a_t(s) \le \bar{C}_t, \quad \forall s_t \in [s_t^{LB}, s_t^{UB}], \ \forall t = 1, \ldots, T$$ → Response cost threshold

$$\underline{s}_t^{LB} \le s_t^{LB} \le s_t^{UB} \le \bar{s}_t^{UB}, \quad \forall t = 1, \ldots, T$$ → Limitation on the range

Size of the largest range of uncertainty $\mathcal{U}_t^{\max}$ at time t:

$$s_t^{\max} = e^T (s_t^{UB} - s_t^{LB})$$

$Fledex_t = s_t^{\max} / s_t^{\text{target}}$

Size of the target range of uncertainty $\mathcal{U}_t^{\text{target}}$ at time t:

$$s_t^{\text{target}} = e^T (\bar{s}_t^{UB} - \underline{s}_t^{LB})$$

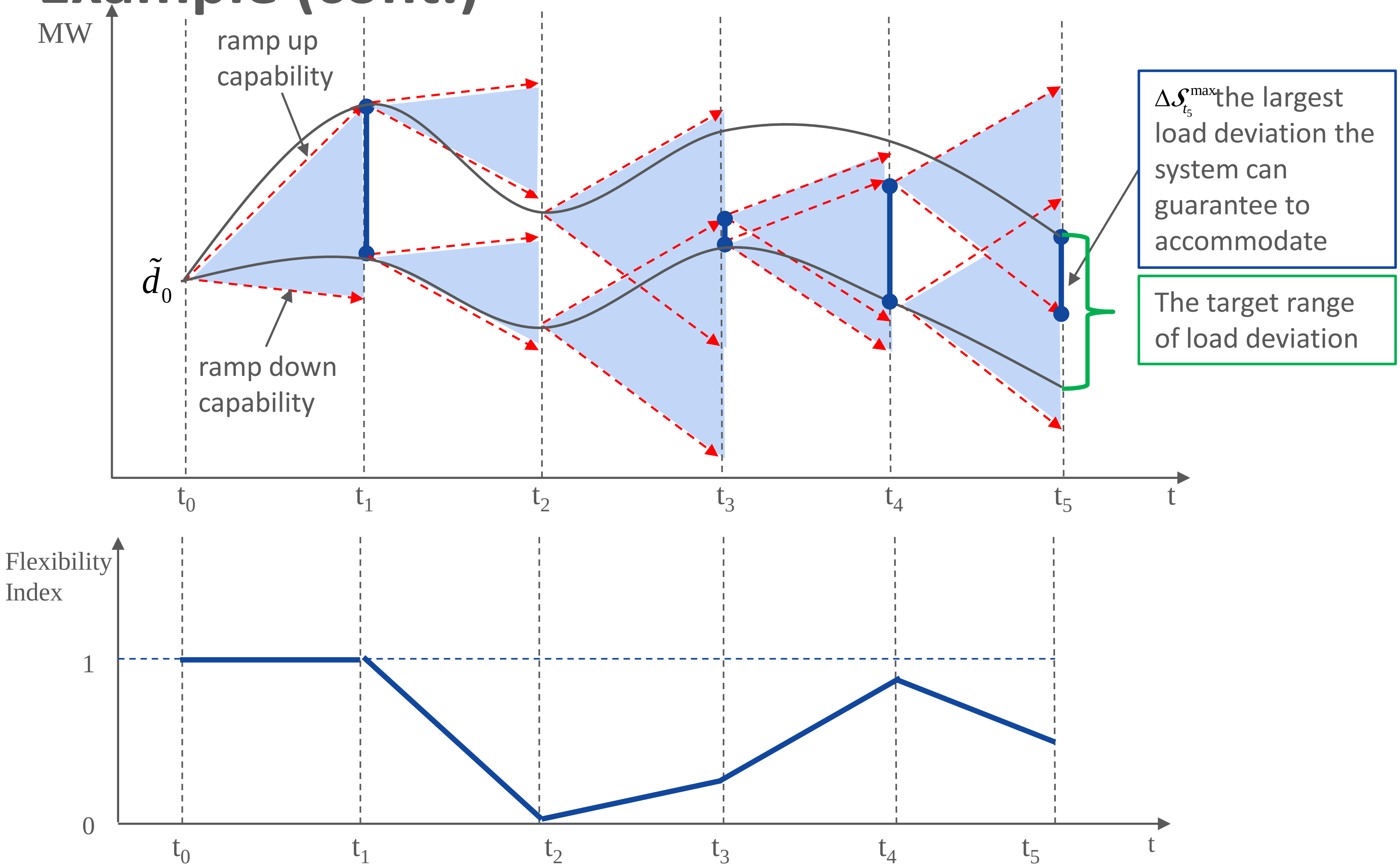
Not a Standard Robust Optimization Problem

- A standard robust optimization problem:
  - Given a range of uncertainty, would I be able to accommodate the worst case?
- Our problem:
  - Given what I can do, what is the largest range of uncertainty I can accommodate?

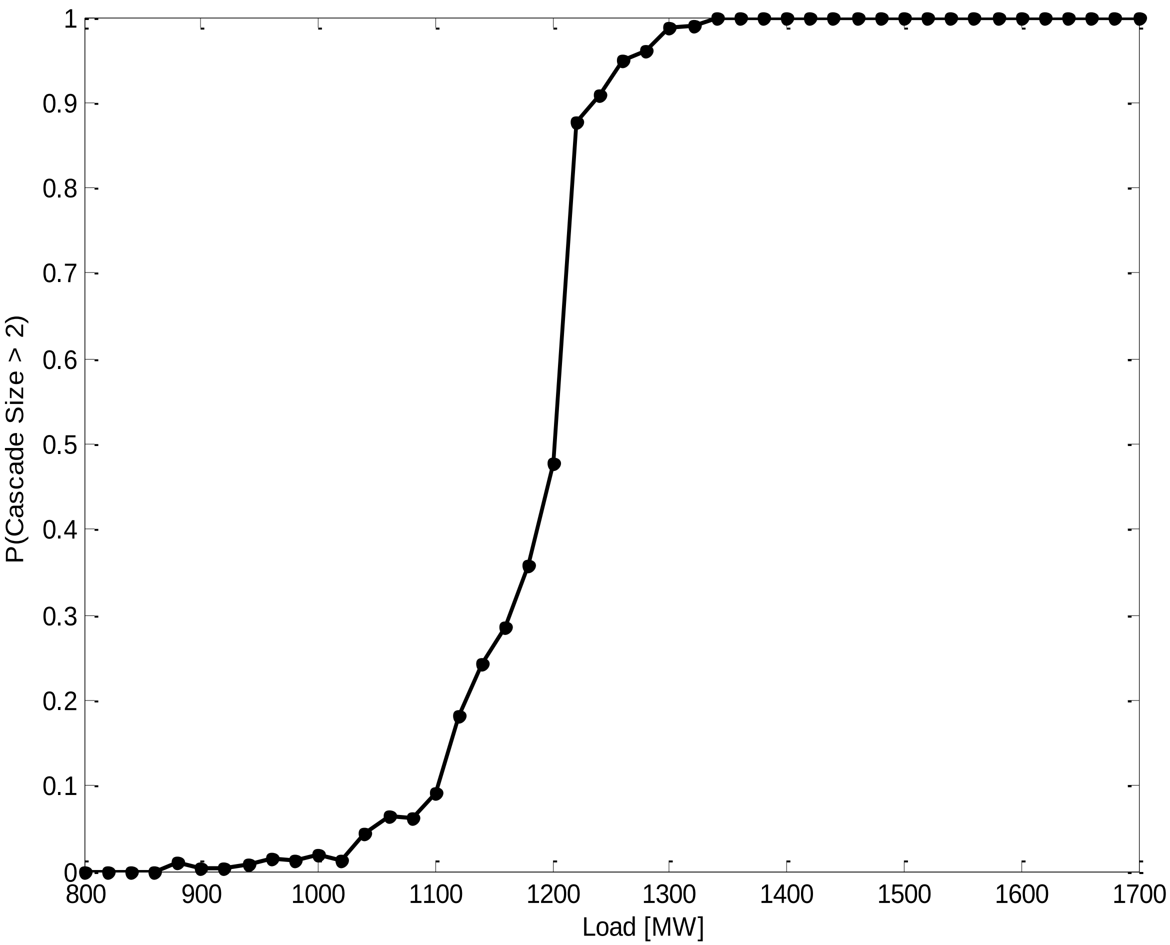
Example

- Do we have sufficient ramping capability to follow system load deviation?
- Use the flexibility index to reflect the possibility and magnitude of the ramping problem in the look-ahead horizon.
- Assumptions:
  - Response time window is 5 minutes
  - No cost threshold
  - Only consider re-dispatch as corrective action
  - Uncertain state deviation is a range of possible future load realizations in the load-ahead horizon
- No transmission constraints are modeled.

Example (cont.)



Probability of Cascading Failure Under System Stress



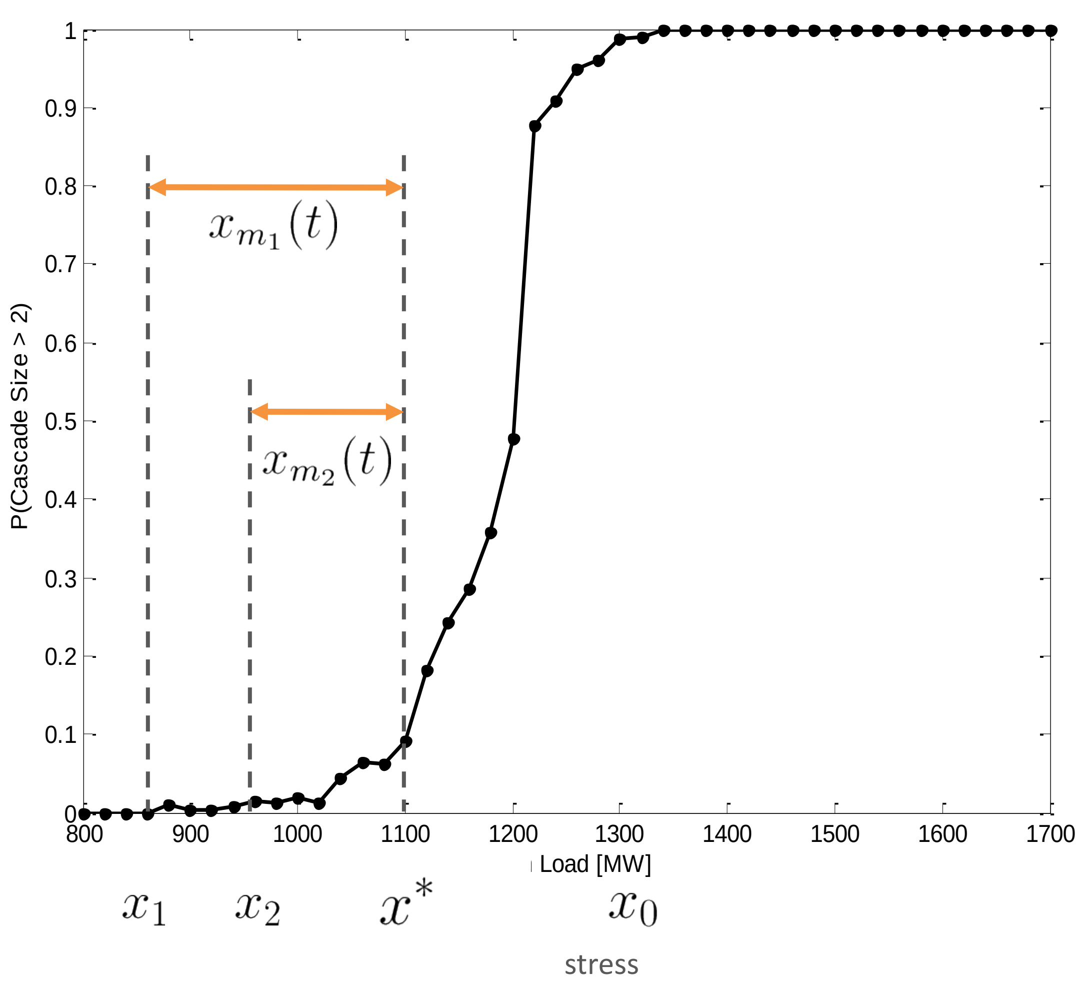
stress

The load (a measure of system stress) is varied from 800 MW to 1700 MW and the system is subjected to:

- Independent generator forced outages
  - FOR = 0.08 (NERC GADS)
- Independent line forced outages
  - FOR = 0.00434 (NERC TADS)

The ordinate is the probability of a cascade in excess of 2 lines (or a loss of load of 20% or more)

Metrics of system stress, resilience, and flexibility:
Flexibility Metric



$x(t)$ = system stress at time t

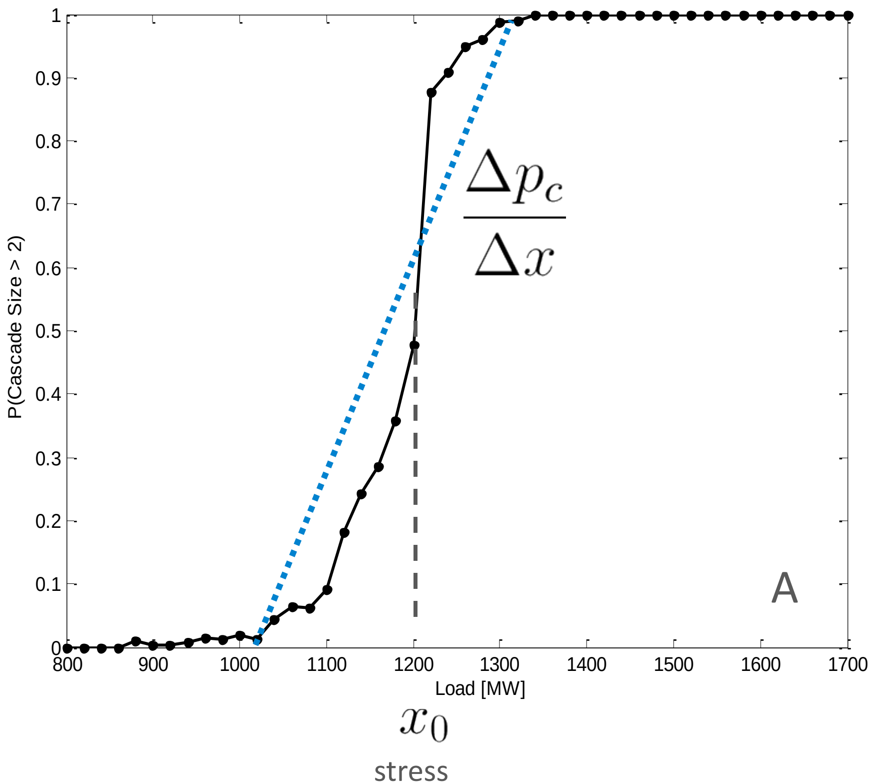
x^* = Phase change threshold for system stress

$x_m(t)$ = Stress margin at time t

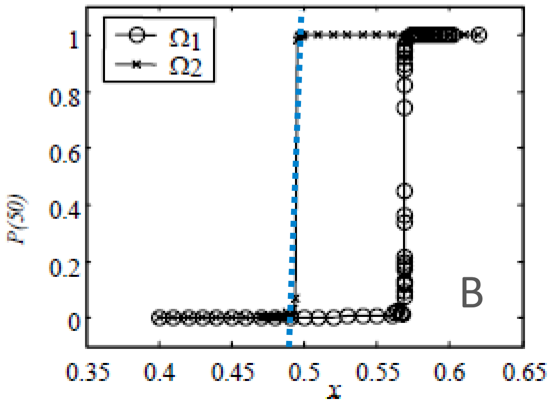
In this case, the system operating at x_1 has a greater margin to work with than x_2. The stress margin can be thought of as a metric of flexibility

Metrics of system stress, resilience, and flexibility

Resilience Metric



Compare the example, A, to the example from [1], B:



It should be clear that: $\xi_B > \xi_A$

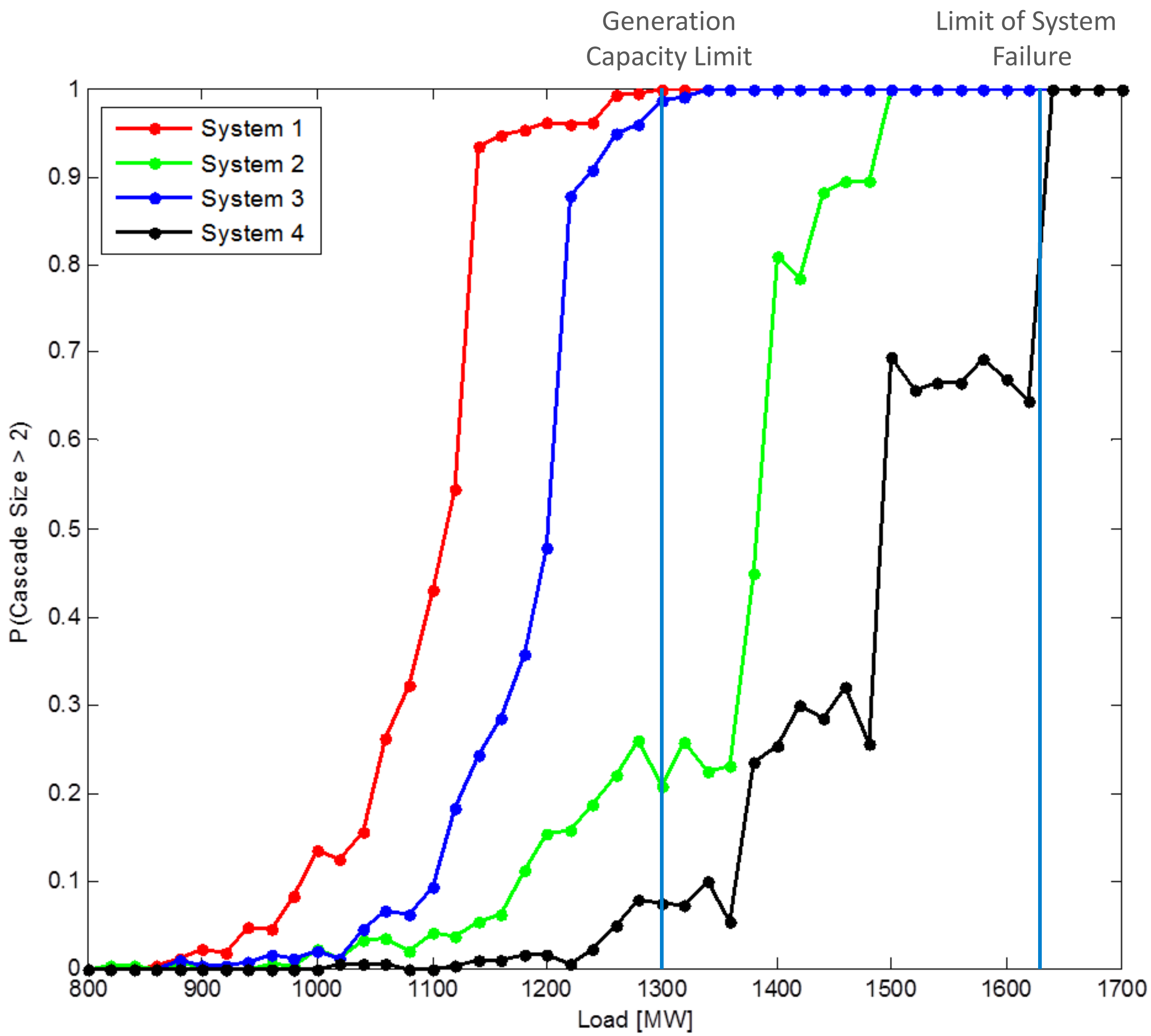
Since the smaller the slope the more gracefully the system degrades, this metric can be thought of as a measure of system resilience

$p_c = P(C \geq c)$ = Probability of a cascade of size *c* or greater

$$\xi = \frac{\Delta p_c}{\Delta x}$$ = Rate of change in the cascade probability with respect to system stress

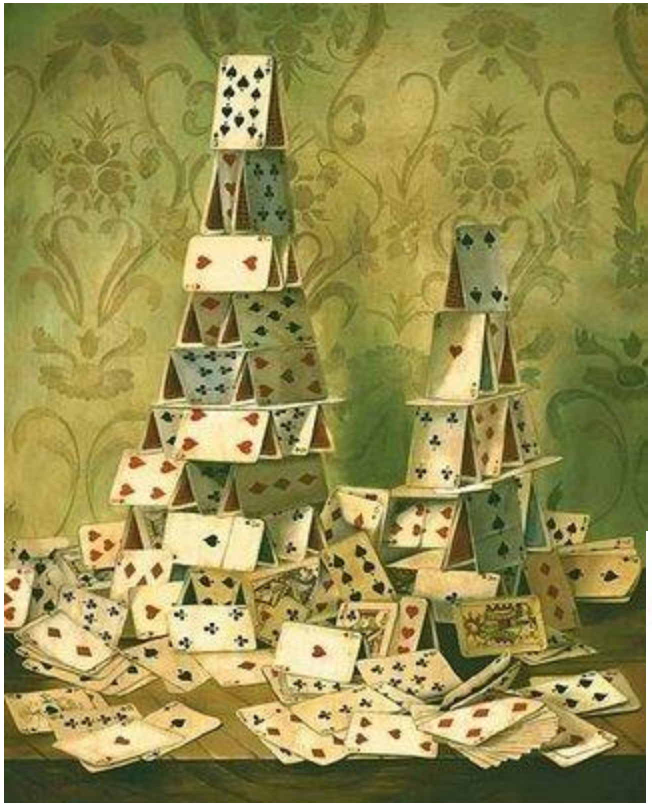
[1] Liao, Apt, and Talukdar, "Phase Transitions in the Probability of Cascading Failures," 2004.

Comparing Test Systems



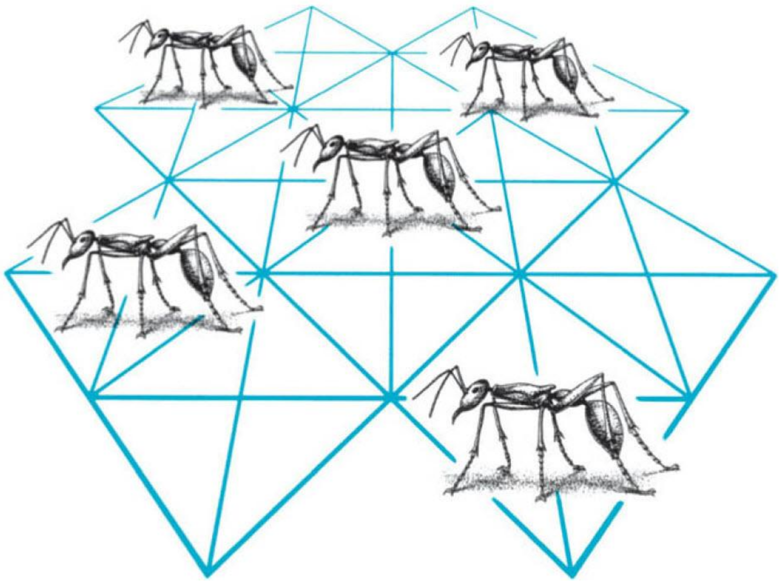
- Each of the systems were identical, except for the location of generators and loads
- Even with such similarity, each system has a substantially different cascade probability profile

System Complexity and Vulnerability



New Control Architecture

- Decentralized, loosely coupled system is more resilient
- Cooperation vs. Coordination among subsystems
- Methods and algorithms to support spontaneous ad-hoc cooperation between subsystems
- Complexity must be measured and controlled during design
- Corrective vs. Preventive control
- Wide-area SPS, RAS, SIP – not less reliable than DR



Questions

Optimal Charging of Vehicle-to-Grid Fleets via PDE Aggregation Techniques

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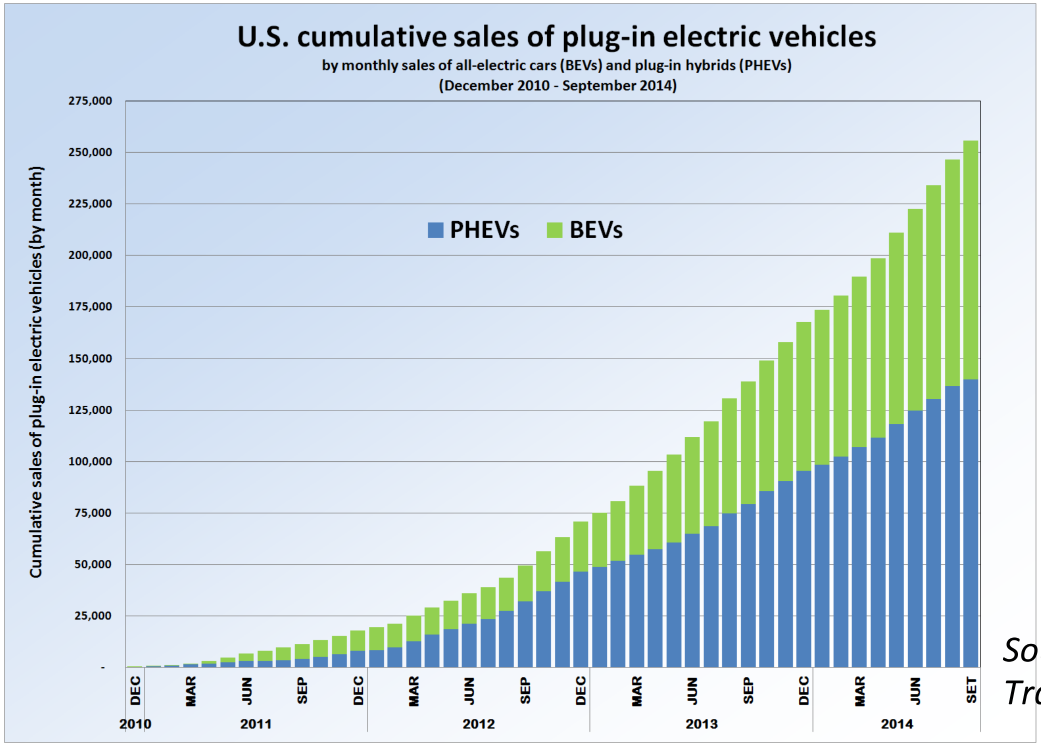
LANL Grid Science 2015

Introduction

Why are "EVs", "Vehicle to Grid (V2G)", hot topics?

Vehicle Electrification

Their number grows…



Source: Electric Drive Transportation Association

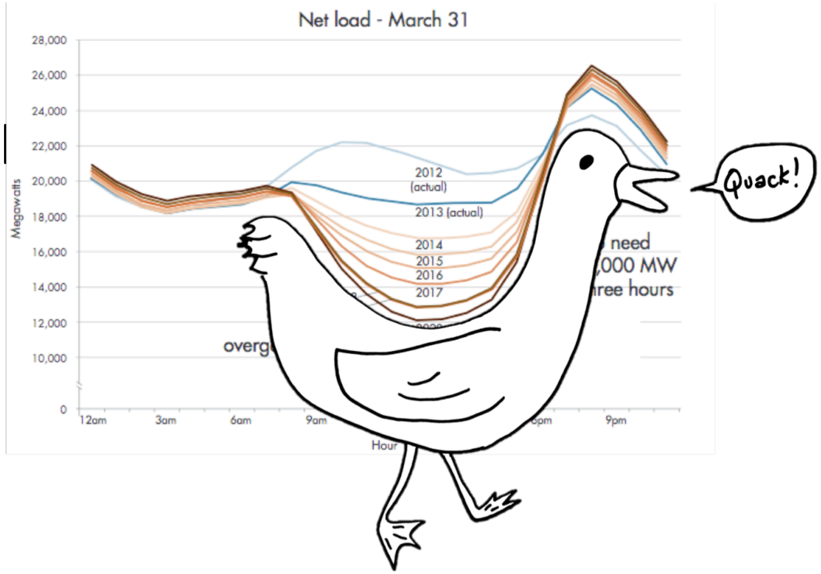
Introduction

Why are "EVs", "Vehicle to Grid (V2G)", hot topics?

Vehicle Electrification

<u>If not controlled</u>, represents an additional risk for Grid Resilience

- Additional loads during peak hours
- Extra investment in grid infrastructure



Axsen, J., & Kurani, K. S. (2010). *Transportation Research Part D: Transport and Environment*, *15*(4), 212-219.

Hadley S. Oak Ridge, *TN: Oak Ridge National Laboratory*; 2006.

Introduction

Why are “EVs”, “Vehicle to Grid (V2G)”, hot topics?

Vehicle Grid Integration

<u>If controlled</u>, represents a great opportunity for

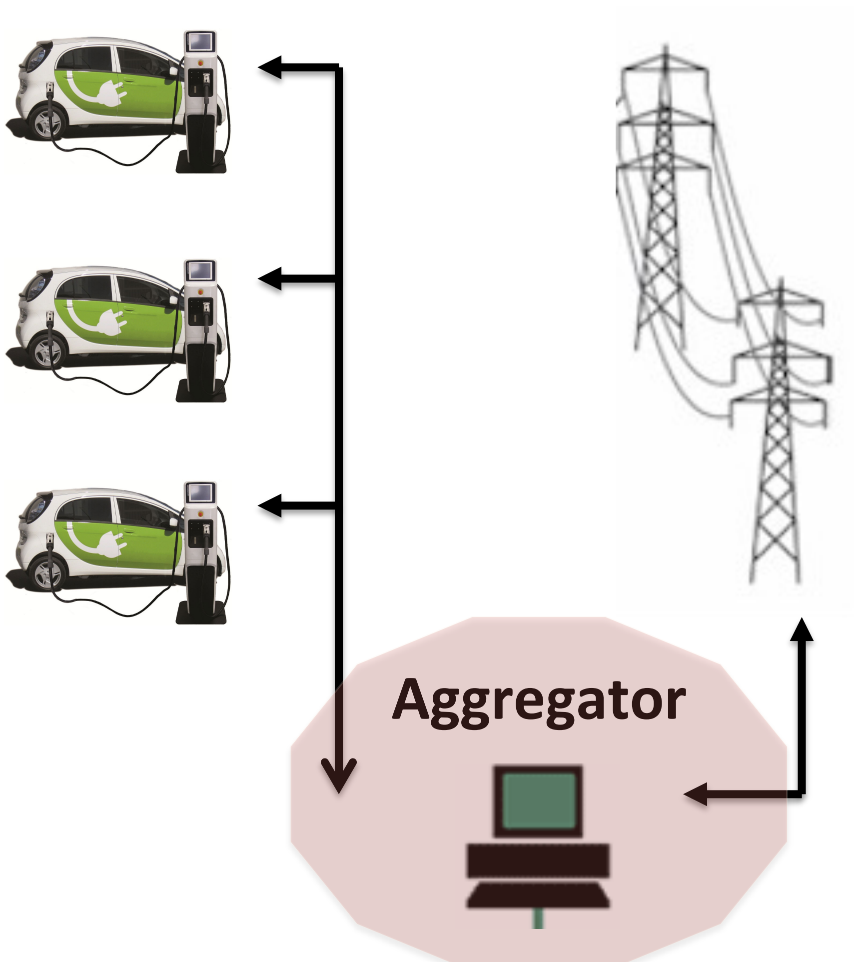
- Demand Response
- Storage
- …

US personal vehicles are parked 96% of time!

(A. Langton and N. Crisostomo, *California Public Utilities Commission, Tech. Rep*, 2013)

How can we model and control PEV loads during this available time?

EV aggregator



Vehicle-To-Grid (V2G):
Cars communicate with the Grid
Can "sell" energy

Aggregator:
Single PEV~ 5-20 kW
The aggregator collectively charges, discharges cars.
The aggregator may participate in the electricity market

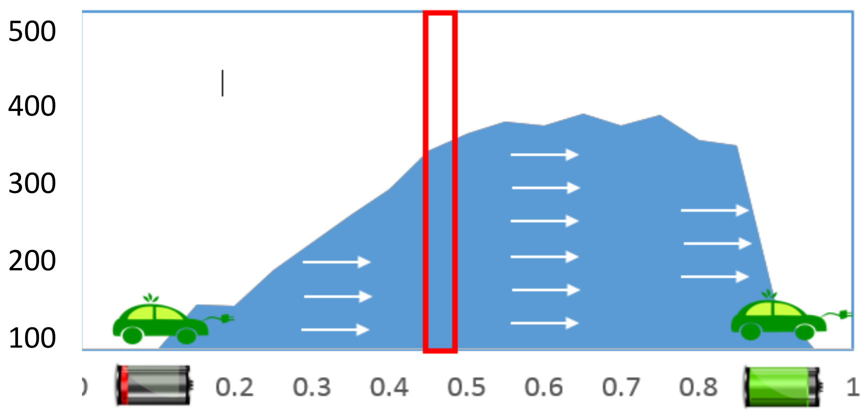
Challenge: Controlling large population of EVs

- **Participate in the electricity market**
- **Satisfy drivers and needs for mobility**
- **Be profitable for the aggregator**

PDE aggregation model

Fleet of EVs

Nb of Cars



X (SOE)

u(x,t): number of cars, which are plugged-in and charging at time t and SOE x.

Charging dynamics of vehicle i:

$$
\begin{aligned}
\dot{x}_i(t) &= \frac{\eta^m(x_i)}{E_{\max}} P_i(t), \qquad i = 1, \cdots, N, \\
m &= \begin{cases} 1 & \text{if } P_i(t) \geq 0, \\ -1 & \text{if } P_i(t) < 0, \end{cases}
\end{aligned}
$$

X_i: State of Energy (SOE)
η: Conversion efficiency
E_{max}: battery energy capacity

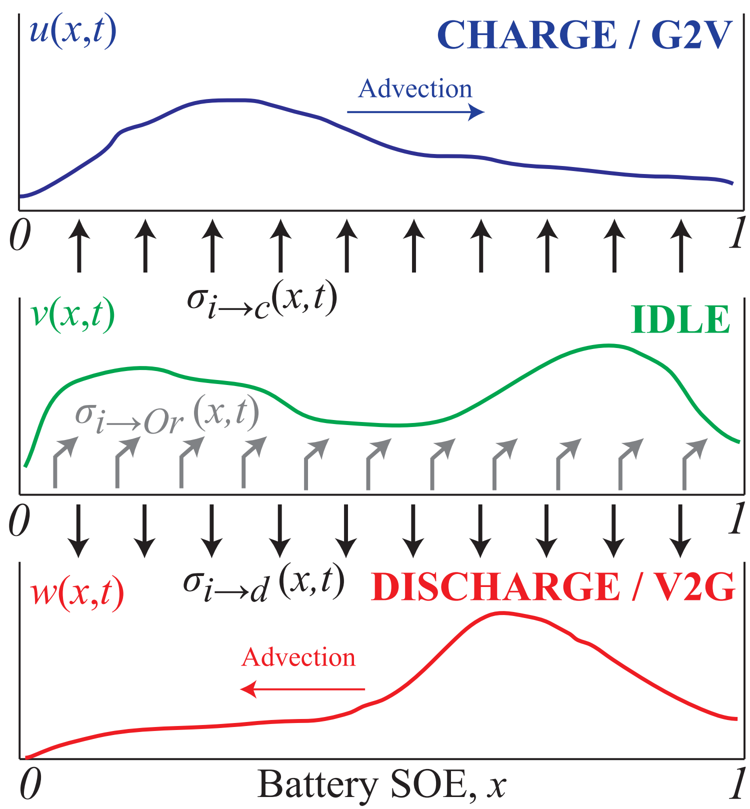
Cars charge at rate $\frac{\eta^m(x_i)}{E_{\max}} P_i(t)$,

$$\boxed{\frac{\partial u}{\partial t}(x,t) = -\frac{\partial}{\partial x}[q_c(x,t)u(x,t)] + \sigma_{i \to c}(x,t).}$$

External flows

PDE aggregation model

Fleet of EVs = 3 states



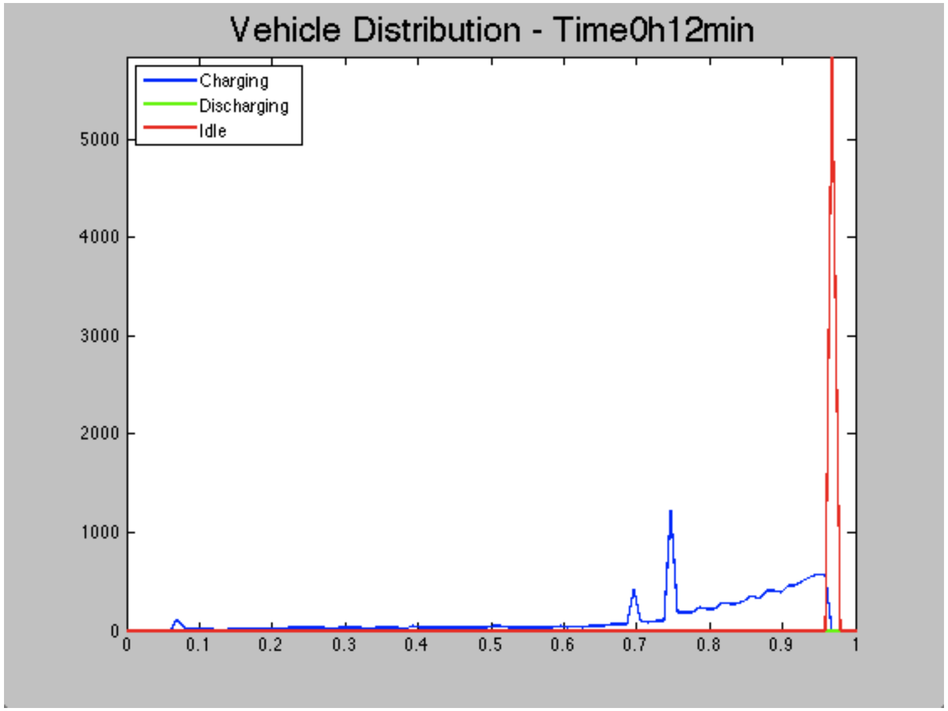
$$\frac{\partial u}{\partial t}(x,t) = -\frac{\partial}{\partial x}[q_c(x,t)u(x,t)] + \sigma_{i\to c}(x,t),$$
$$\frac{\partial v}{\partial t}(x,t) = -\sigma_{i\to or}(x,t) - \sigma_{i\to c}(x,t) - \sigma_{i\to d}(x,t),$$
$$\frac{\partial w}{\partial t}(x,t) = \frac{\partial}{\partial x}[q_d(x,t)w(x,t)] + \sigma_{i\to d}(x,t).$$

System of 3 coupled PDEs

PDE aggregation model

EVs stop charging at 97% SOC.

EVs discharge (V2G) between 6pm and 9pm



Why is this model interesting?

- Computation doesn't depend on the number of cars
- Nice Parallel with TCLs
- Large number of analysis and control methods for PDEs

Optimal Charging of V2G fleets

(Validation of the model with V2Gsim)

V2G-SIM

$$\min_{\sigma_{i\to d},\sigma_{i\to c},Dep} C = \int_0^{T_{max}} C_{elec}(t) \int_0^1 q_c(x,t)u(x,t)\ dx\ dt,$$

subject to

$$\frac{\partial u}{\partial t}(x,t) = -\frac{\partial}{\partial x}[q_c(x,t)u(x,t)] + \sigma_{i\to c}(x,t),$$
$$\frac{\partial v}{\partial t}(x,t) = -\sigma_{i\to c}(x,t) - \sigma_{i\to d}(x,t)$$
$$+Arr(x,t) - Dep(x,t),$$
$$\frac{\partial w}{\partial t}(x,t) = \frac{\partial}{\partial x}[q_d(x,t)w(x,t)] + \sigma_{i\to d}(x,t),$$

$$u(0,t) = 0,\ w(1,t) = 0,$$
$$u(x,0) = u_0(x),\ v(x,0) = v_0(x),\ w(x,0) = w_0(x),$$

$$-w(x,t) \le \sigma_{i\to d}(x,t) \le v(x,t),$$
$$-u(x,t) \le \sigma_{i\to c}(x,t) \le v(x,t),$$

$$u(x,t) = 0, \quad \forall\ x \ge X_{max},$$
$$v(x,t) = 0, \quad \forall\ x \le X_{min},$$
$$w(x,t) = 0, \quad \forall\ x \le X_{min}.$$

$$\int_0^1 q_d(x)w(x,t)dx \ge P^{des}(t), \quad \forall\ t.$$

$$\int_{X_{dep}}^1 Dep(x,t)\ dx = Dem(t), \quad \forall\ t.$$

$$\int_{X_{dep}}^1 (u+v+w)(x,T_{max})\ dx \ge N_{min}.$$

Optimal Charging of V2G fleets

(Validation of the model with V2Gsim)

V2G-SIM

$$\min_{\sigma_{i\to d},\sigma_{i\to c},Dep} C = \int_0^{T_{max}} C_{elec}(t) \int_0^1 q_c(x,t)u(x,t)\ dx\ dt,$$

subject to

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$$\frac{\partial v}{\partial t}(x,t) = -\sigma_{i\to c}(x,t) - \sigma_{i\to d}(x,t) + Arr(x,t) - Dep(x,t),$$

$$\frac{\partial w}{\partial t}(x,t) = \frac{\partial}{\partial x}[q_d(x,t)w(x,t)] + \sigma_{i\to d}(x,t),$$

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$$-u(x,t) \le \sigma_{i\to c}(x,t) \le v(x,t),$$

$$u(x,t) = 0, \quad \forall\ x \ge X_{max},$$

$$v(x,t) = 0, \quad \forall\ x \le X_{min},$$

$$w(x,t) = 0, \quad \forall\ x \le X_{min}.$$

$$\boxed{\int_0^1 q_d(x)w(x,t)dx \ge P^{des}(t), \quad \forall\ t.}$$

$$\boxed{\int_{X_{dep}}^1 Dep(x,t)\ dx = Dem(t), \quad \forall\ t.}$$

$$\int_{X_{dep}}^1 (u+v+w)(x,T_{max})\ dx \ge N_{min}.$$

Optimal Charging of V2G fleets

(Validation of the model with V2Gsim)

$$\min_{u,v,w,Dep} \Delta t \ \Delta x \sum_{n=0}^{N} \sum_{j=0}^{J} C_{elec}^{n} q_j^n w_j^n$$

subject to

$$[u+v+w]^{n+1} + \frac{Dep^{n+1}}{\Delta x} = M_c u^n + M_d w^n + \frac{Arr^{n+1}}{\Delta x}$$

$$u_0^n = 0, \quad v_J^n = 0,$$
$$u_j^0 = u_{0,j}(j\Delta x), \ v_j^0 = v_{0,j}(j\Delta x), \ w_j^0 = w_{0,j}(j\Delta x), \ \forall j,$$

$$u^n, v^n, w^n, Dep^n \geq 0,$$
$$u_j^n = 0 \ \forall \ j \geq X_{max} \cdot J$$
$$v_j^n = 0 \ \forall \ j \leq X_{min} \cdot J$$
$$w_j^n = 0 \ \forall \ j \leq X_{min} \cdot J$$

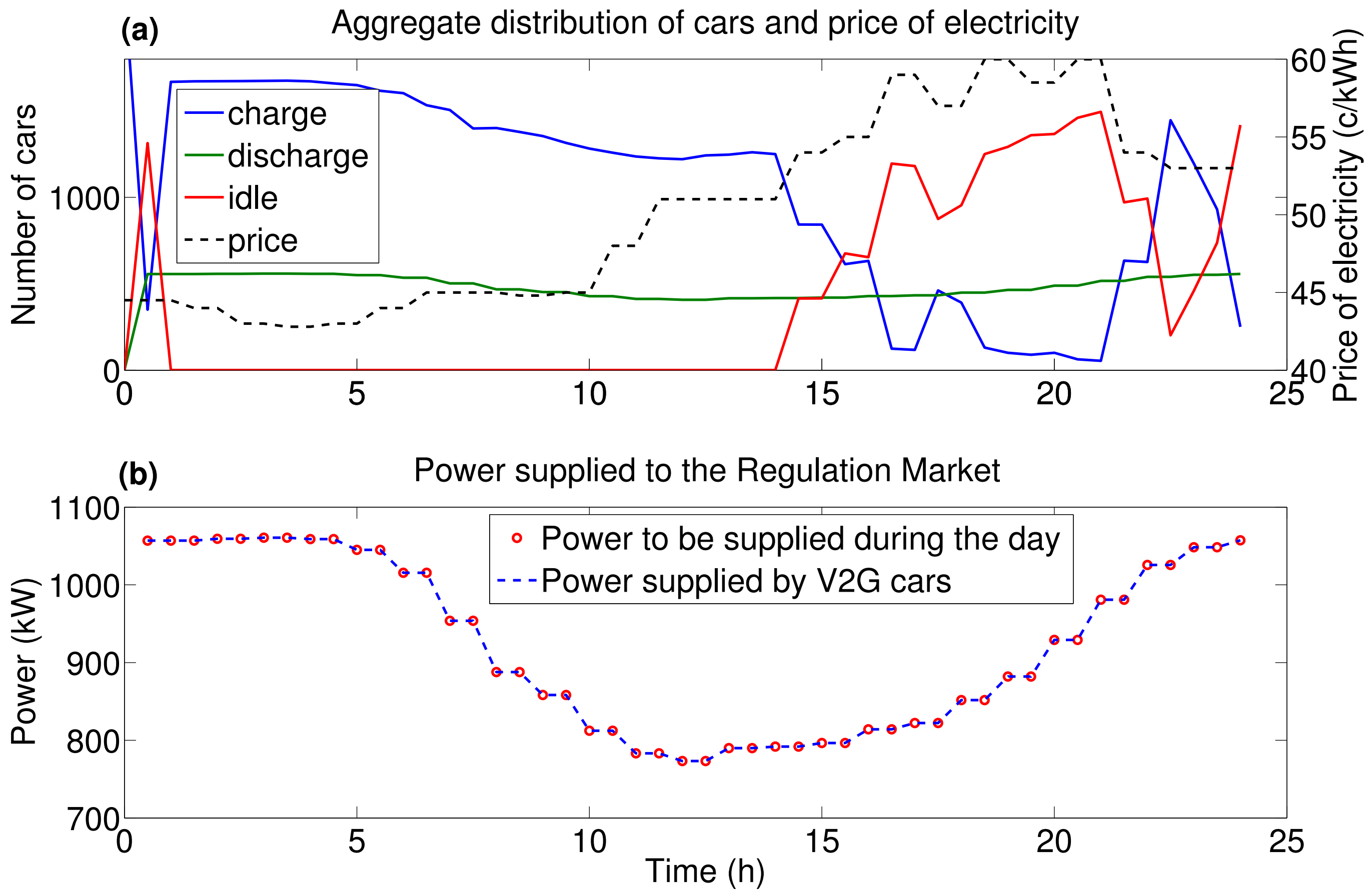
$$\Delta x \sum_{j=0}^{J} q_{d,j}^n w_j^n \geq P^{des,n}$$

$$\sum_{j=X_{dep}\cdot J}^{J} Dep_j^n = Dem^n$$

$$\Delta x \sum_{j=X_{dep}\cdot J}^{J} u_j^N + v_j^N + w_j^N \geq N_{min}$$

Optimal Charging of V2G fleets

Result :



Conclusion

Conclusion

- Model is well suited to handle large population of EVs
- We gave an example for using this model to control a EV fleet

Ongoing work

- Heterogeneity and stochasticity
- Grid constraints
- Different optimization objectives

Thank you!

Optimally integrating renewables

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Grid Science Winter
Conference
LANL
Santa Fe
January 16, 2015

Uncertainty of renewable power

- Wind power is stochastic, not dispatchable

Power (MW)

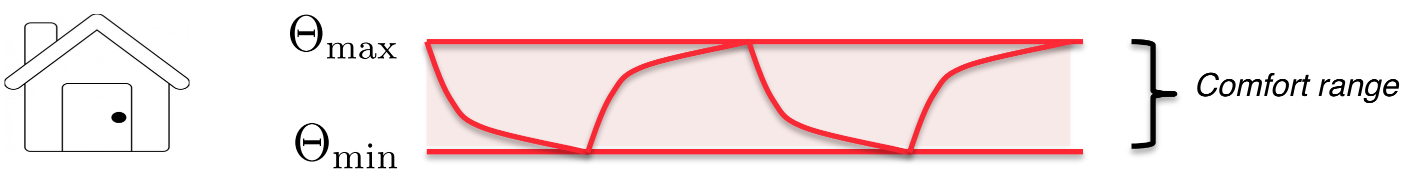
- How to integrate wind?

Demand response

- Adjust *demand* to match supply

- Some loads can be switched off for a while without being noticed

  - E.g., Air conditioners under thermostatic control

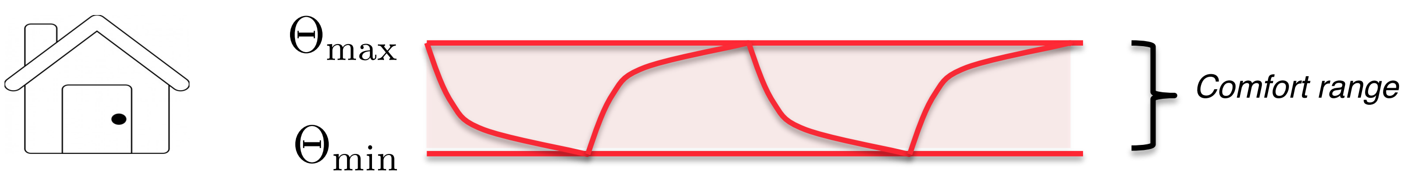


- Inertial thermal loads can absorb fluctuations in available wind power

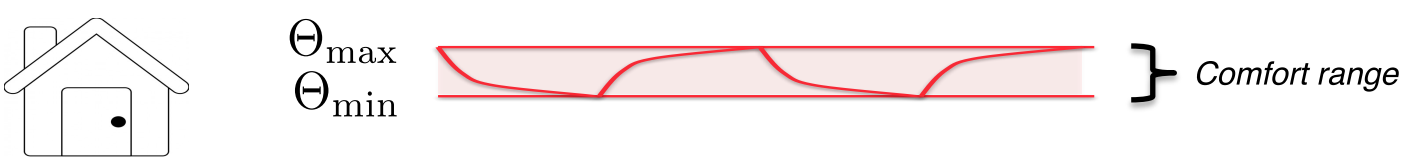
Flexibility of load requirements

- Amount of demand response will depend on how flexible the loads are with respect to their requirements

- More demand response possible



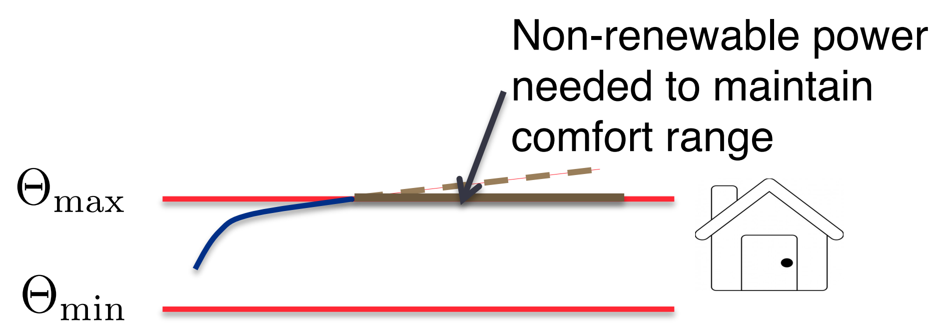
- Lesser scope for demand response



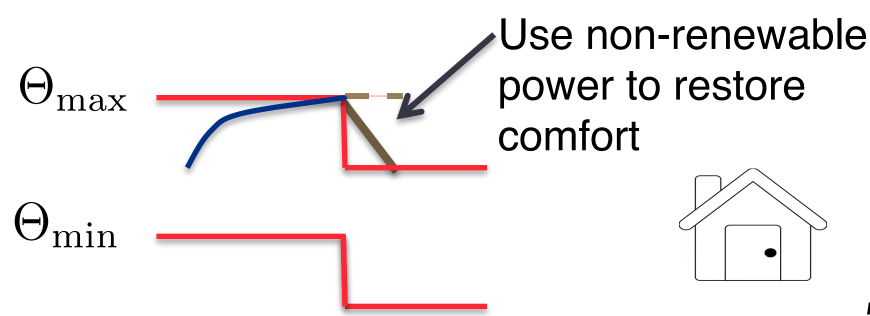
Renewable power is not enough to fully satisfy load requirements

- Renewables can help *reduce* need for non-renewables
- However, they *cannot eliminate* need for non-renewables

- Non-renewables still required
  - When wind stops blowing



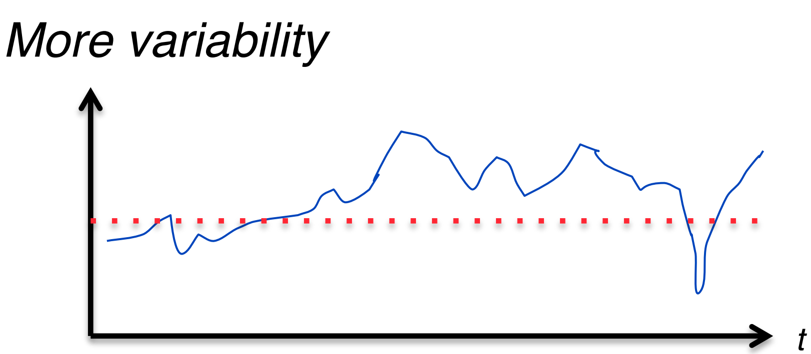
  - After sudden comfort-setting change

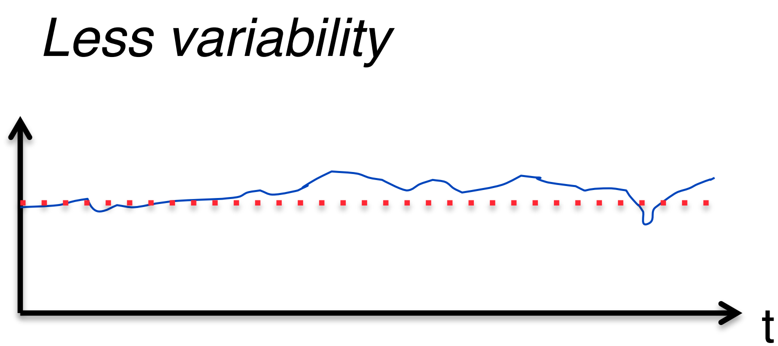


Reduce peak-to-average non-renewable power generation

- Non-renewables still required

- Need to reduce peak-to-average of non-renewable power





  - Reduce expensive spinning/other reserves, capital, etc

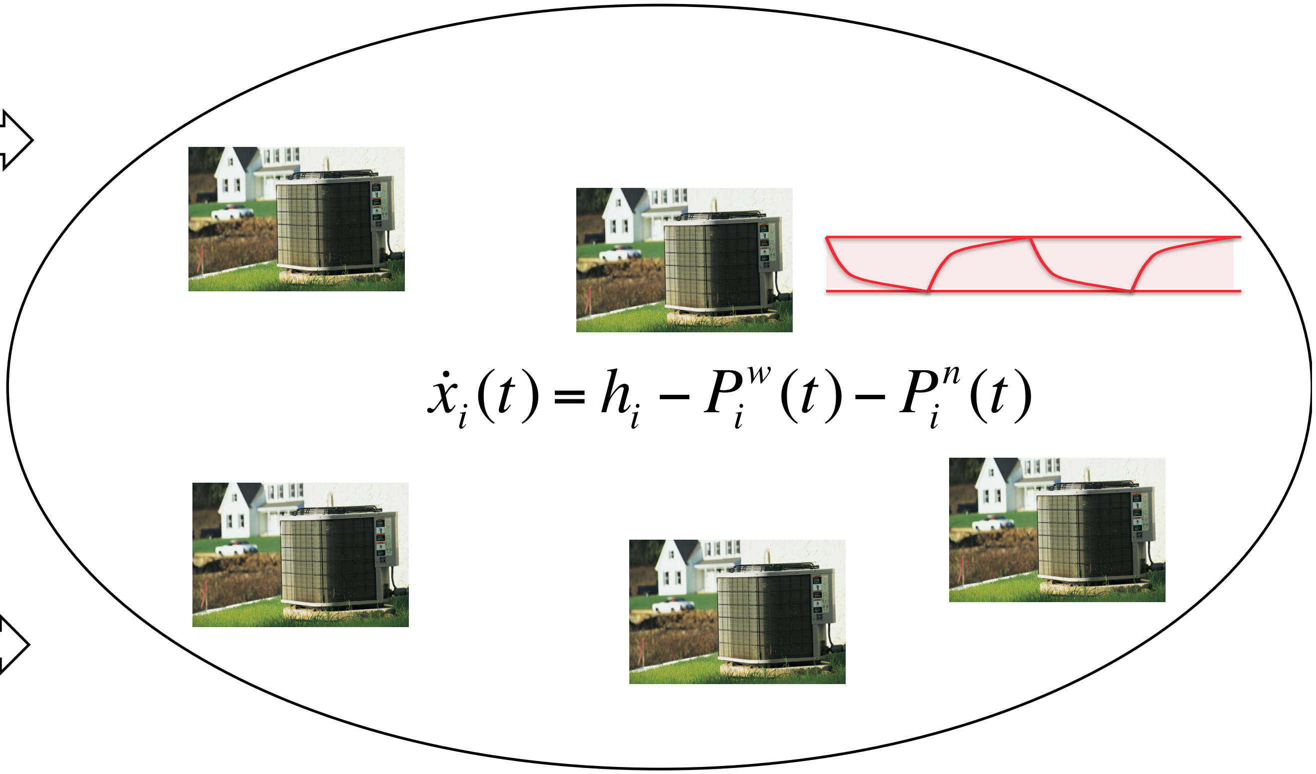
Concavity and desynchronization

A stochastic control problem

- Collection of loads

$P_{wind}(t)$

$P_{non-renewable}(t)$



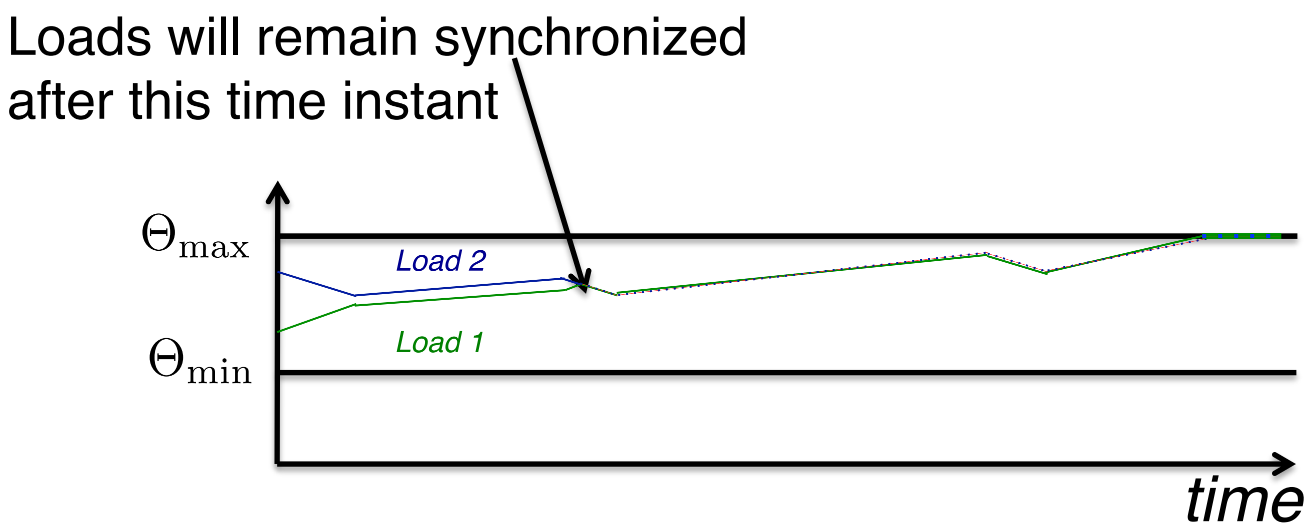
$$\dot{x}_i(t) = h_i - P_i^w(t) - P_i^n(t)$$

Stochastic control model

- Wind process $\sum P_i^w(t) \sim \text{Markov process}$
- Temperature dynamics $\dot{x}_i(t) = h_i - P_i^w(t) - P_i^n(t)$
- Non-renewable power $P_i^n(t) \geq 0$
- Temperature constraint $x_i(t) \in [\Theta_{\min}, \Theta_{\max}], \forall i$
- Quadratic cost to reduce variability $$\lim_{T\to\infty} \frac{1}{T} \int_0^T [\sum_i P_i^n(t)]^2 dt$$

Optimal solution: Synchronization

- Theorem: The optimal policy synchronizes loads

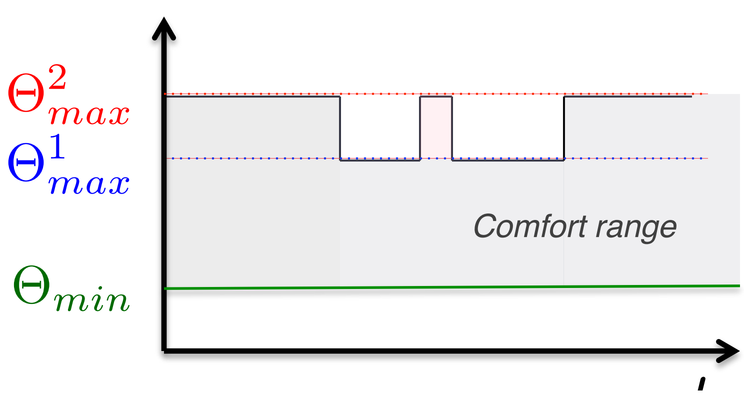


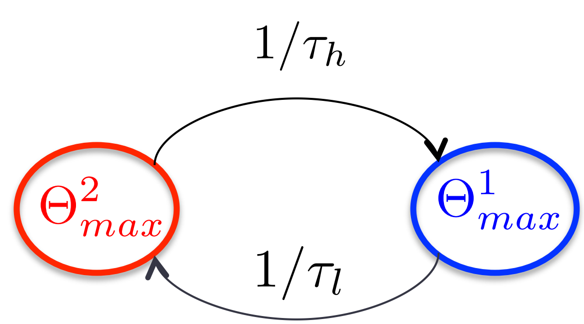
- Is there some modification in the model or cost function which leads to de-synchronization ?

Stochastic model for $\Theta_{\max}$

- Suppose users occasionally change Θ_{max} settings at the same time
  - E.g. Super Bowl Sundays @ game time

- Model changes in Θ_{max} as a two state Markov process





Resulting stochastic control problem

- Wind process: $\sum P_i^w(t) \sim \text{Markov process}$
- Temperature dynamics: $\dot{x}_i(t) = h_i - P_i^w(t) - P_i^n(t)$
- Non-renewable power $P_i^n(t) \geq 0$
- Stochastic comfort level $\Theta_{max}(t) \sim \text{Markov process}$, $\Theta_{max}(t) \in \{\Theta_{max}^1, \Theta_{max}^2\}$
- Temperature constraint: $x_i(t) \in [\Theta_{min}, \Theta_{max}^2], \forall i$
- Maximum cooling rate: $P_i^n(t) = M \text{ If } x_i(t) > \Theta_{max}(t)$
- Quadratic cost: $\lim_{T\to\infty} \frac{1}{T} \int_0^T [\sum_i P_i^n(t)]^2 dt$

HJB equation and optimal solution

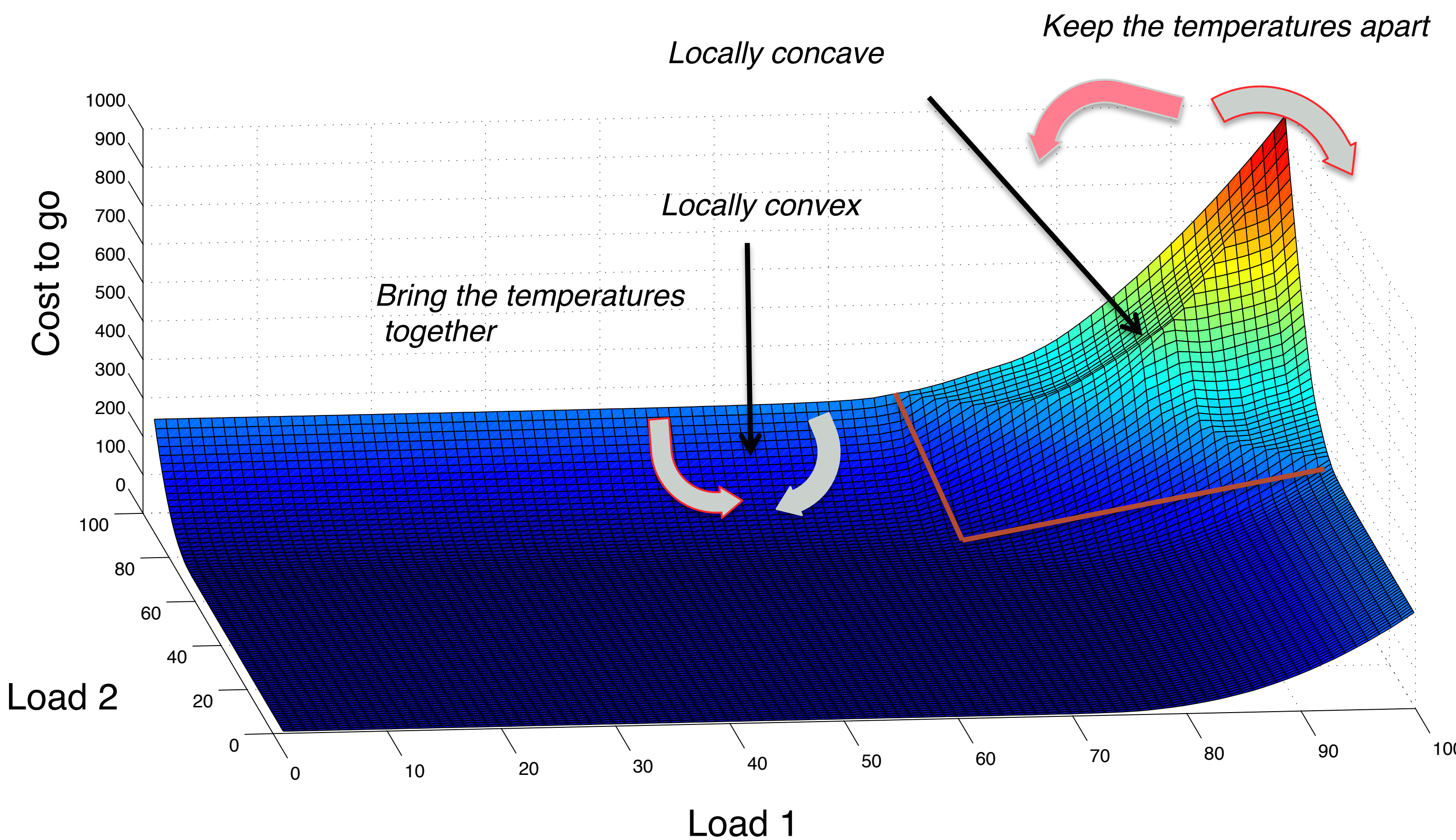
- Cost to go function $V^{ij}(x,t) := \min_{P_i^n, P_i^w \in U} \mathbb{E}\left[\int_t^T (P_1^n + P_2^n)^2 | w(t) = i, th(t) = j, x(t) = x\right]$

- HJB equation
$$\min_{P_1^n, P_2^n \in U}\{(P_1^n + P_2^n)^2 - \frac{\partial V^{ij}}{\partial x_1} P_1^n - \frac{\partial V^{ij}}{\partial x_2} P_2^n\} - \max_{P_1^w, P_2^w \in U}\{\frac{\partial V^{ij}}{\partial x_1} P_1^w + \frac{\partial V^{ij}}{\partial x_2} P_2^w\}\chi_{\{i=1\}}$$
$$= q_{ii'}(V^{ij} - V^{i'j}) + q_{jj'}(V^{ij} - V^{ij'}) - h_i(V_{x1}^{ij} + V_{x2}^{ij}) - \dot{V}^{ij}$$

- Optimal Solution
$$(\overset{\star}{P}{}_1^w(\vec{x}, j), \overset{\star}{P}{}_2^w(\vec{x}, j)) = \begin{cases} (\text{W,0}) & \text{if } \frac{\partial V_{1j}^*}{\partial x_1} > \frac{\partial V_{1j}^*}{\partial x_2} \\ (\text{0,W}) & \text{if } \frac{\partial V_{1j}^*}{\partial x_1} < \frac{\partial V_{1j}^*}{\partial x_2} \end{cases}$$
$$(\overset{\star}{P}{}_1^n(\vec{x}, i, j), \overset{\star}{P}{}_2^n(\vec{x}, i, j)) = \begin{cases} (\frac{1}{2}\frac{\partial V_{ij}^*}{\partial x_1}(\vec{x}), 0) & \text{if } \frac{\partial V_{ij}^*}{\partial x_1} > \frac{\partial V_{ij}^*}{\partial x_2} \\ (0, \frac{1}{2}\frac{\partial V_{ij}^*}{\partial x_2}(\vec{x})) & \text{if } \frac{\partial V_{ij}^*}{\partial x_1} < \frac{\partial V_{ij}^*}{\partial x_2} \\ (\frac{1}{2}\frac{\partial V_{ij}^*}{\partial x_1}(\vec{x}), \frac{1}{2}\frac{\partial V_{ij}^*}{\partial x_2}(\vec{x})) & \text{if } \frac{\partial V_{ij}^*}{\partial x_1} = \frac{\partial V_{ij}^*}{\partial x_2} \end{cases}$$

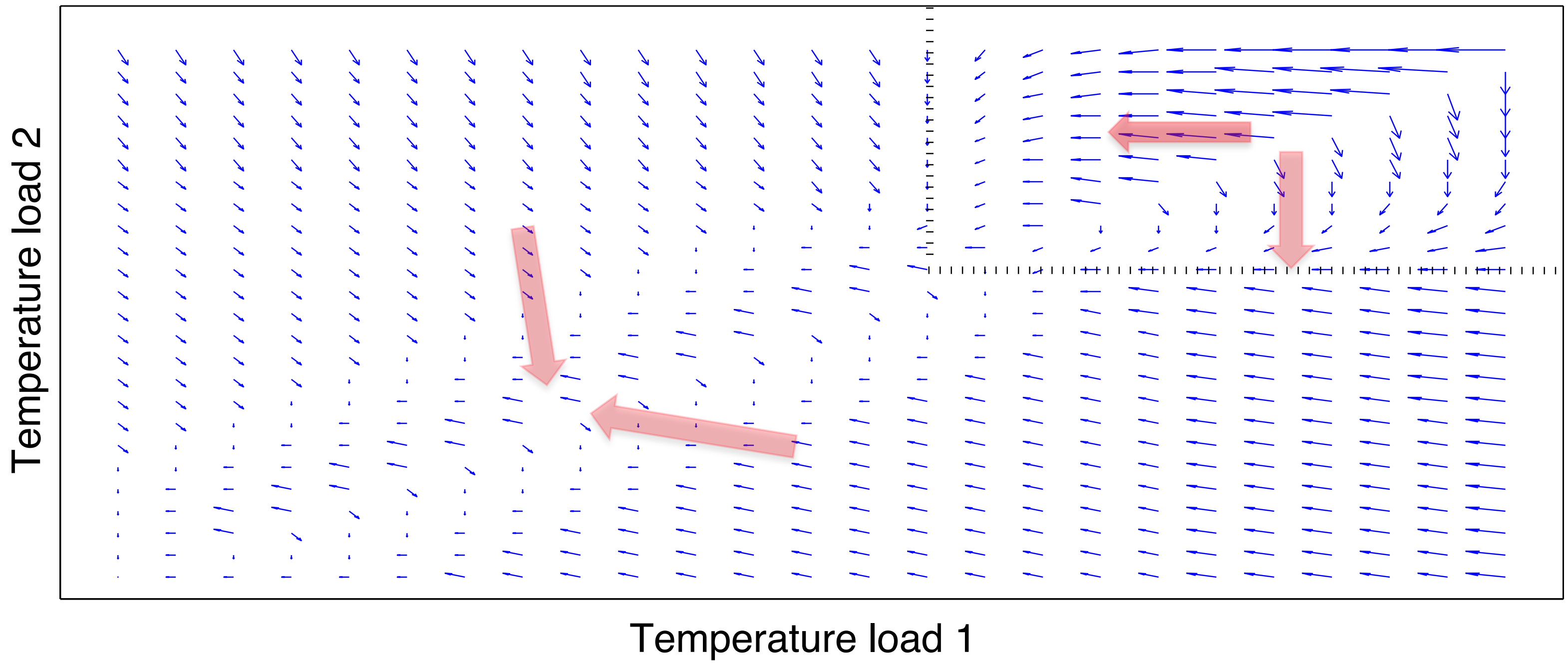
- Optimal power allocation depends upon $\frac{\partial V_{ij}^*}{\partial x_1} \lesseqgtr \frac{\partial V_{ij}^*}{\partial x_2}$ when $x_1 \leq x_2$

Local concavity in stochastic $\Theta_{\max}$ variational model



Optimal solution for stochastic $\Theta_{\max}$ variation model

- Nature of the optimal solution

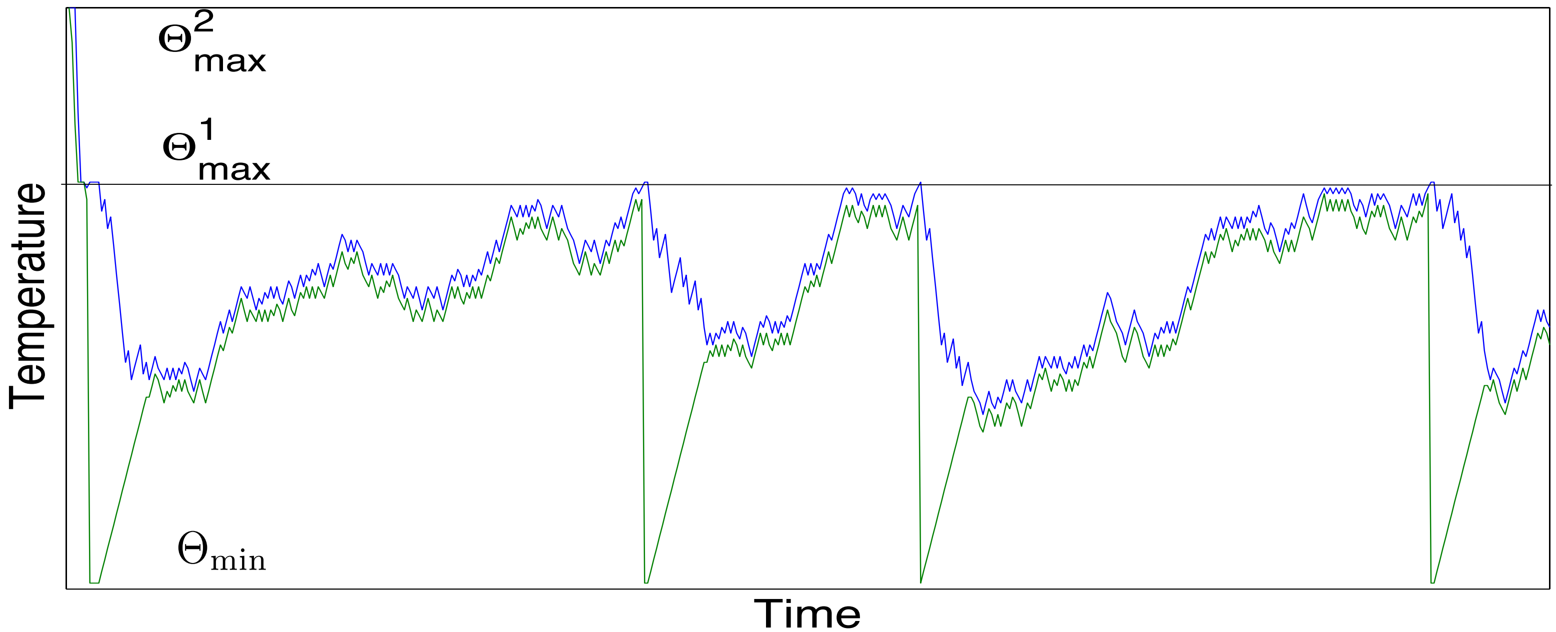


Vector field of temperature changes

- De-synchronization at high temperatures
- Re-synchronization at low temperatures

De-synchronization/Re-synchronization in solution

- It is optimal to separate at high temperatures



  - Hedges against the future eventuality that the thermostats are switched low

Issues in designing an architecture and solution for demand response

Need for demand side and supply side information exchange

- Loads need to know *when* to invoke demand response

- Supply side needs to know *how much* demand response will provide

- Need for two-way communication between demand side and supply side
  - Volume of data
  - Delay requirements of data

Need to respect privacy

- How to control demand without intrusive sensing of temperatures of homes?

Need to reduce communication requirements

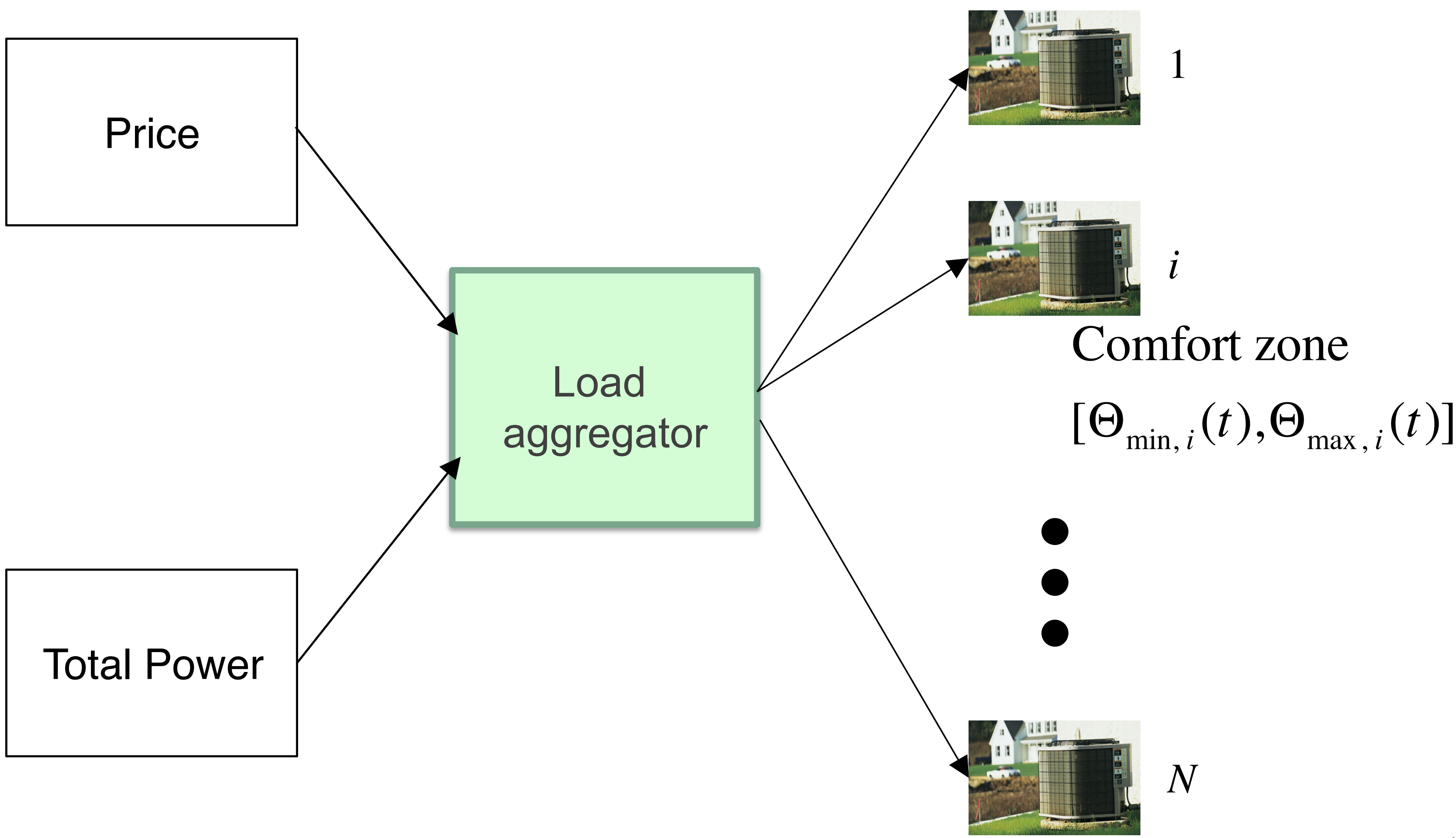
- How to minimize communication requirements for measurements and actuation signals?

Challenges

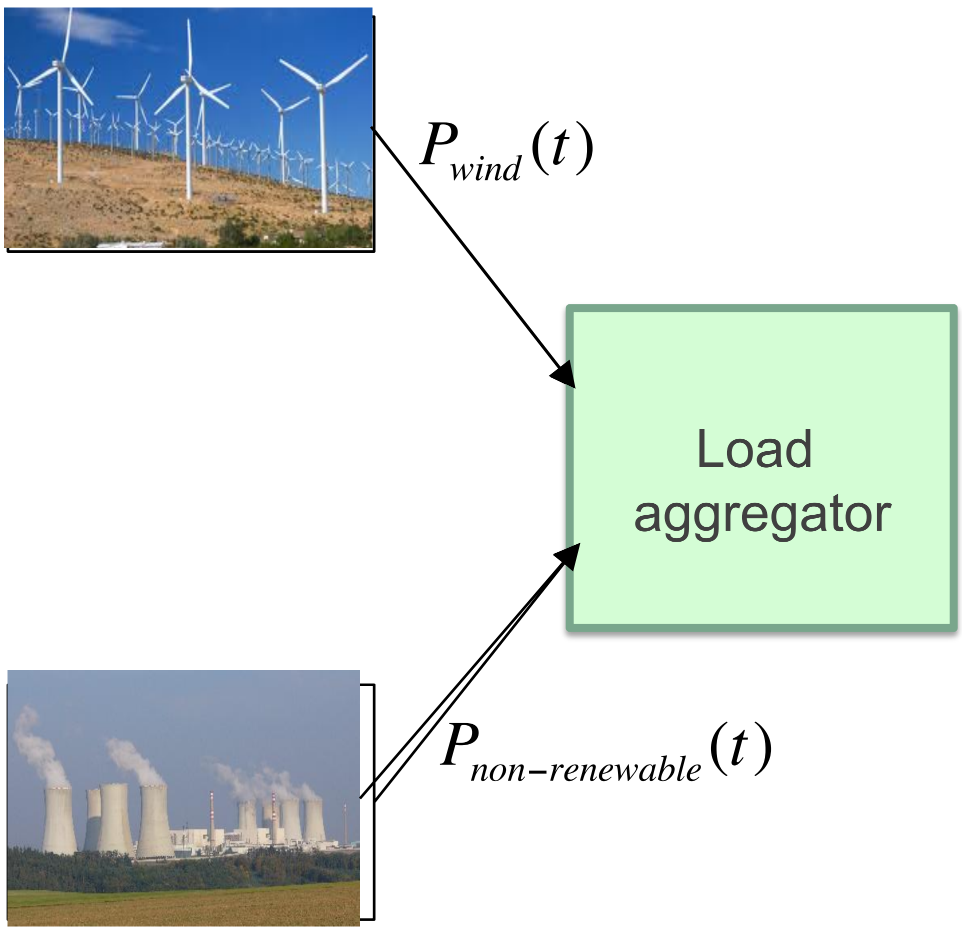
- Goals
  - Maximize utilization of renewable energy
  - Minimize variability of non-renewable power required
  - Respect comfort constraints of homes
- Architecture
  - How to achieve demand *pooling*?
  - Respect privacy: No intrusive sensing
  - Minimize communication requirements
    - Volume and latency of data
- Solution
  - "Optimal" – efficient in some sense
  - Computationally tractable for large number of homes

Architecture of the solution

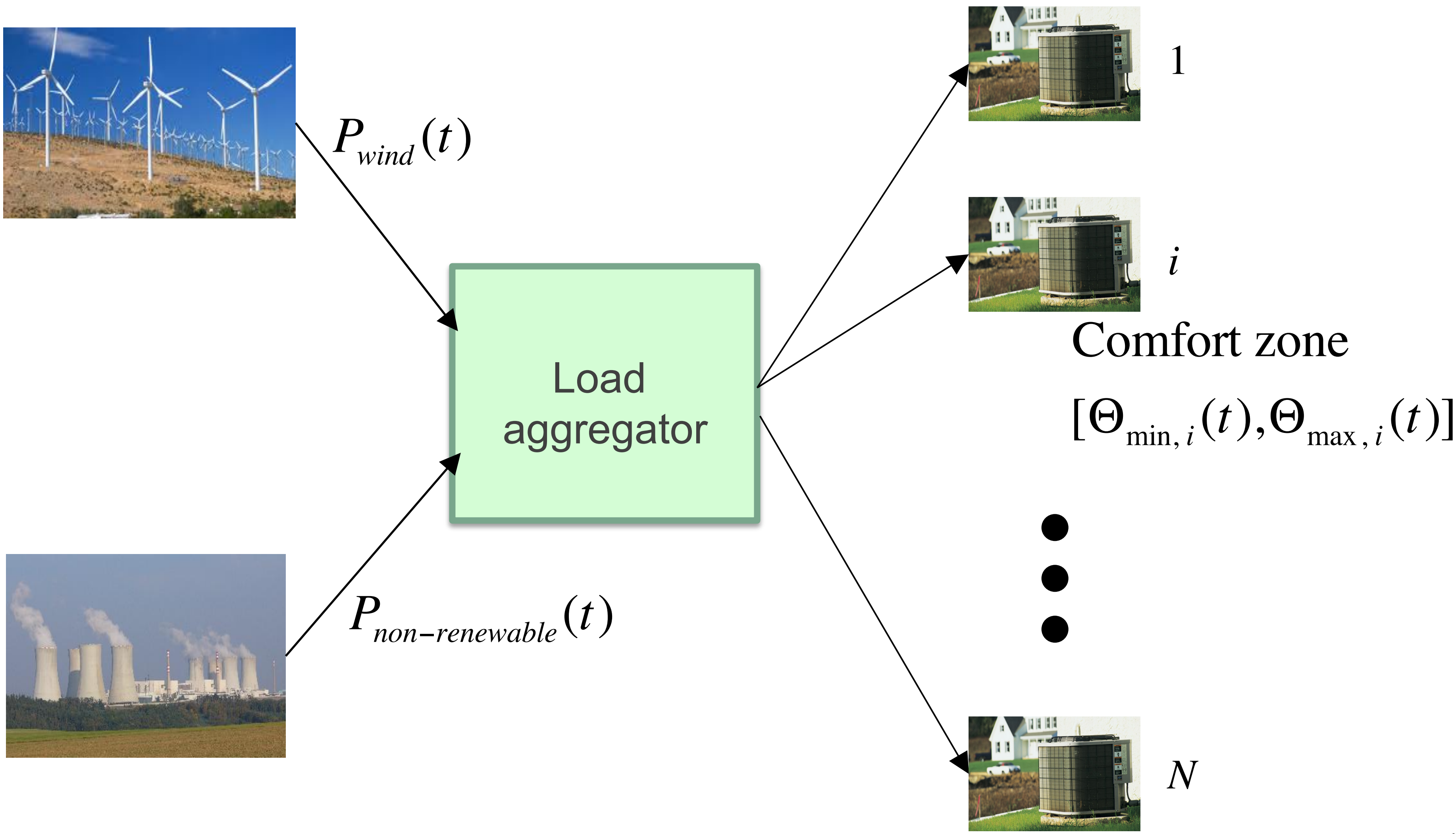
Load aggregator: Price based aggregation



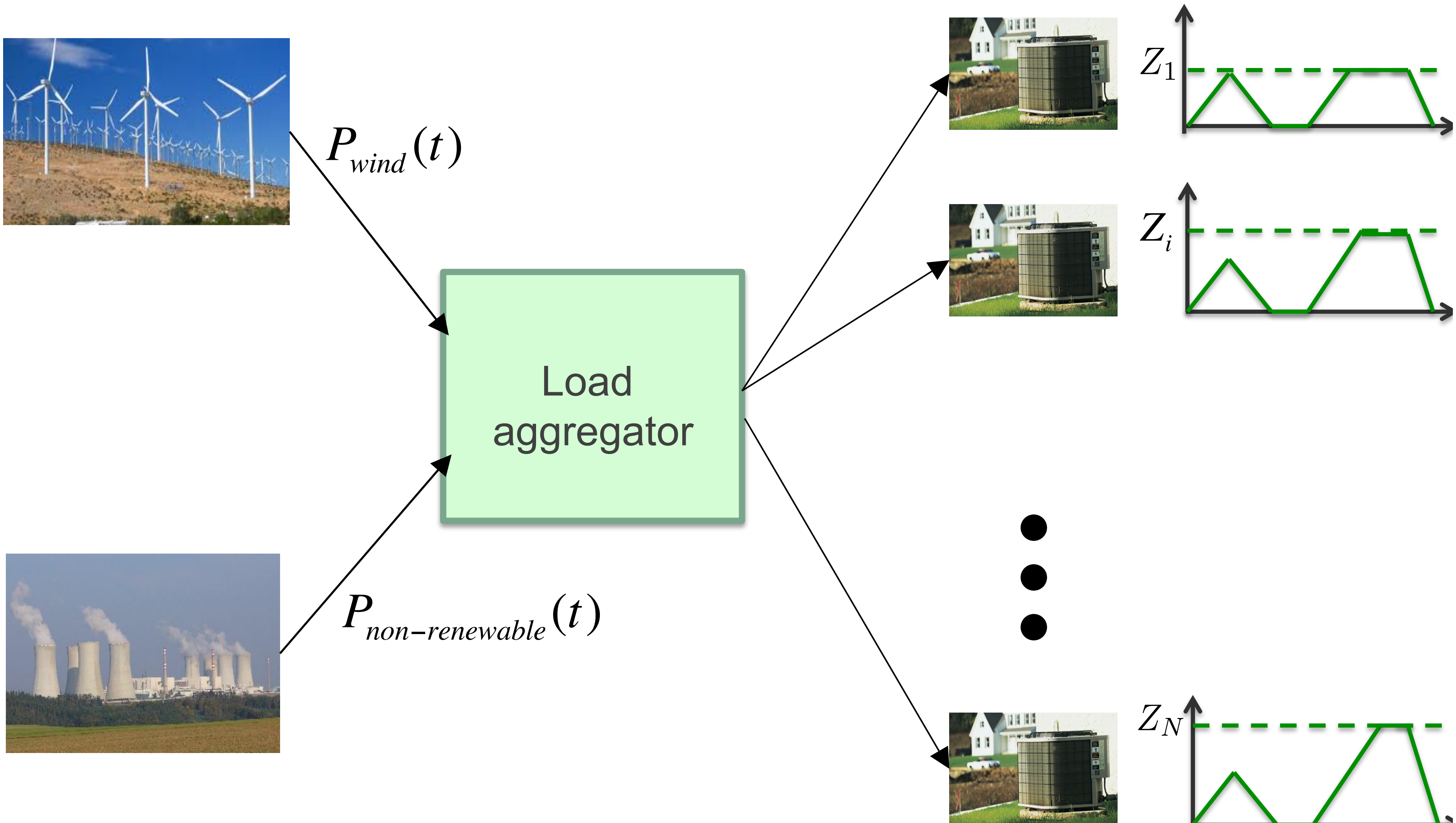
Load aggregator: Price based aggregation



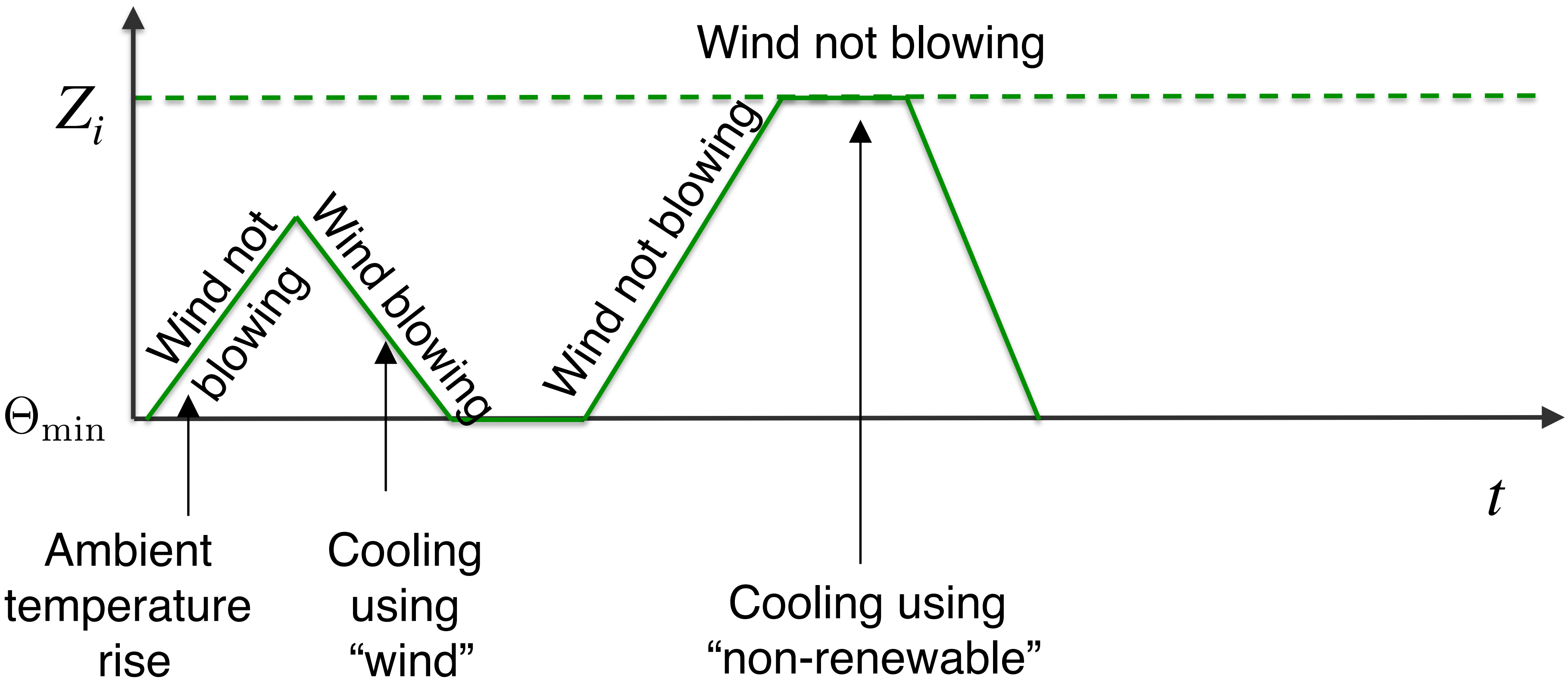
Load aggregator: Microgrid with renewable energy supply



Thermostatic control with set points Z_i



Thermostatic set-point based control policy

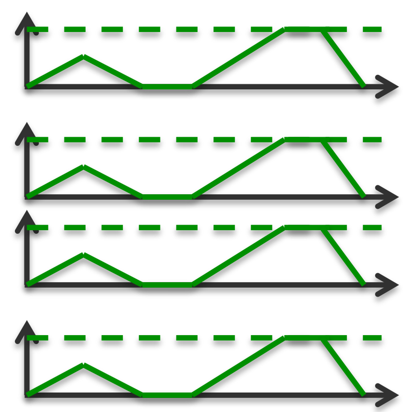


$$\dot{x} = h \qquad \dot{x} = -c \qquad \dot{x} = 0$$

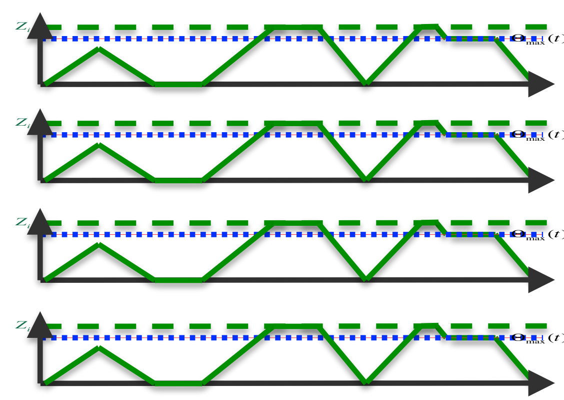
$$P^n = 0 \qquad P^w = (h+c) \qquad P^n = h$$

Problem: Synchronization of demand response

- Optimal solution: All users behave alike
- Loads synchronize and move in lock-step

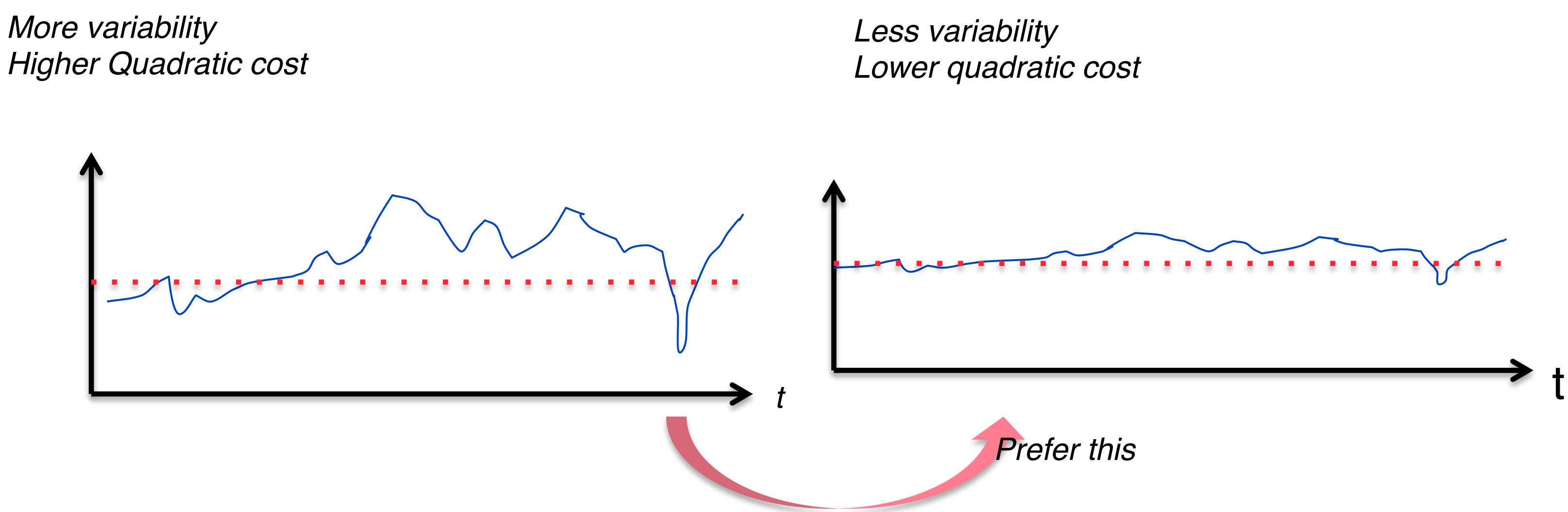


- Robustness problem: Suppose users change comfort level settings at same time
  - Super bowl Sundays @ game time
- Demand suddenly rises, causing large peak in nonrenewable power required
  - Model cost as $\lim_{T\to\infty}\frac{1}{T}\int_0^T \left(P^n(t)\right)^2 dt$



Reduce peak-to-average ratio of non-renewable power

- Low variability in non-renewable power consumption is desired

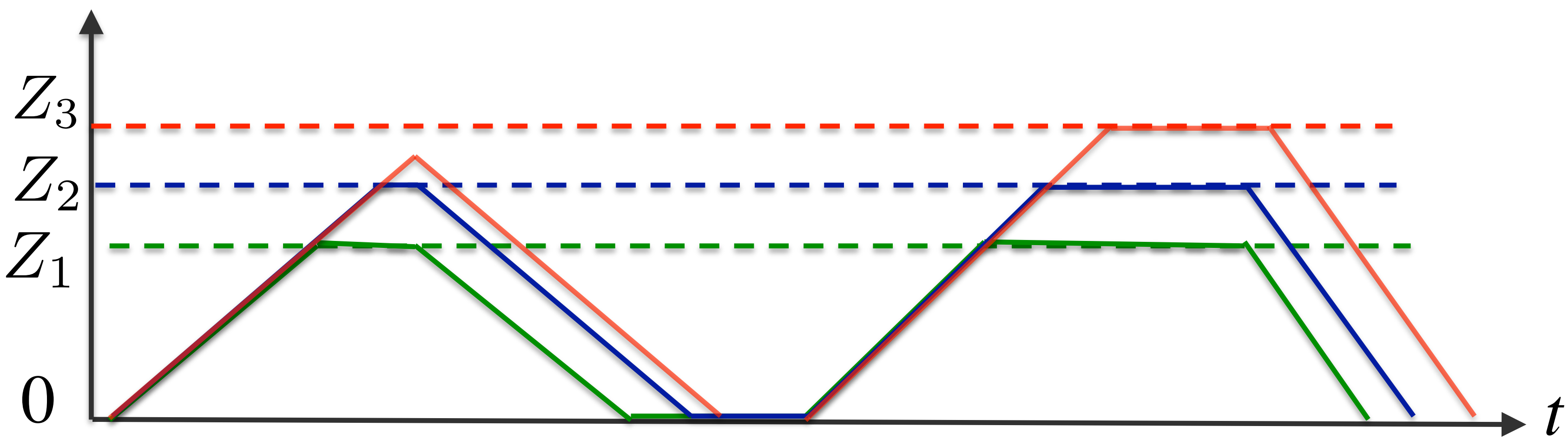


  - Lowers operating reserve requirements

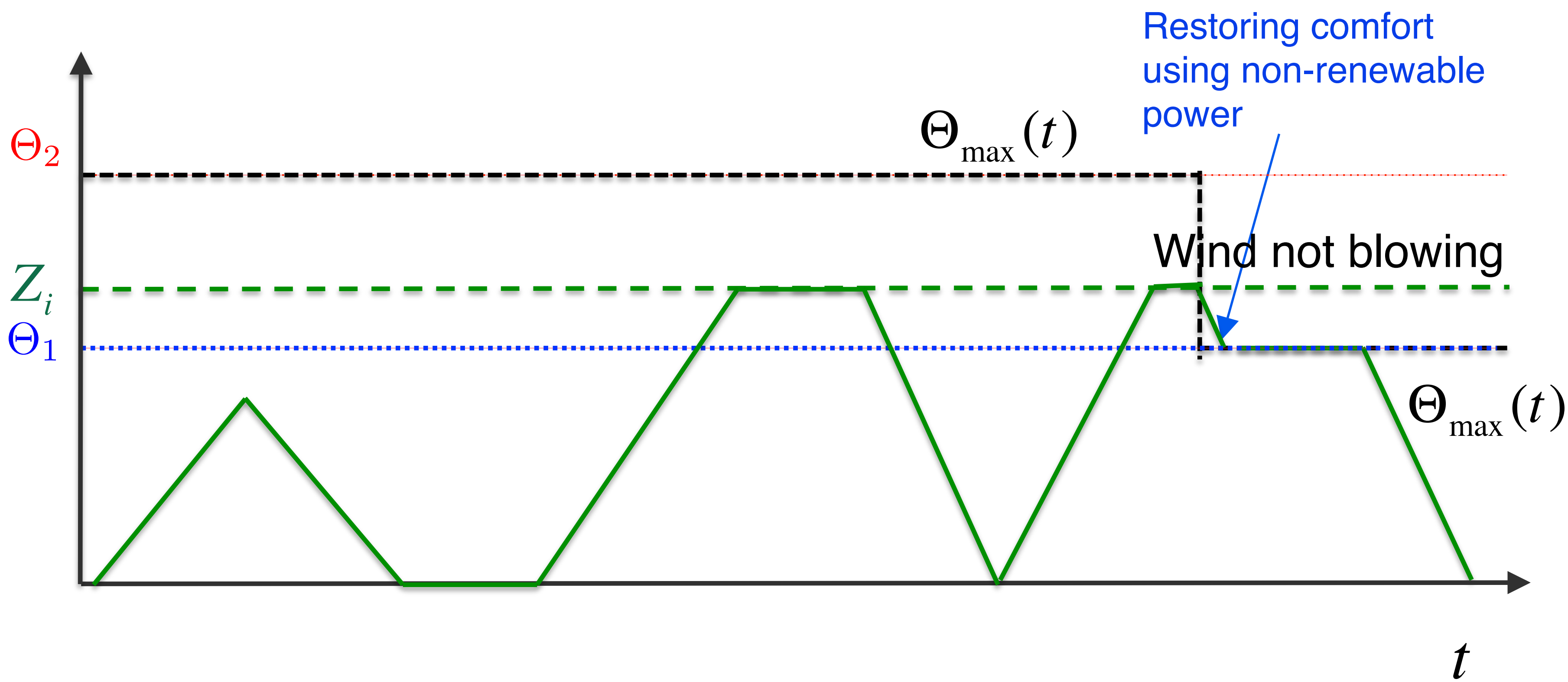
- Impose a quadratic cost on non-renewable power usage $\int P^2_{\text{non-renewable}}(t)dt$

Staggered set-points

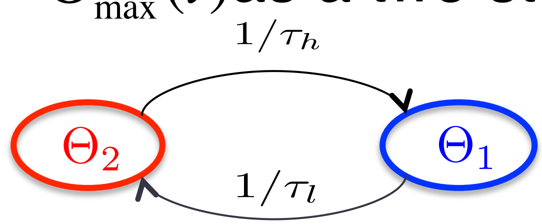
- De-synchronize load behaviors
- Choose different set-points $(Z_1, Z_2, ..., Z_N)$ for different loads



Discomfort: Maximum cooling when comfort range is violated



- Model changes in $\Theta_{max}(t)$ as a two state Markov process



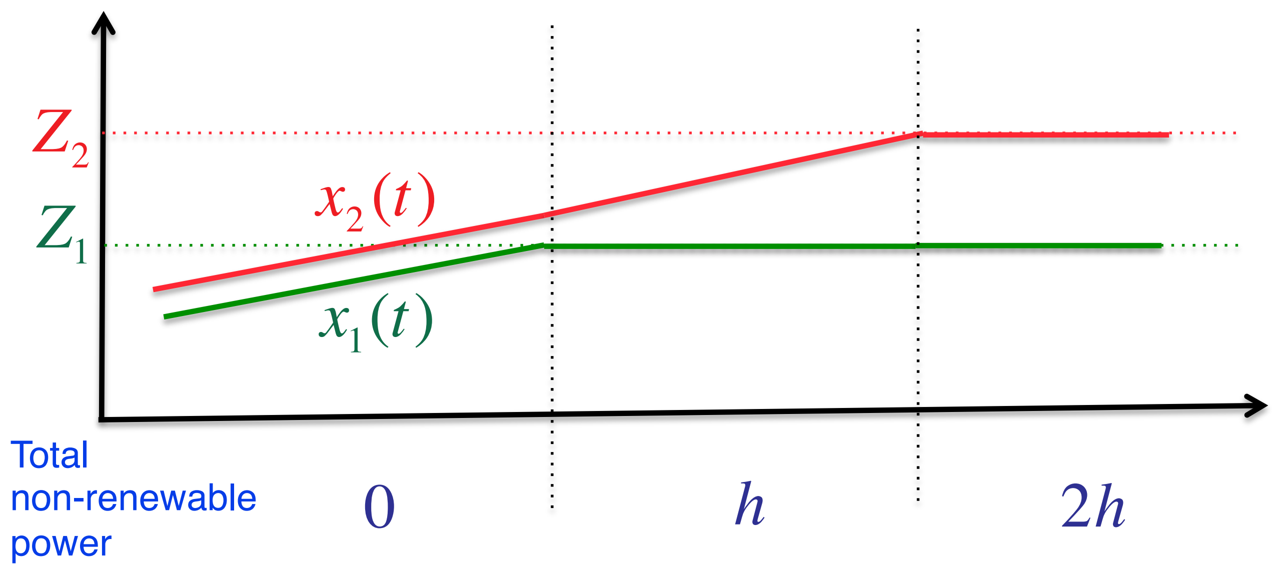
Stochastic optimization problem for $\{Z_1, Z_2, \ldots, Z_N\}$

- Stochastic wind process: $P^w(t)$
- Temperature dynamics: $\dot{x}_i(t) = h - P_i(t)$

  $P_i(t) = P_i^w(t) + P_i^n(t)$
- Comfort specification: $\dot{x}_i(t) \in [0, \Theta_{\max}(t)]$
- Robustness model: Stochastic process $\Theta_{\max}(t)$
- Set-point control: $P_i^n(t) = \begin{cases} h \text{ if } x_i(t) = Min(Z_i, \Theta_{\max}(t)) \\ 0 \text{ otherwise} \end{cases}$
- Cost: $C_N = \underbrace{\lim_{T\to\infty} \frac{1}{T}\int_0^T \left(P^n(t)\right)^2 dt}_{\text{Variation}} + \underbrace{\gamma_N \sum_{i=1}^{N} \left((x_i(t) - \Theta_{\max}(t))^+\right)^2 dt}_{\text{Discomfort}}$

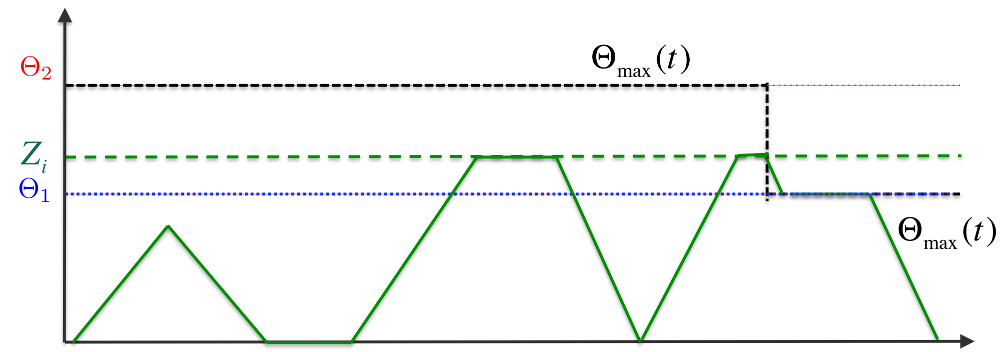
Evaluating the cost: Stochastic coupling

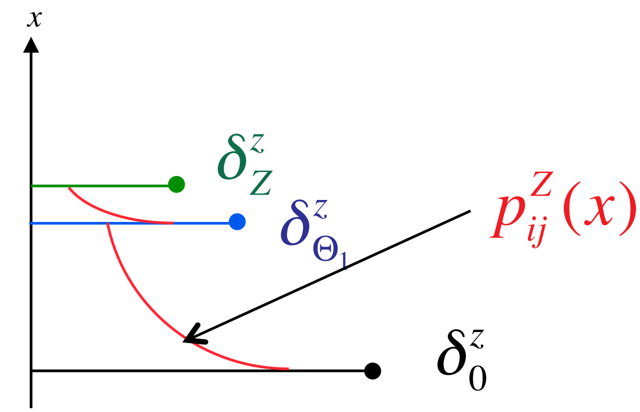
- Evaluation of cost $\lim_{T\to\infty} \frac{1}{T}\int_0^T (\sum_{i=1}^{N} P_i^n)^2 dx$ is difficult
- Needs N-dimensional joint probability distribution of temperature states $(x_1, x_2, \ldots, x_N)$
- Can use stochastic coupling to solve this



$$\underbrace{\begin{aligned}&\mathrm{Prob}(P_1^n(t) + P_2^n(t) = 2h)\\ &= \mathrm{Prob}(x_2(t) = Z_2)\end{aligned}}_{\text{Needs only marginal distribution of } x_2(t)}$$

The marginal probability distribution of a load





$$\frac{d\mathbf{p}^z(x)}{dx} = \begin{bmatrix} -\frac{q_0+r_0}{k(x)} & \frac{r_1}{k(x)} & \frac{q_1}{k(x)} & 0 \\ \frac{q_0}{h} & -\frac{q_0+r1}{h} & 0 & \frac{q_1}{h} \\ -\frac{q_0}{c} & 0 & \frac{q_1+r_1}{c} & -\frac{r_1}{c} \\ 0 & -\frac{q_0}{c} & -\frac{r_0}{c} & \frac{q_1+r_1}{c} \end{bmatrix} \mathbf{p}^z(x).$$

vhere $k(x) = \begin{cases} h & x < \Theta_1 \\ -c & x > \Theta_1 \end{cases}$. The boundary conditions are

$$\begin{bmatrix} h/q_1 & h/q_1 & 0 & 0 \\ 0 & 0 & c/q_1 & c/q1 \end{bmatrix} \mathbf{p}^z(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \delta_0,$$

$$\begin{bmatrix} \cdot r_0 & 0 & 0 & 0 & \frac{h}{q_0+r_0} & 0 & 0 & 0 \\ & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ & -\frac{c}{q_0} & 0 & 0 & 0 & \frac{c}{q_0} & 0 & 0 \\ & \frac{h}{r_0} & 0 & 0 & 0 & -\frac{h}{r_0} & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{p}^z(\Theta_1-) \\ \mathbf{p}^z(\Theta_1+) \end{bmatrix} = \begin{bmatrix} \delta_{\Theta_1}^z \\ 0 \\ \delta_{\Theta_1}^z \\ \delta_{\Theta_1}^z \end{bmatrix},$$

$$\begin{bmatrix} \frac{c}{r_1} & 0 & 0 & 0 \\ 0 & \frac{h}{q_0+r_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{c}{q_0} \end{bmatrix} \mathbf{p}^z(z) = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \delta_z^z,$$

$$\int_0^z (\mathbf{1}^T \mathbf{p}^z(x))dx + \delta_0^z + \delta_z^z + \delta_{\Theta_1}^z = 1,$$

$$\int_0^z p_{01}^z(x)dx + \delta_z^z = \frac{q_1 r_0}{(q_1+q_0)(r_1+r_0)}.$$

Marginal probability distribution can be determined through solution of linear system equations

The optimization problem for a finite number of loads

- Minimize

$$C^N(Z_1,\ldots,Z_N) = \sum(\text{Power level})^2 \times \text{Prob(Power level)} + \gamma_N \sum \text{Expected Discomfort}$$

- Subject to

$$0 \le Z_1 \le Z_2 \ldots \le Z_N \le \Theta_2$$

- Difficult
  - High dimensional when N is large
  - Complex
  - Need to solve different problems for different N's

Continuum limit as $N \to \infty$.

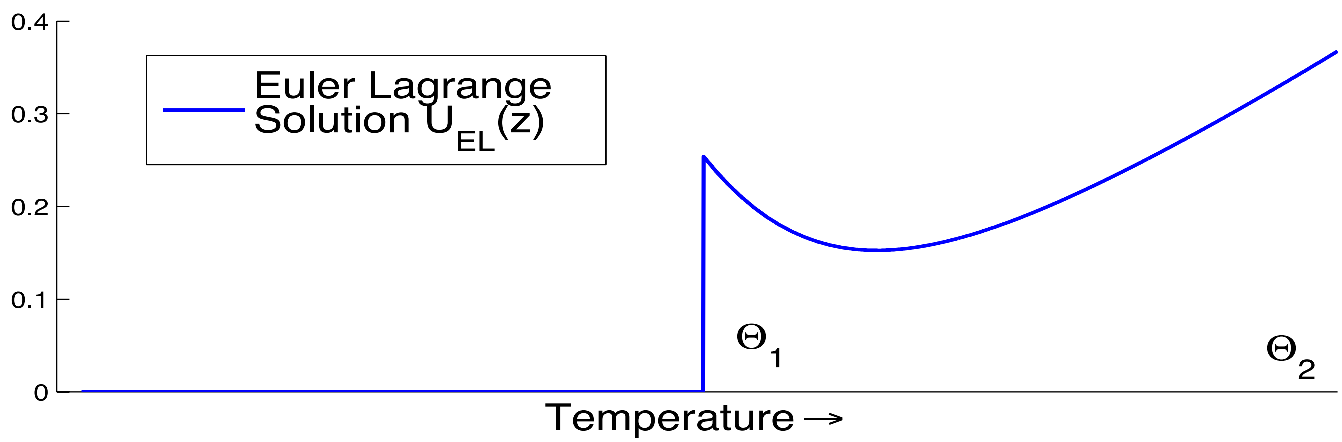
- Solution
  - Study asymptotic limit as $N \to \infty$.
  - Consider Set of loads = $[0,1]$
  - Can solve using analytical methods
    - Pontryagin Minimum Principle
  - Solution is explicit!
  - Also asymptotic solution is also nearly optimal even for small N!
  - Essentially this solves the problem for all N's

Difficulty with Euler Lagrange method

- Calculus of variation problem $J[u] = \int_0^{\Theta_2} F(u, u', z)dz$
  - Euler-Lagrange solution

$$u_{EL}(z) = \frac{\gamma\Phi'(z) + 2c(c+h)D_2(z)}{2(h^2 D_1(z) + c^2 D_2(z))}$$

- This is not an increasing function, and does not satisfy boundary condition



Optimal solution via Pontryagin's minimum principle

- Use Pontryagin's Minimum principle

Control $v(z)$

State (non-decreasing): $\dfrac{d}{dz}u(z) = f(u, v, z) = v^2(z) \geq 0$

Hamiltonian: $H = (u(z) - u_{EL}(z))^2 w(z) + \lambda(z) v^2(z)$

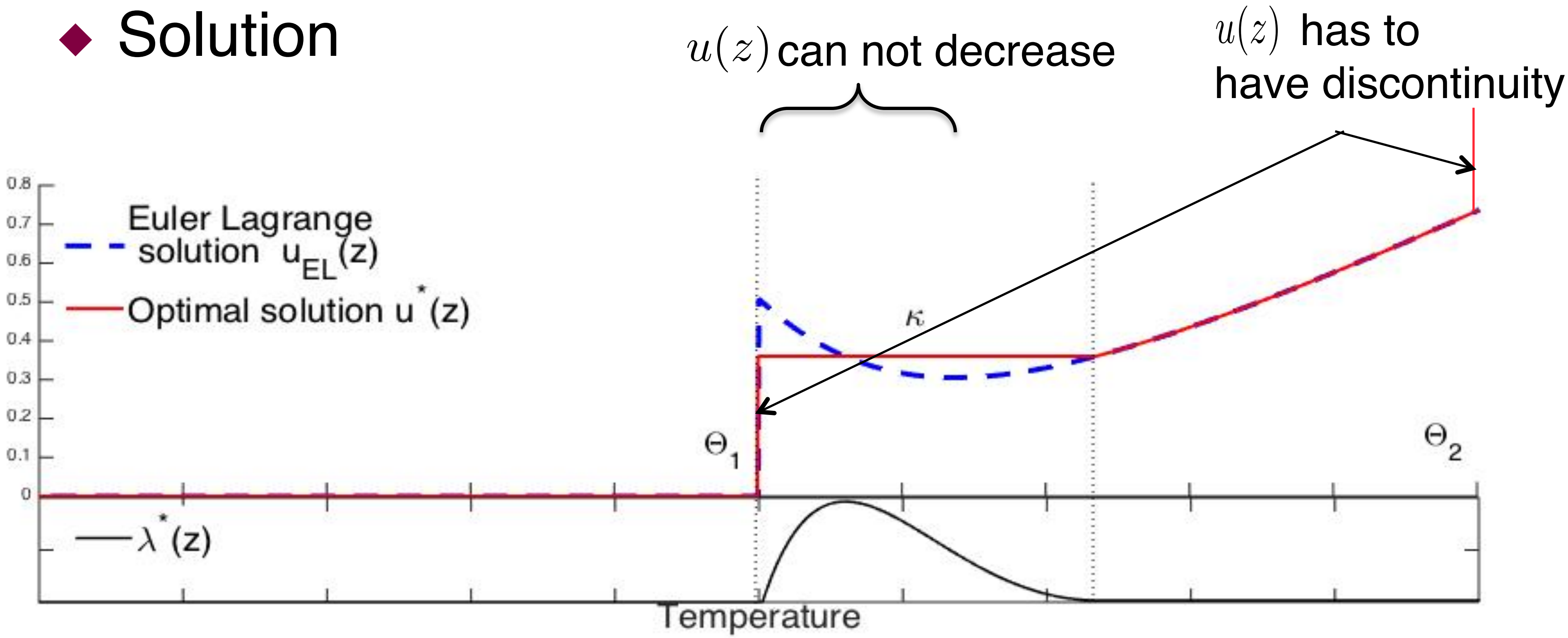
Necessary conditions:

$$\frac{d}{dz}\lambda(z) = -2(u(z) - u_{EL}(z))w(z)$$

$$v(t) = arg \min_{v \geq 0} \left[(u(z) - u_{EL}(z))^2 w(z) + \lambda(z) v^2(z) \right]$$

Optimal solution via Pontryagin's minimum principle

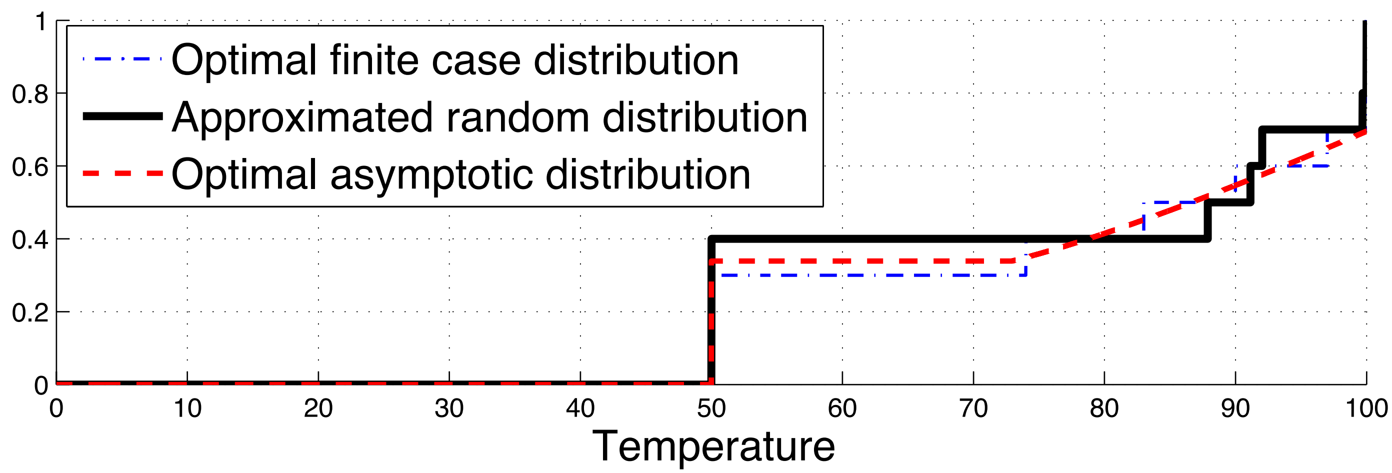
- Solution



- This gives the optimal staggering of set points

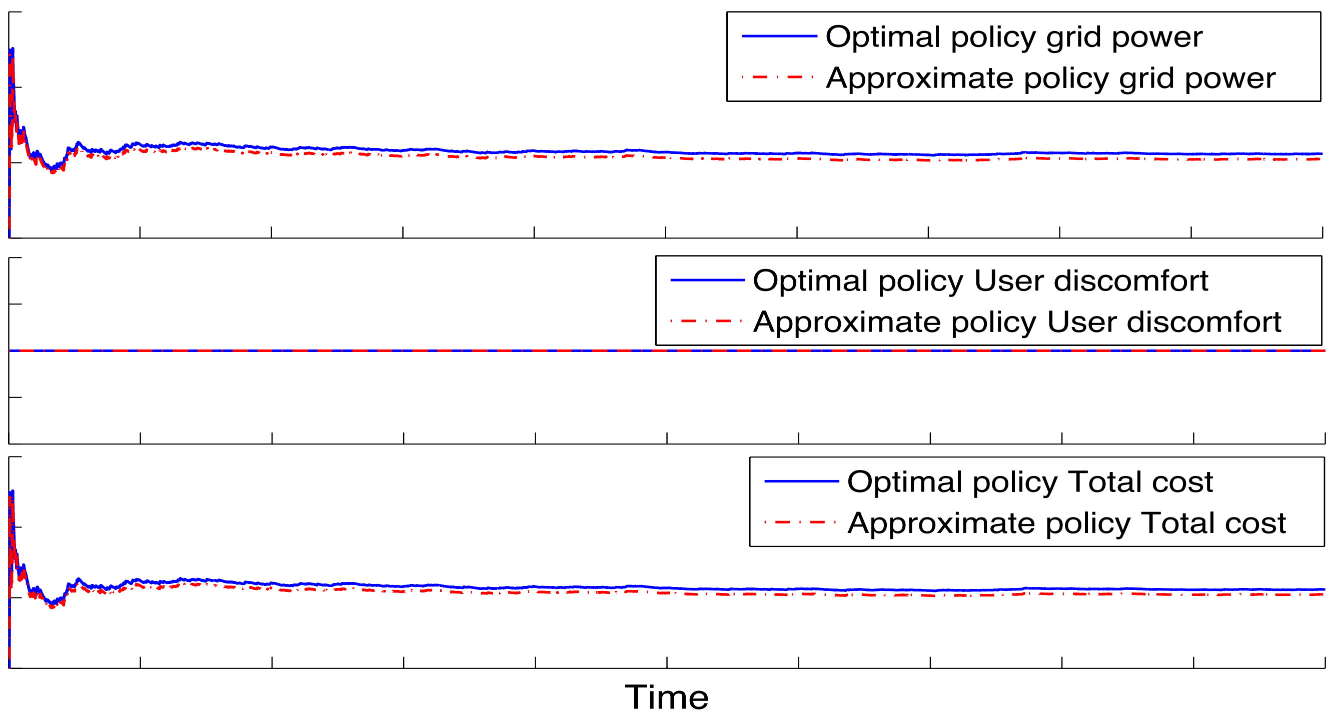
Solving for finite N: Approximation to continuum limit

- We can generate $\{Z_i\}_1^N$ according to continuum limit distribution, to approximate finite optimal distribution



Some simulation results

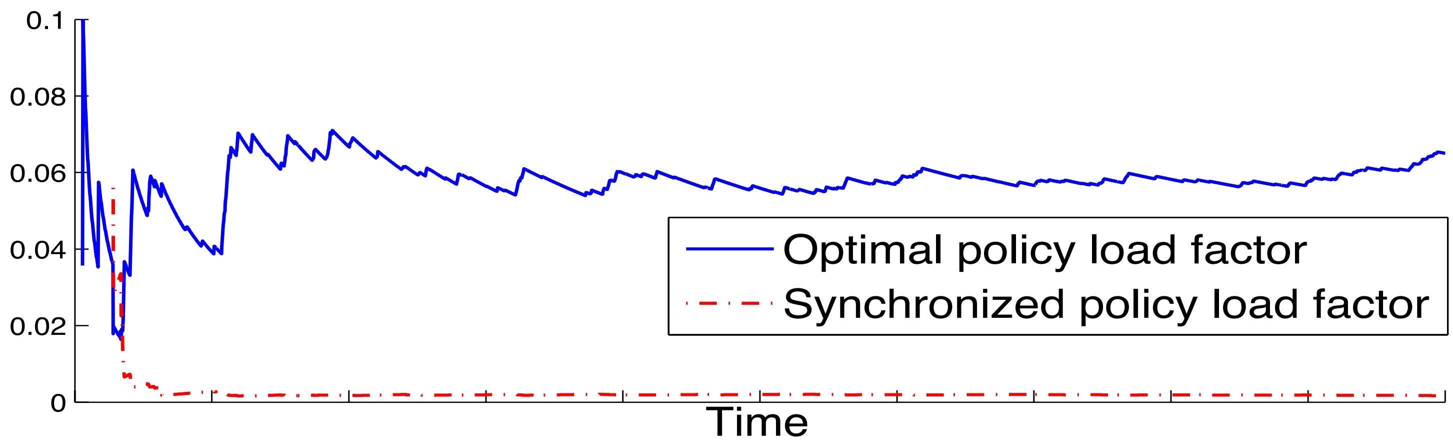
- The random generation method works reasonably well, even when N is small



Some simulation results - 2

$$\text{Load factor} = \frac{\text{Average power}}{\text{Peak power}}$$

- Optimal policy has higher load factor than other naive policies



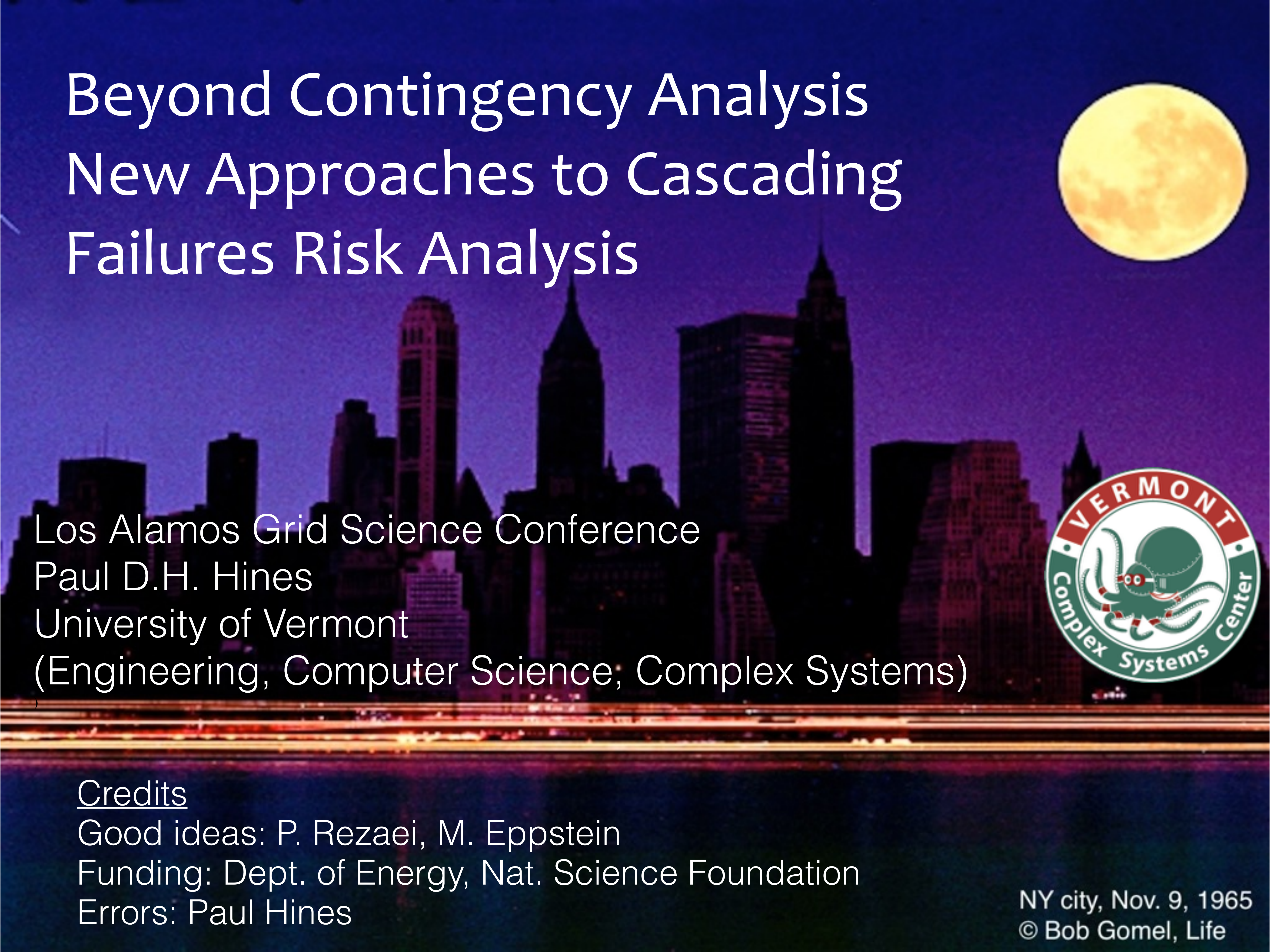
Concluding remarks

- Design and analysis of an architecture and a simple set-point policy
  - Is architecturally simple to implement
  - De-synchronizes the loads to lower non-renewable peak-to average
  - Alleviates privacy concerns
  - Simple to analyze, low communication requirement, decentralized control
- Many extensions are feasible
  - Response to comfort variations
  - Availability of wind power
  - Generalize wind model, temperature dynamics, etc.

Thank you

Beyond Contingency Analysis
New Approaches to Cascading Failures Risk Analysis

Los Alamos Grid Science Conference
Paul D.H. Hines
University of Vermont
(Engineering, Computer Science, Complex Systems)



NY city, Nov. 9, 1965
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Credits
Good ideas: P. Rezaei, M. Eppstein
Funding: Dept. of Energy, Nat. Science Foundation
Errors: Paul Hines

Contingency analysis

- N-1 security has been the core power systems operating principle for >50 years

- While it has served us well, it also has limitations:

    - Not all contingencies are equally **likely**.

    - Not all limit **violations** are equally important—some produce blackouts, others don't.

    - Sometimes components fail in **sets** (e.g., storms) or in **unexpected** ways (Aug. 14 2003 blackout).

    - **Binary**: Imperfect data (e.g., from neighboring areas) can change the apparent state of system from insecure to secure. (2011 SW blackout)

Beyond contingency analysis

- Valuable insight comes from contingency analysis, so replacing it would be unwise.
- However, operators need additional indicators of risk.
- Lots of ongoing work:
  PMU angle difference analysis, statistical indicators (variance, autocorrelation), energy function/Lyaponav methods, …
- Focus: Given a state estimator or day-ahead planning model, quantify ***and explain*** the risk posed by ***all*** potential cascading blackouts.

Beyond contingency analysis

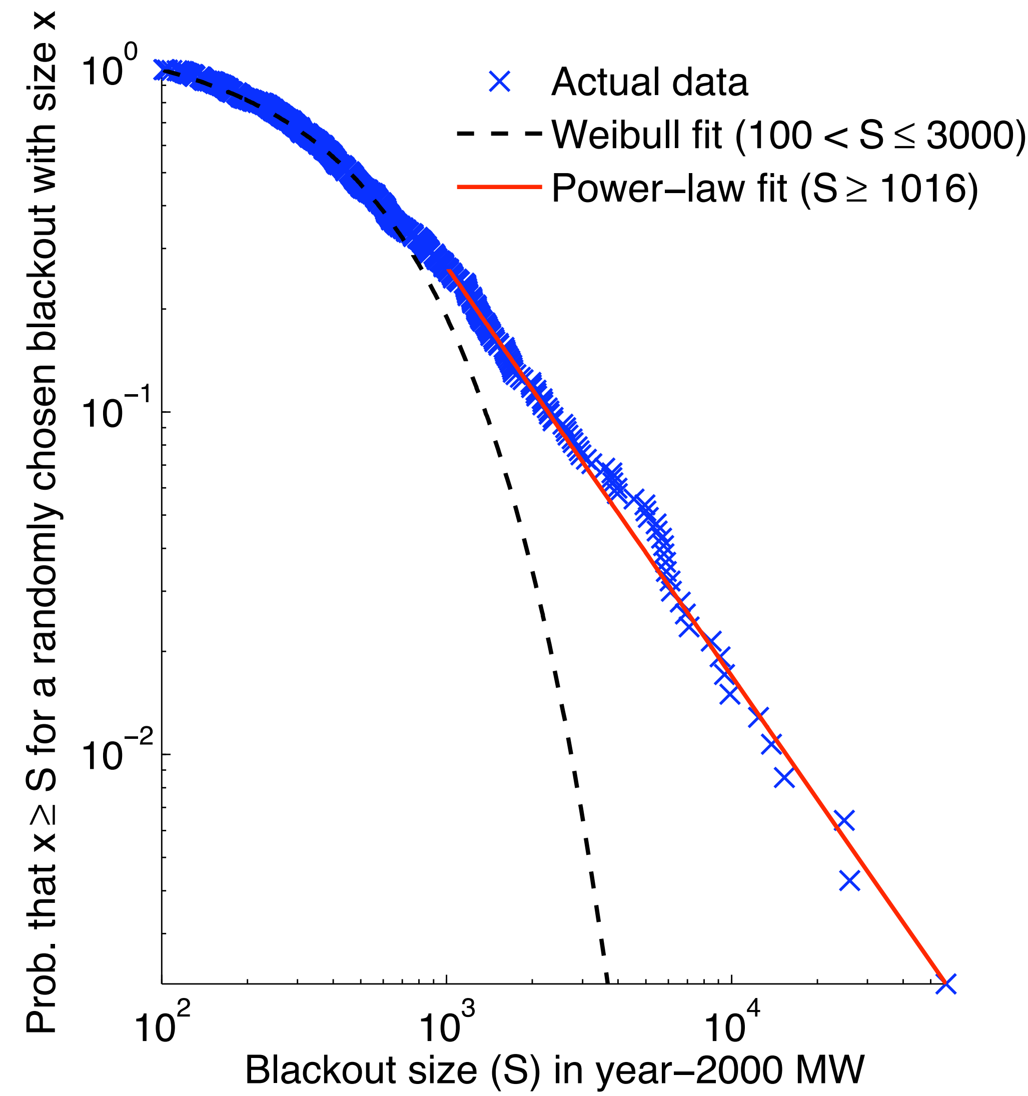
- Focus: Given a state estimator or day-ahead planning model, quantify ***and explain*** the risk posed by ***all*** potential cascading blackouts.

- Why this is hard:

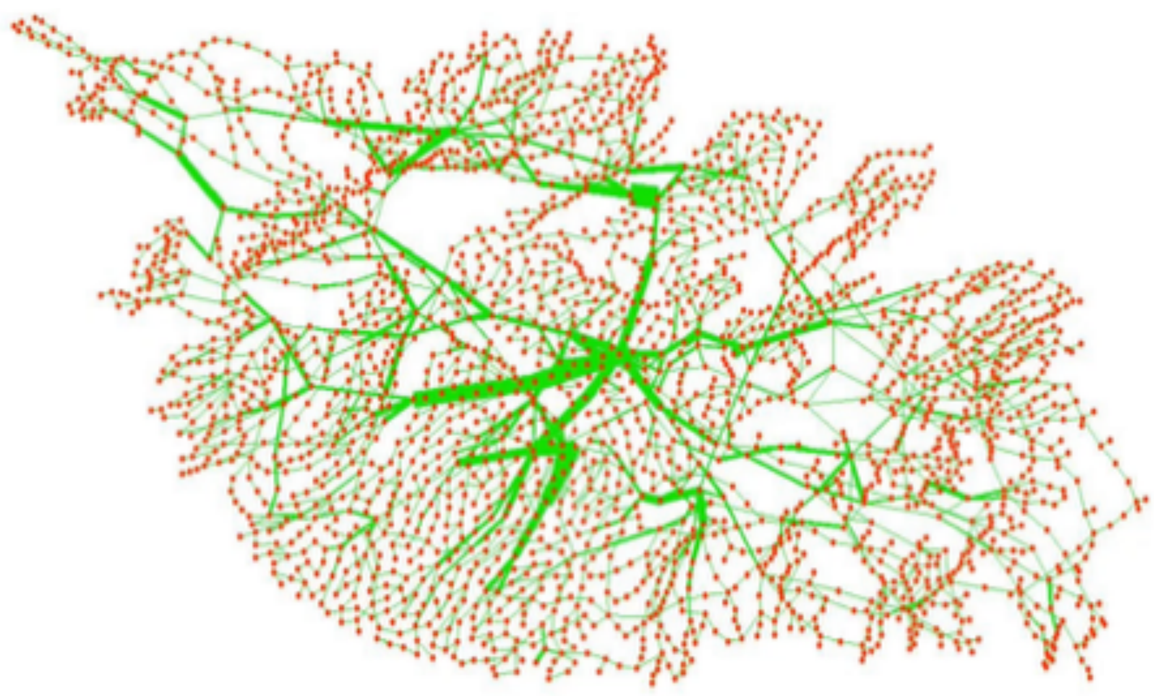
  - All n-1 contingencies and most n-{2,3,4}s do not cause blackouts. Many samples needed to find one blackout.

  - Power-law in blackout sizes means that we need many blackout simulations to describe the risk.

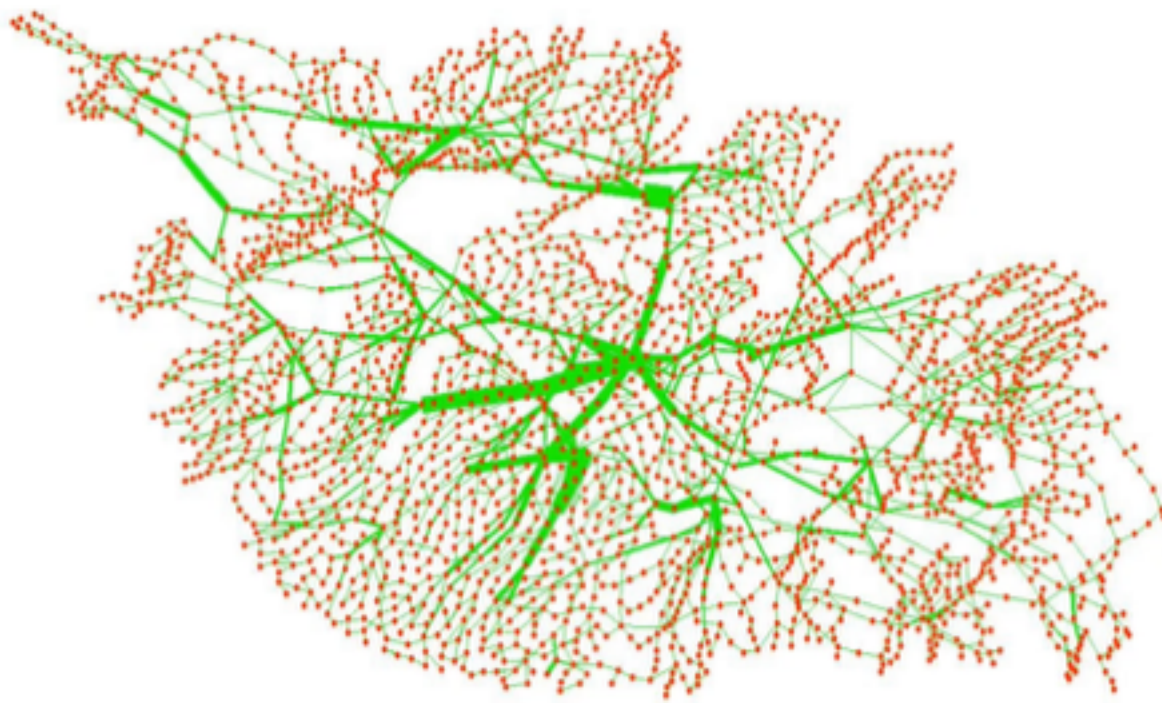
  - Explaining why is always difficult (but probably the most important thing we can do)



Illustration

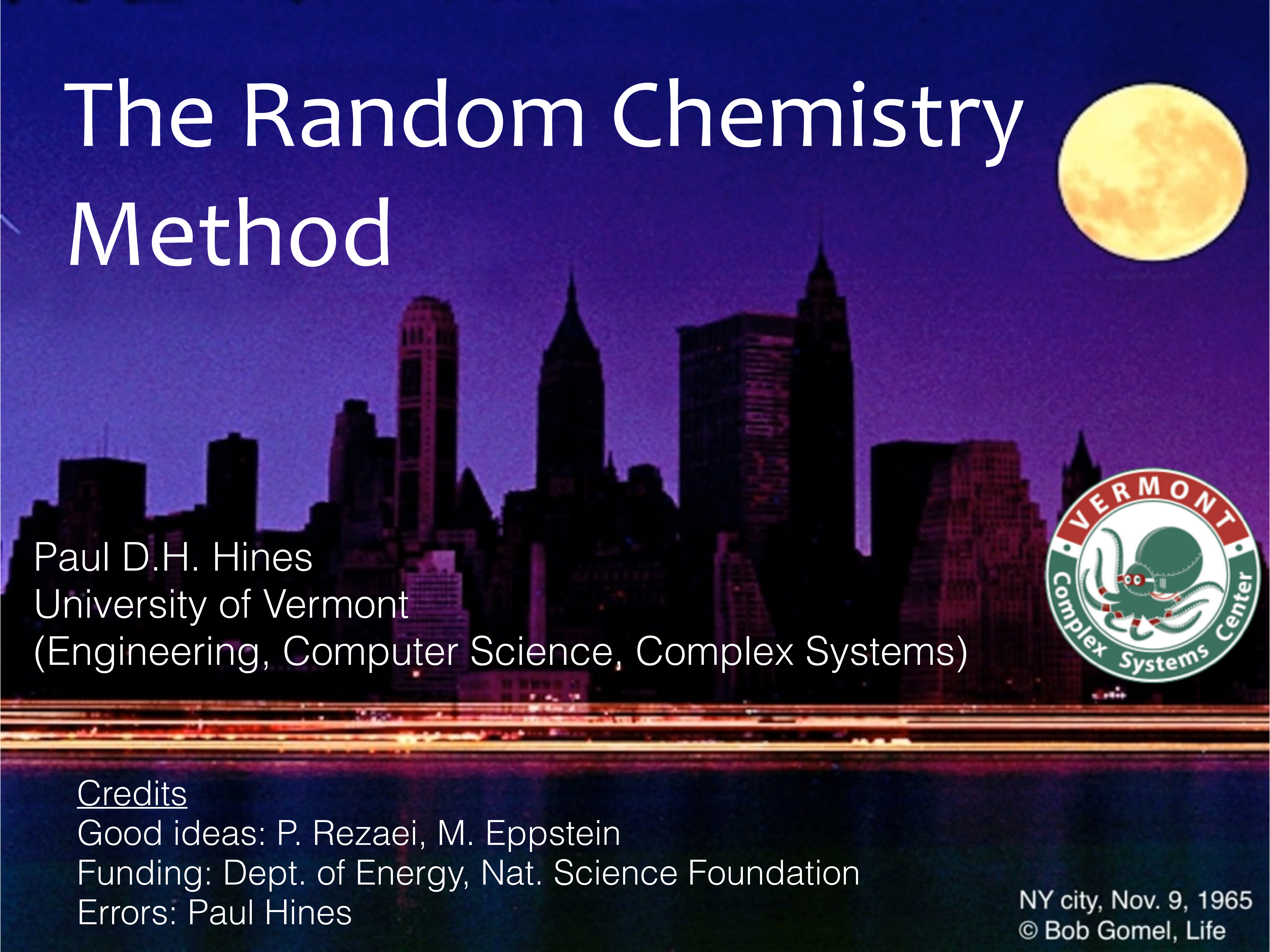


Case 1 (noon tomorrow)
High blackout risk



Case 2 (2 pm tomorrow)
Low blackout risk

Both cases are secure.
What makes the two cases different?
How can we make Case 1 more like Case 2?



The Random Chemistry Method

Paul D.H. Hines
University of Vermont
(Engineering, Computer Science, Complex Systems)

Credits
Good ideas: P. Rezaei, M. Eppstein
Funding: Dept. of Energy, Nat. Science Foundation
Errors: Paul Hines

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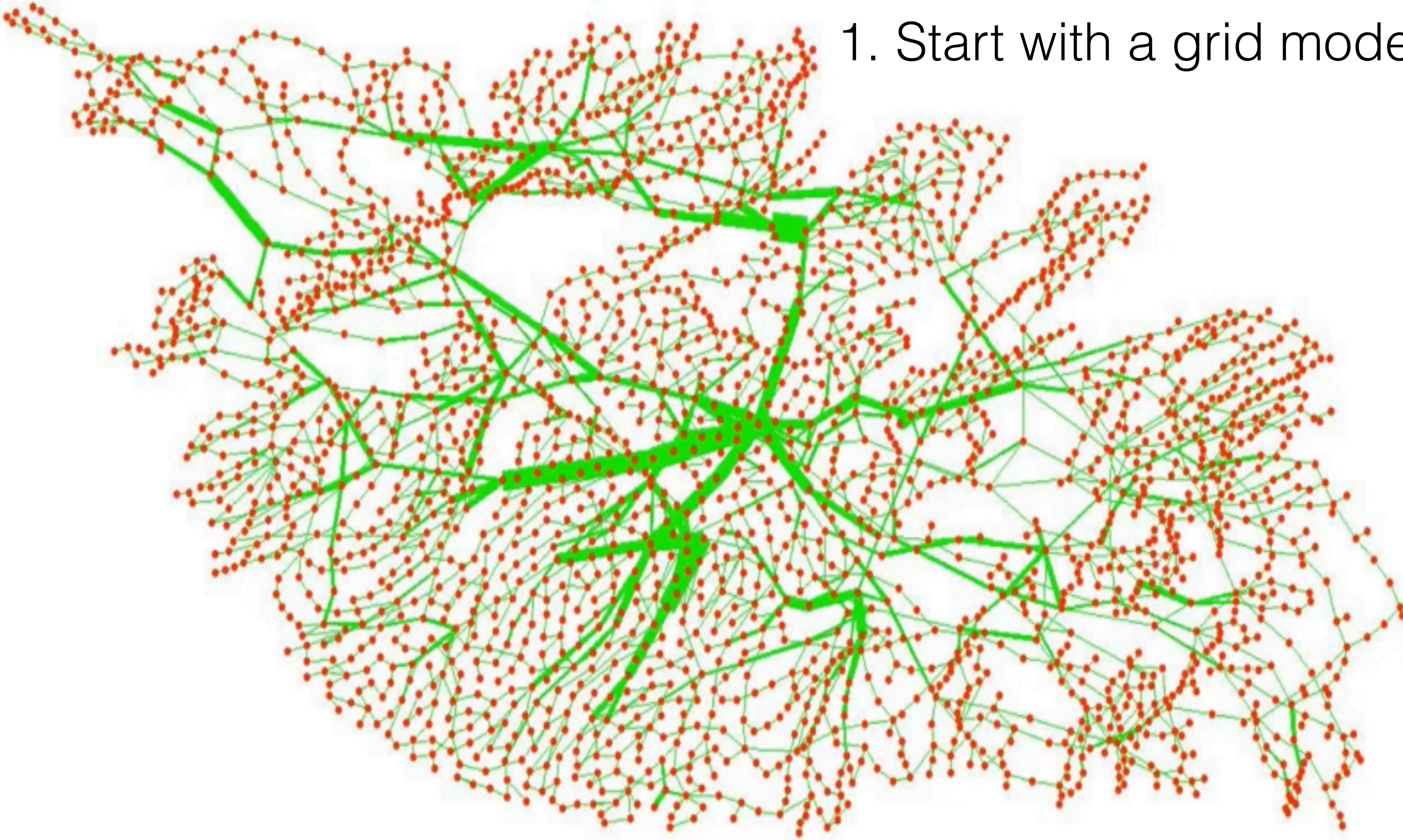
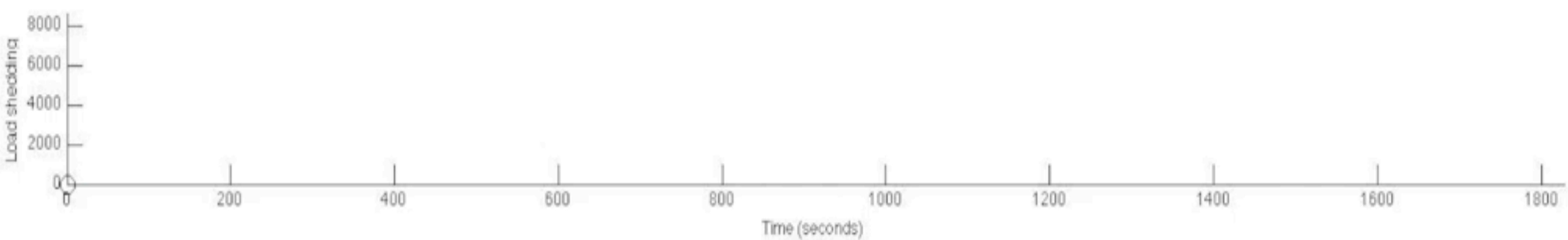
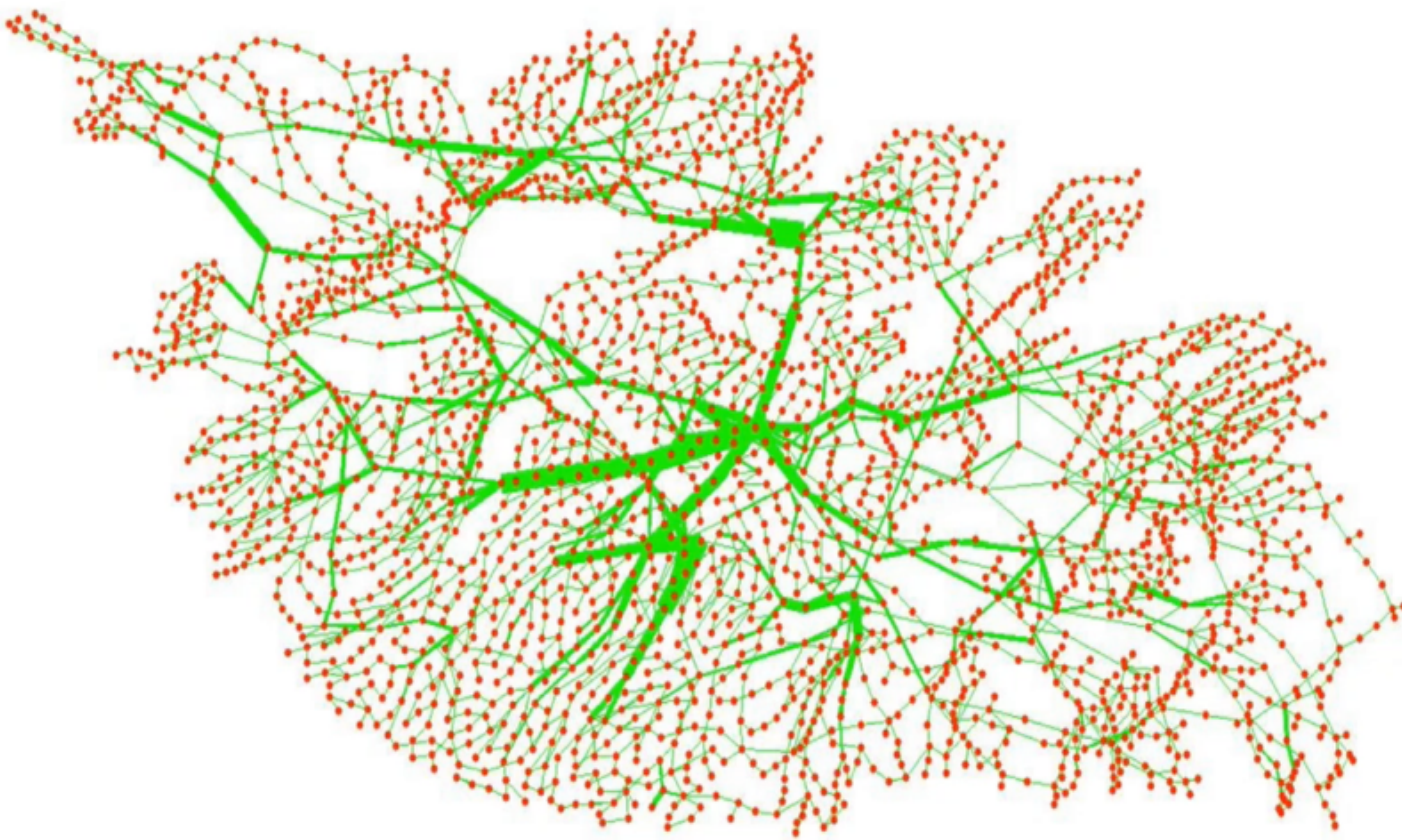
1. Start with a grid model.

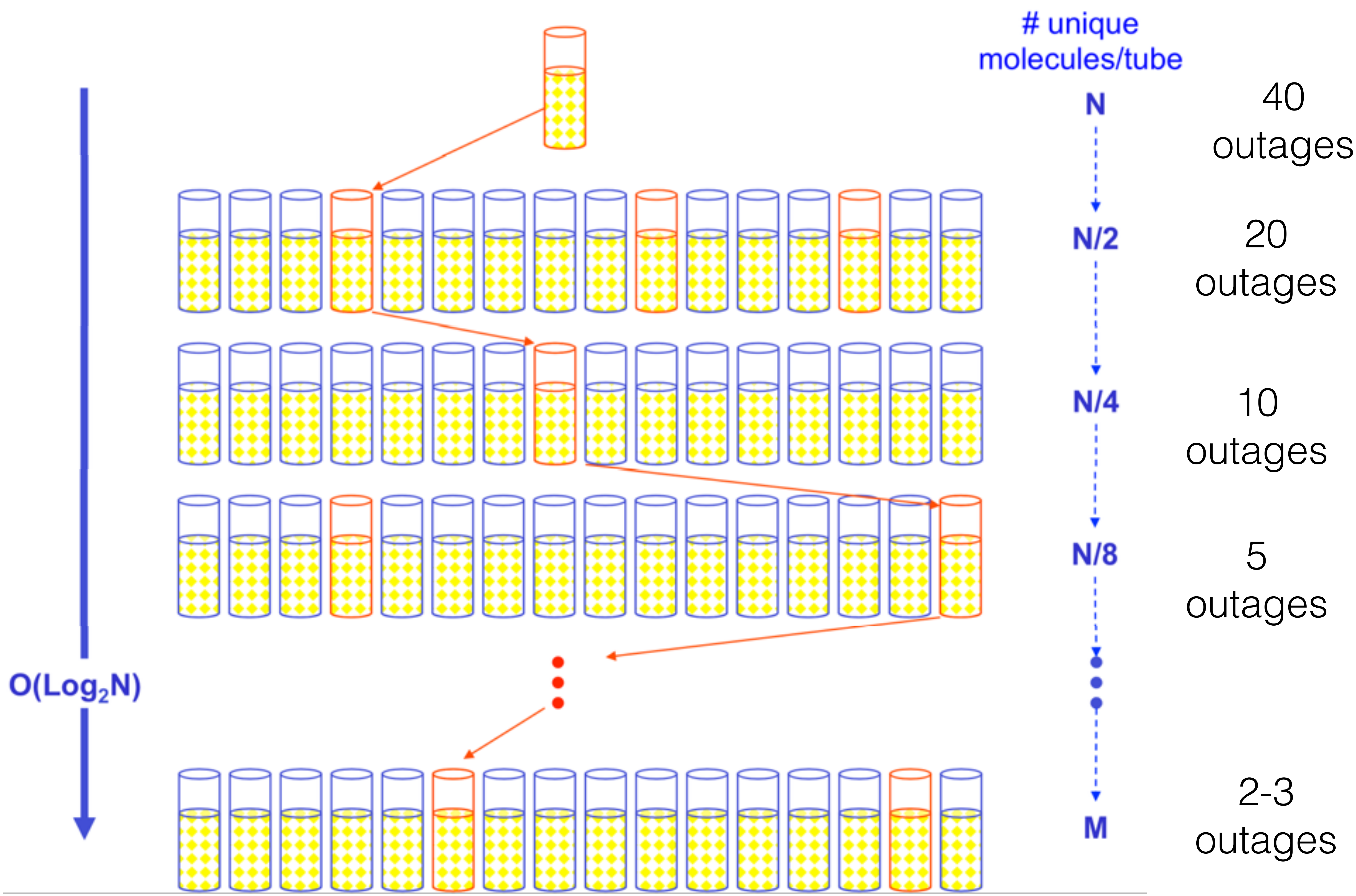




2. Now find many of the outage combinations that cause blackouts (the malignancies)



The Random Chemistry algorithm



3. Use the results to quantify blackout risk

The estimated number of malignancies of size k

Blackout sizes

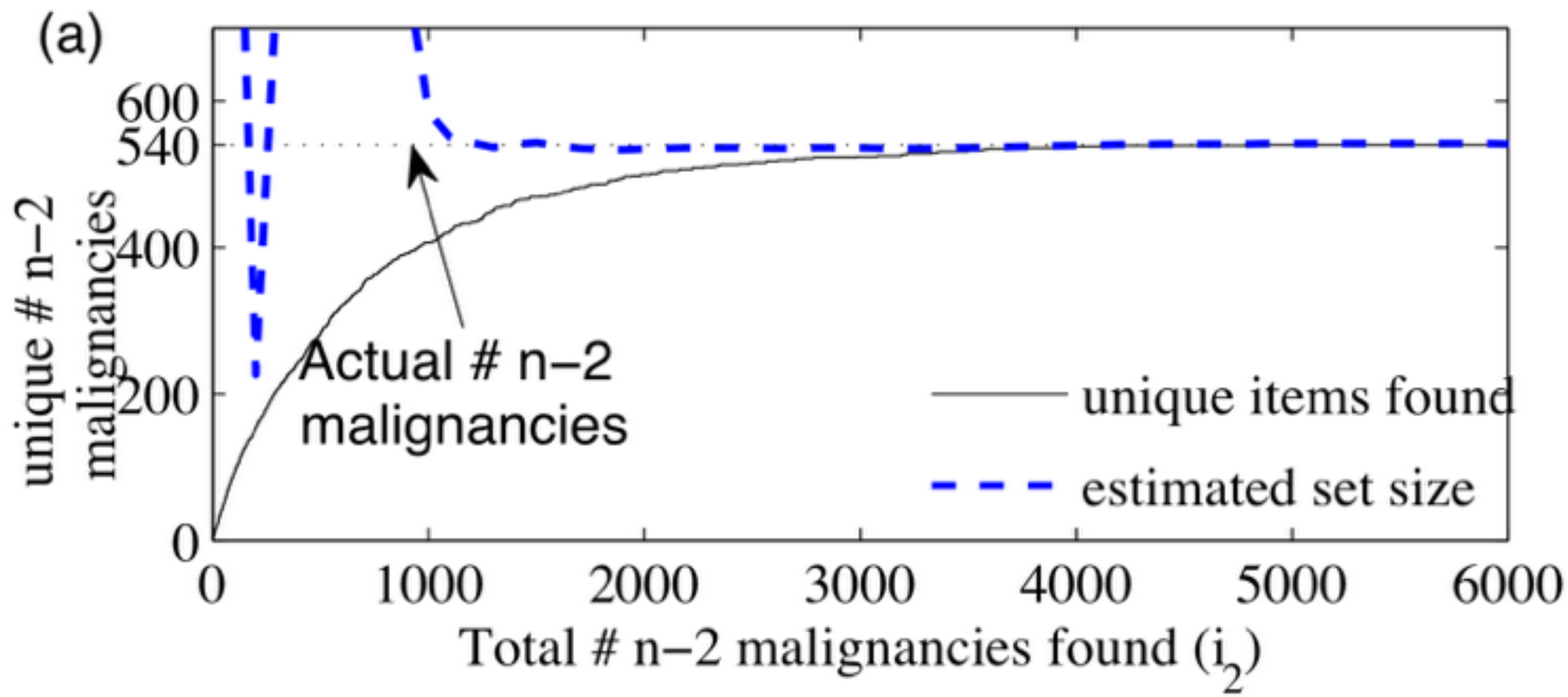
$$\hat{R}_{RC}(x) = \sum_{k=2}^{k_{\max}} \frac{\boxed{\hat{M}_k}}{\boxed{|\Omega_{RC,k}|}} \sum_{m \in \Omega_{RC,k}} \boxed{\Pr(m)}\,\boxed{S(m,x)}$$

Probability of (multiple) contingency

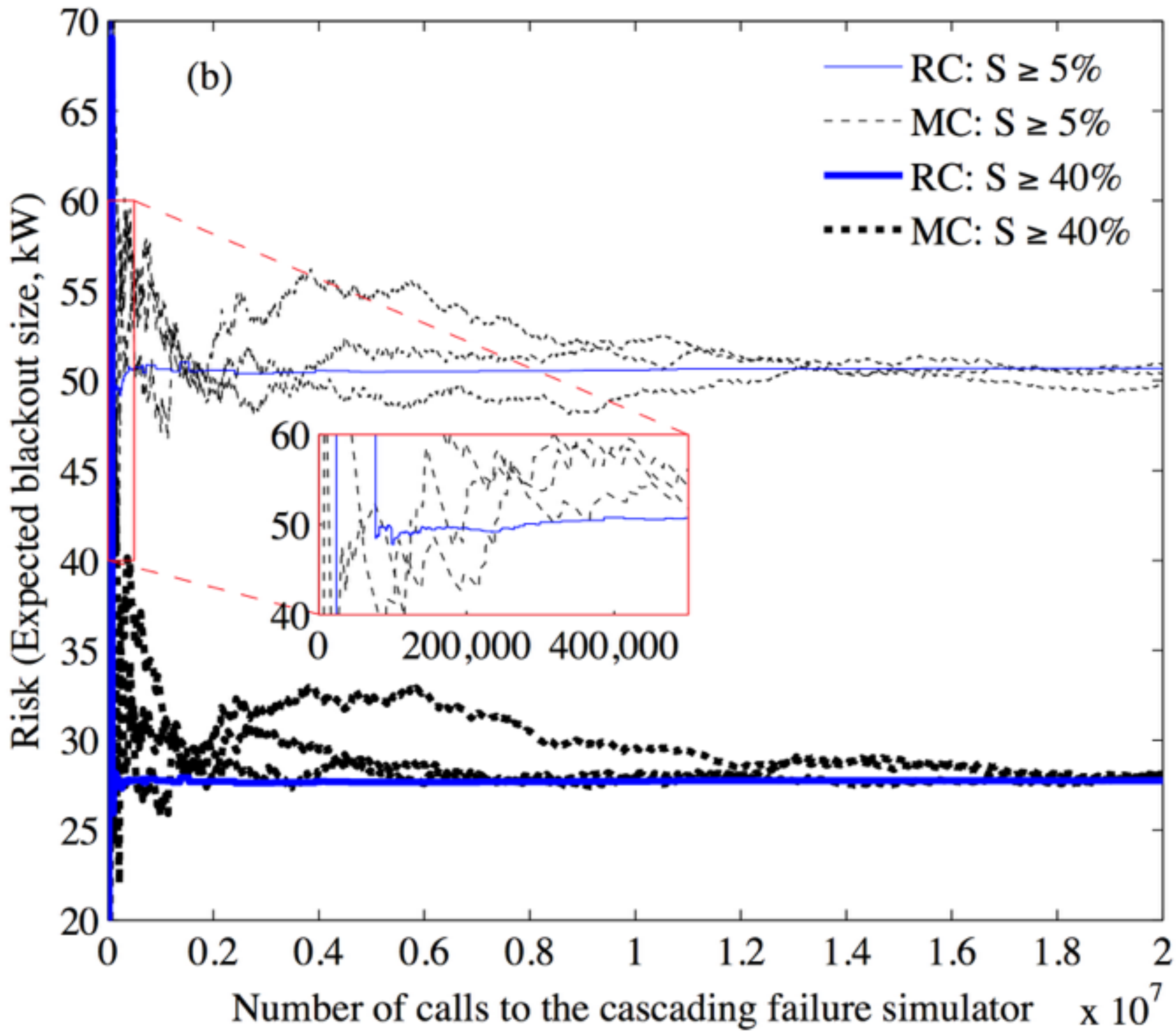
The number of malignancies of size k found by RC

4. Estimating the number of blackout-causing contingencies by modeling the rate at which unique malignancies are found

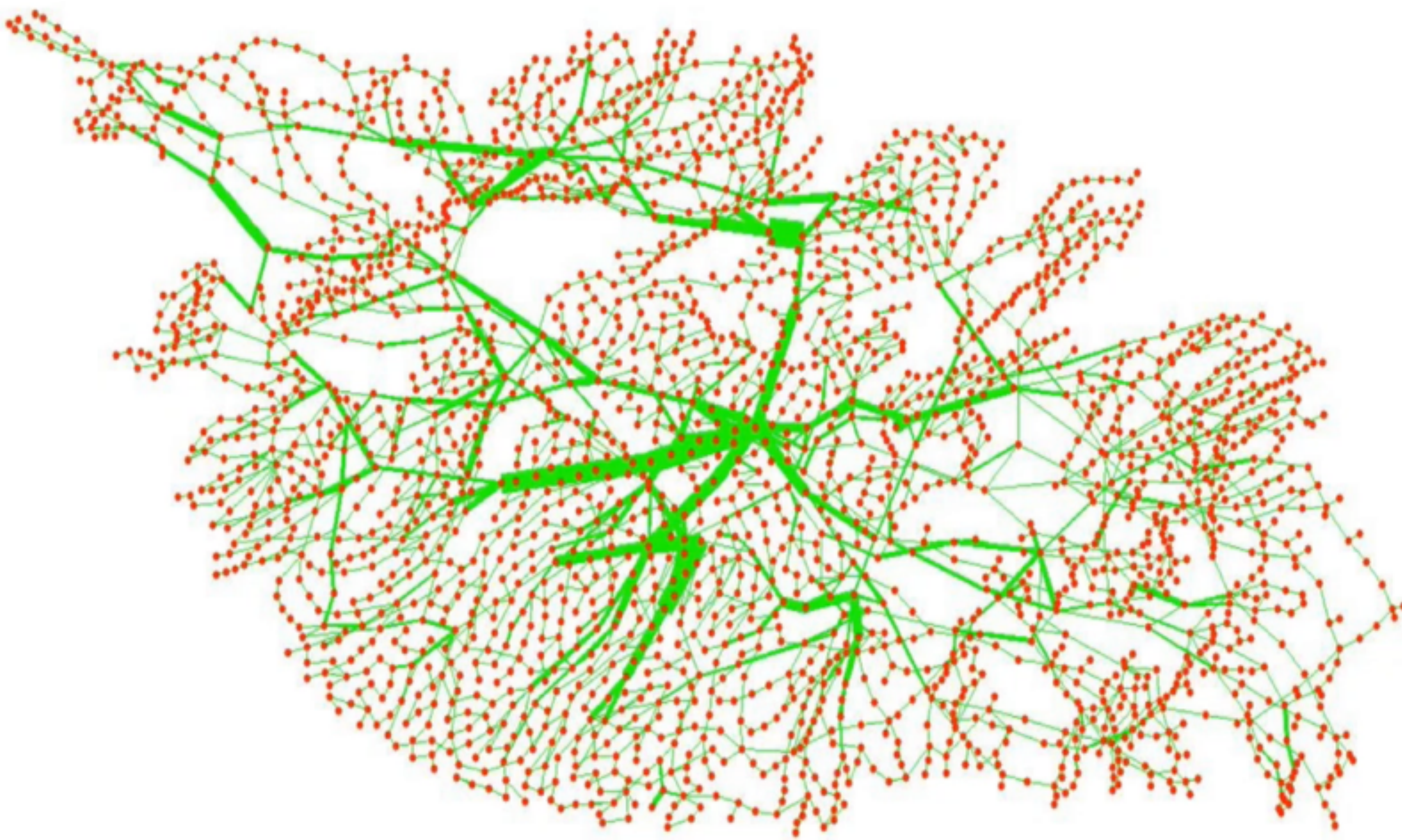
nba.com



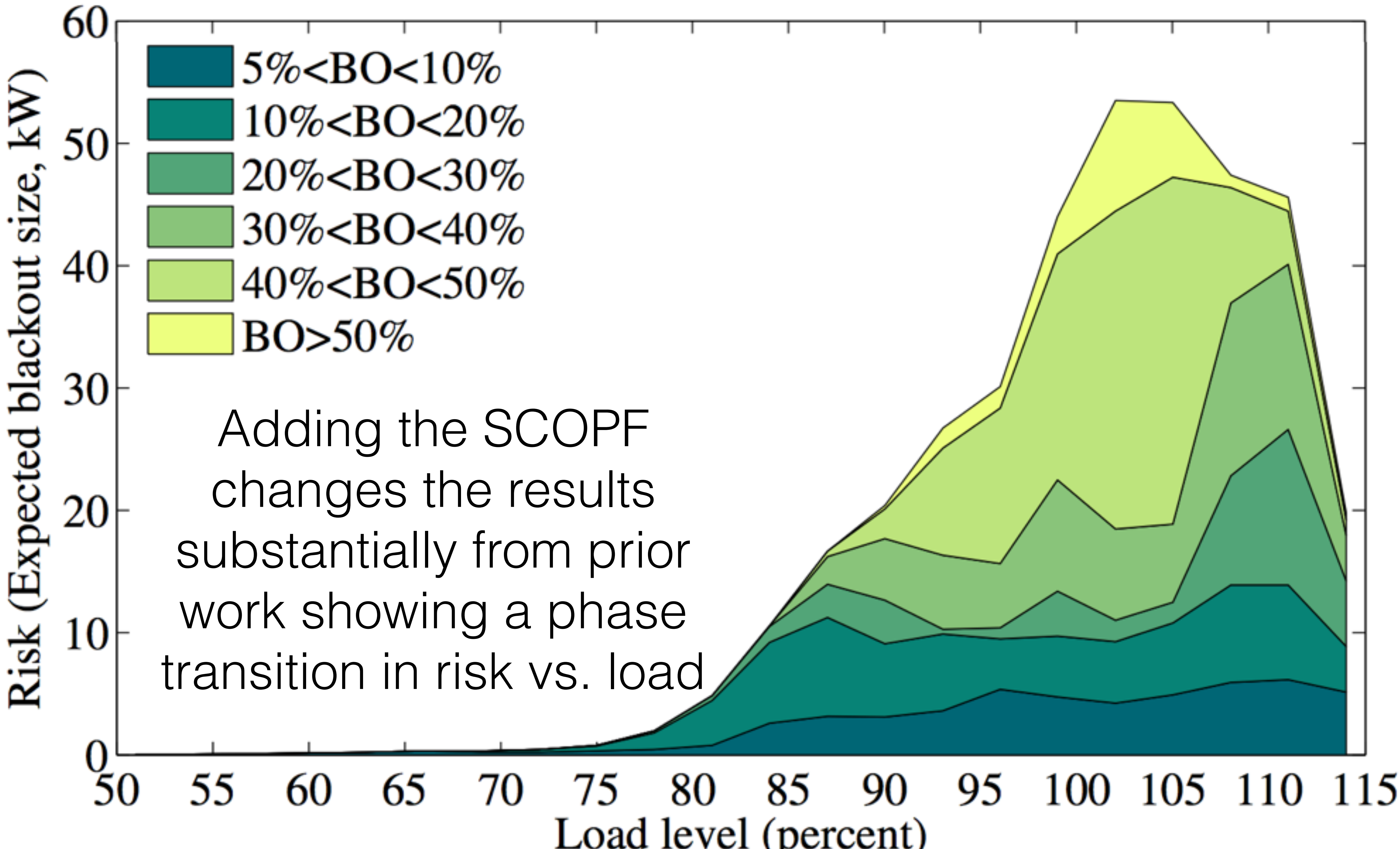
Comparing RC to Monte Carlo



Now that we can estimate blackout risk, what insight can we gain?



Risk vs. load, given SCOPF



Why?

- At high load levels SCOPF leaves larger margins on long inter-area tie lines (to allow for potential contingencies)

Total absolute flow on lines with large (>200MW) base case flow

Load level	95%	100%	105%	110%	115%
MW flow	16,312	17,032	17,102	16,869	15,916

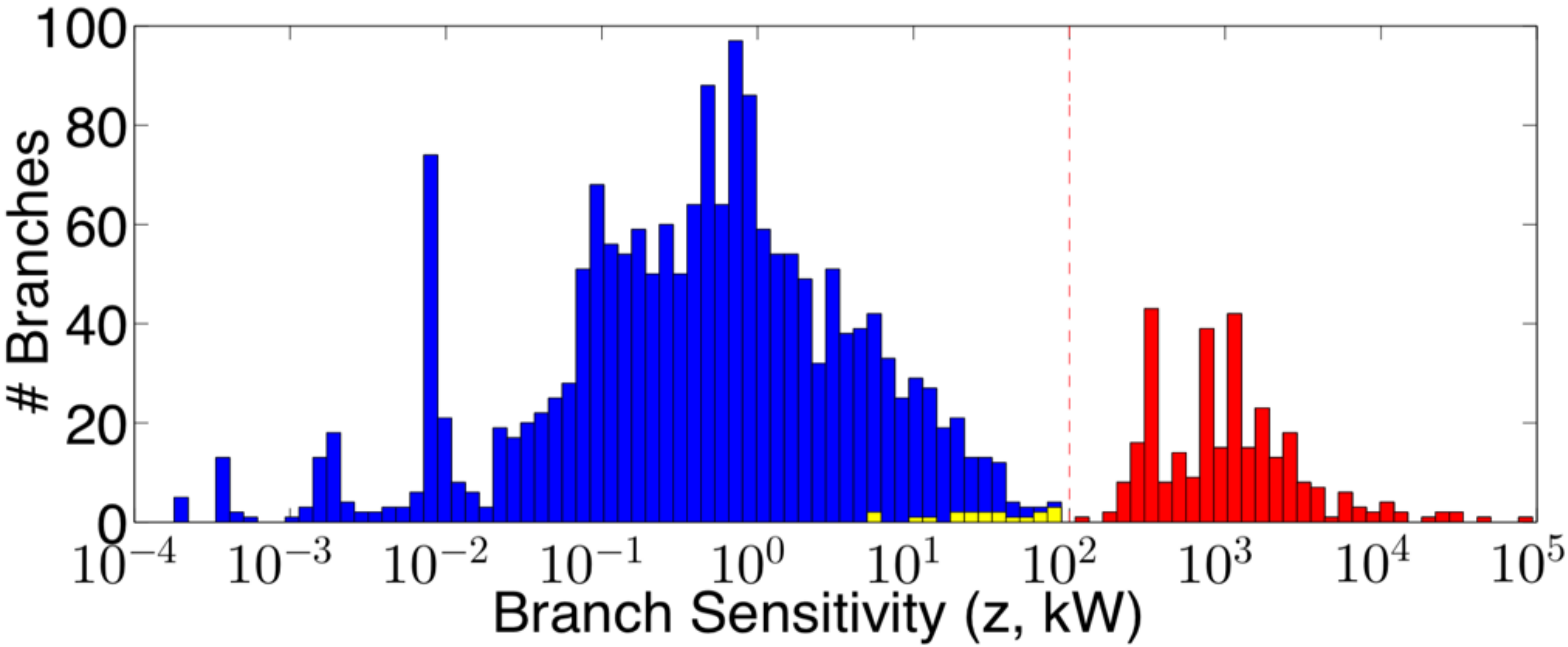
Finding the contribution of elements to risk

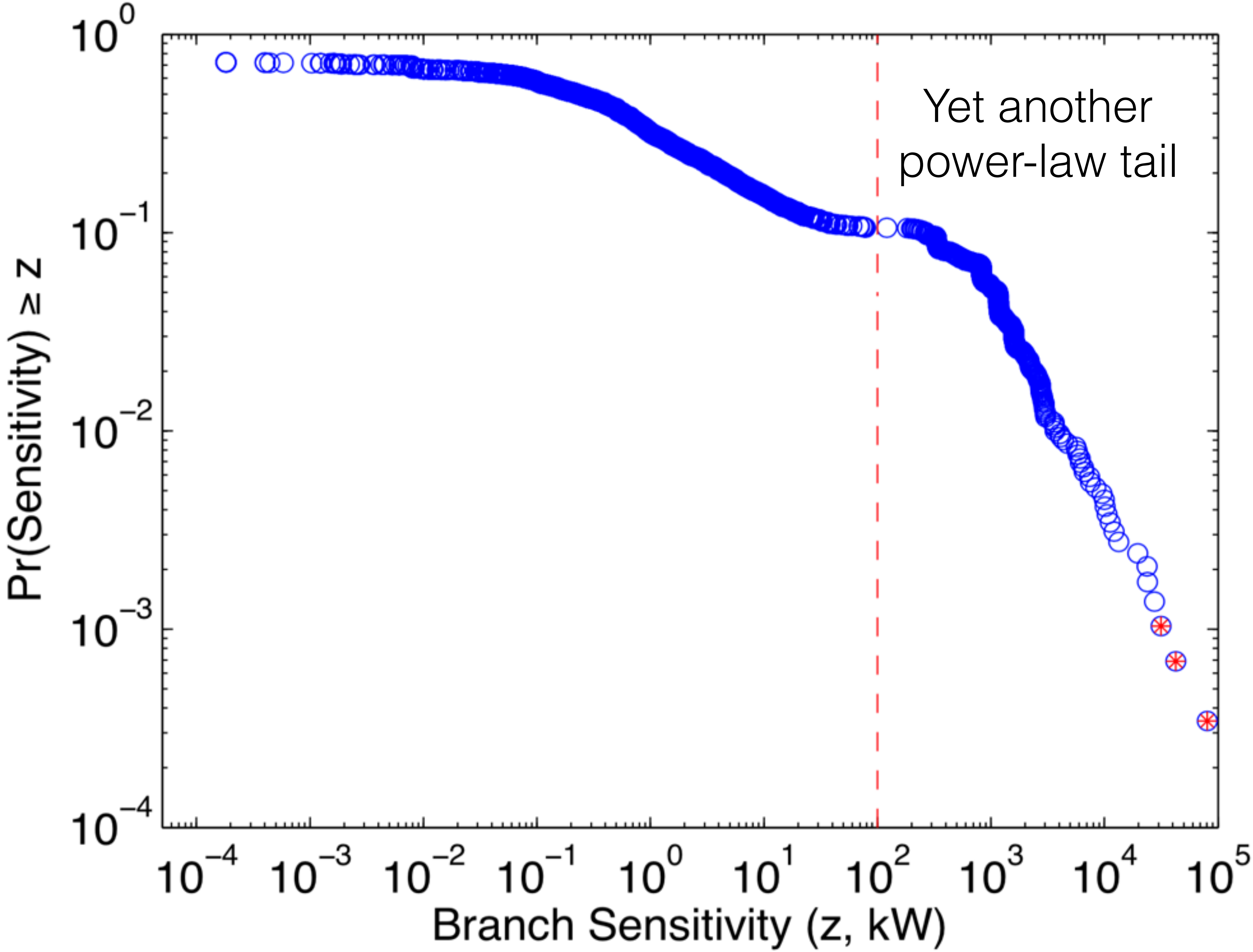
Differentiate the risk equation with respect to element outage probabilities

$$\hat{R}_{RC}(x) = \sum_{k=2}^{k_{\max}} \frac{\hat{M}_k}{|\Omega_{RC,k}|} \sum_{m \in \Omega_{RC,k}} \Pr(m) S(m,x)$$

$$\frac{\partial \hat{R}_{RC,k}}{\partial p_i} = \frac{\hat{M}_k}{|\Omega_{RC,k}|} \sum_{m \in \Omega_{RC,k}} S(m,x) \frac{\partial}{\partial p_i} \Pr(m)$$

Distribution of "risk sensitivity"

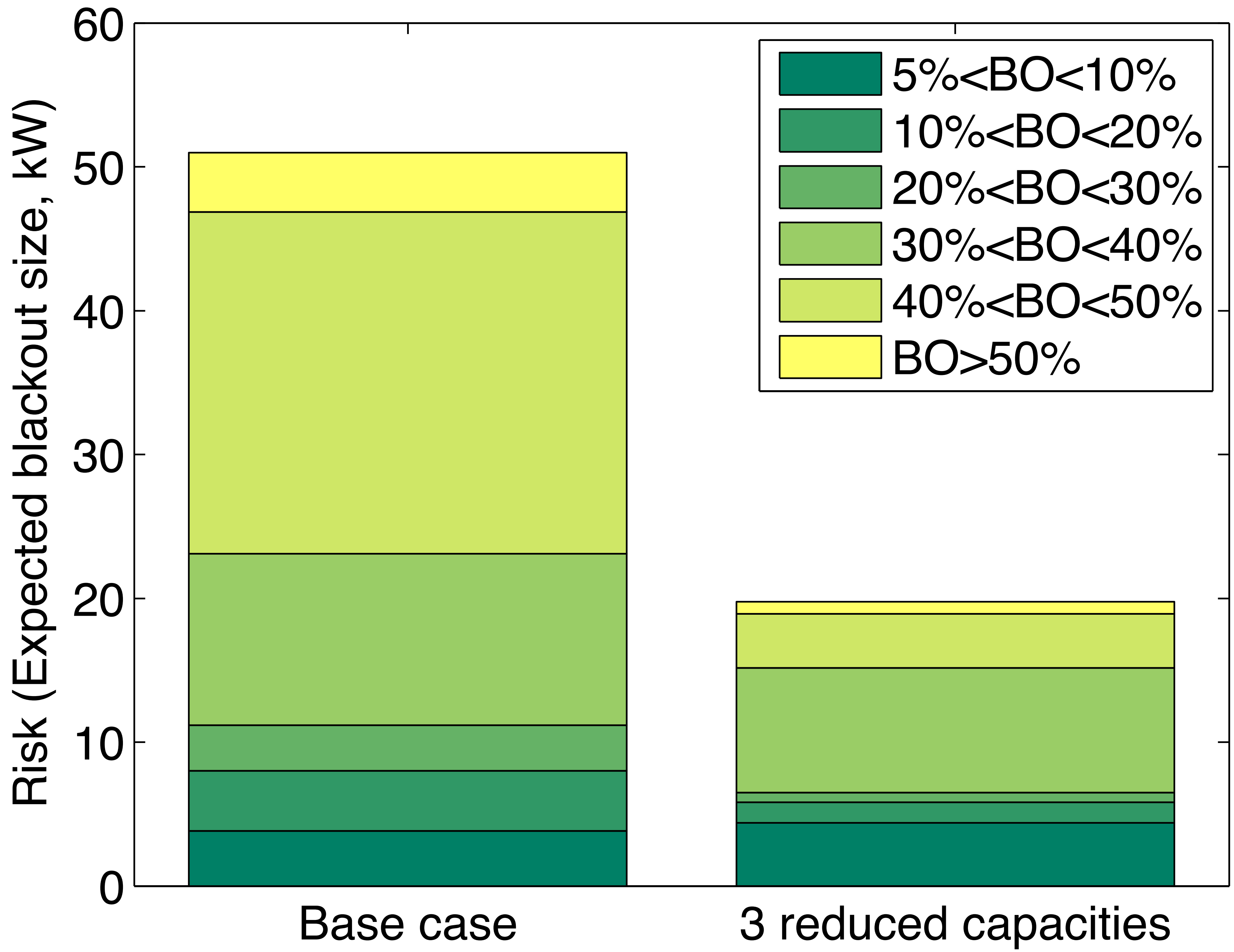



Yet another power-law tail
Pr(Sensitivity) ≥ z
Branch Sensitivity (z, kW)
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^0 10^1 10^2 10^3 10^4 10^5

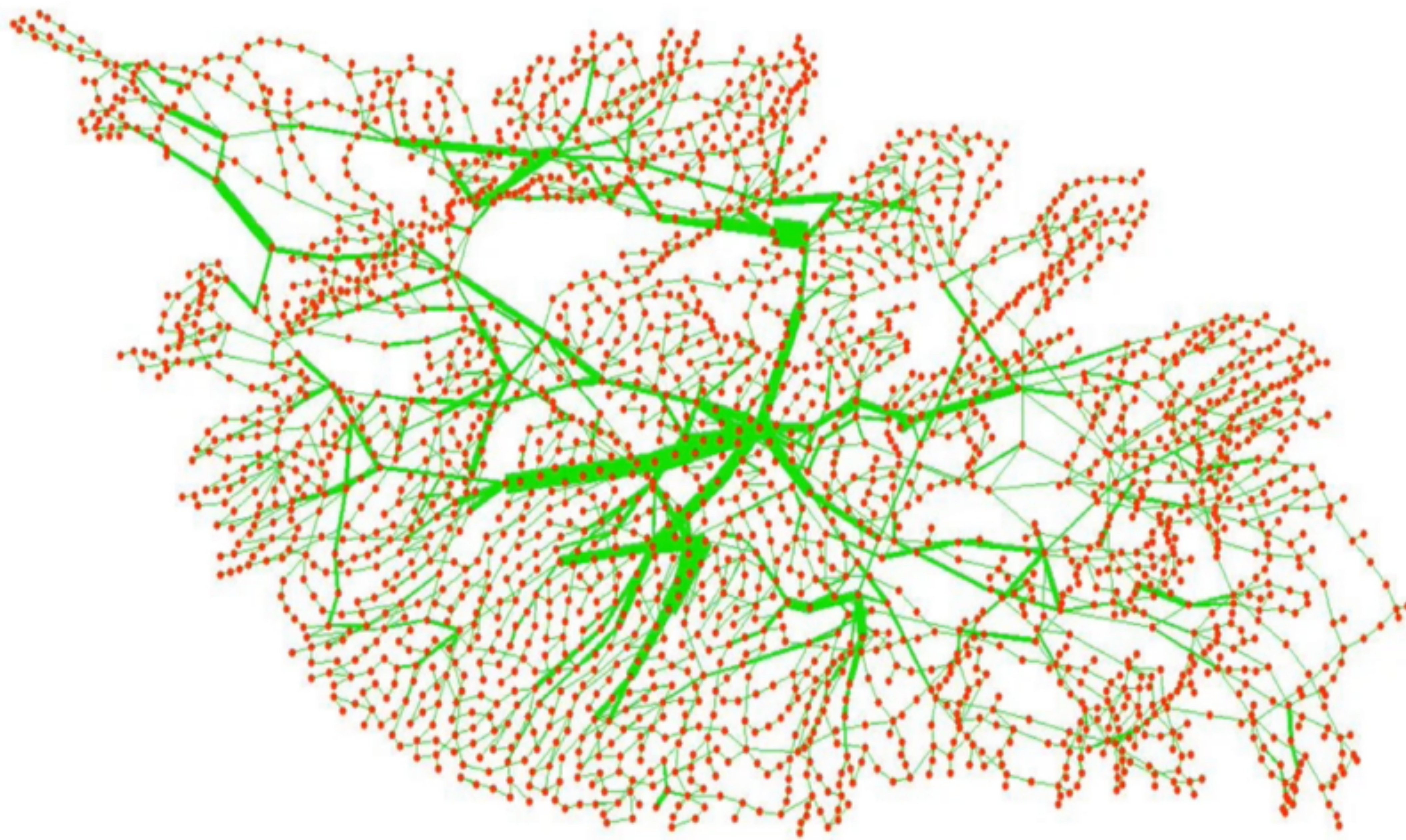
Can we use this insight to reduce risk?

- Take the 3 lines that contribute most to blackout risk
- Re-dispatch generators to leave more margin between the flow on these lines and the limit (cut the limit in half)
- Fuel costs increase by 1.6%
- Large (S>5%) blackout risk decreases by 61%
- Very large (S>40%) blackout risk decreases by 83%
- Perhaps we would be better off without these lines?

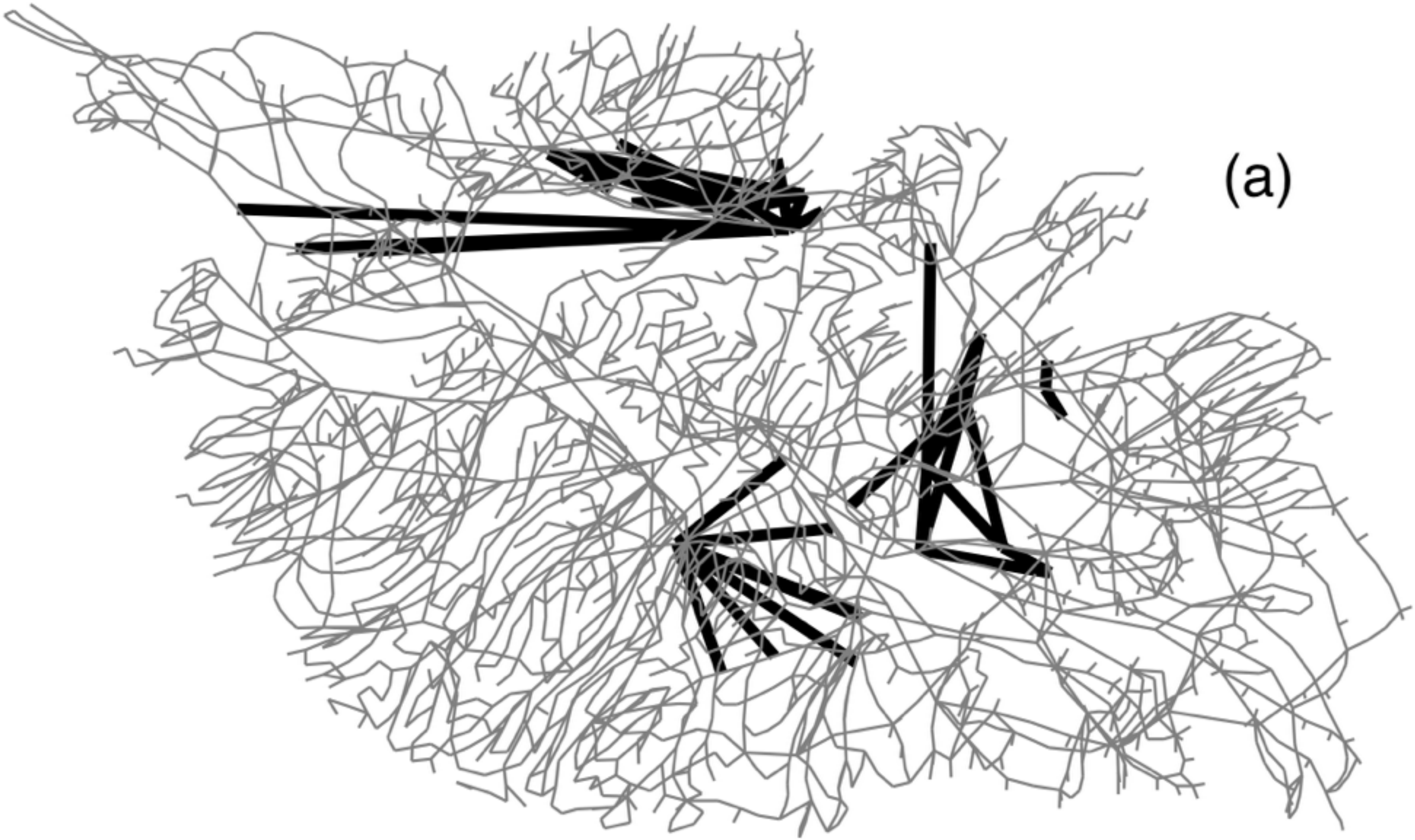
Before and after



Do the blackout-causing n-2 contingencies change at different load levels?

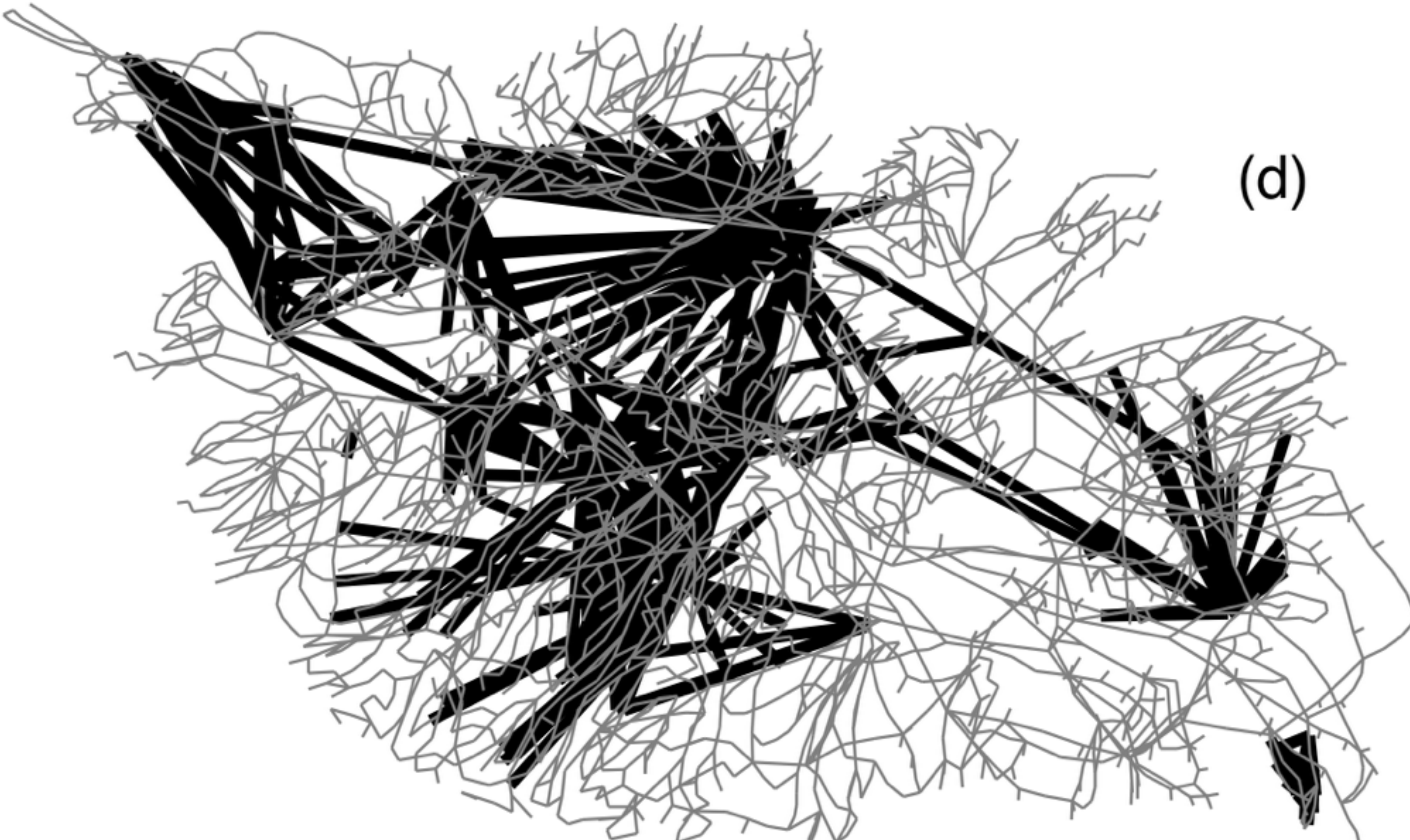


39 n-2 malignancies at 75% load

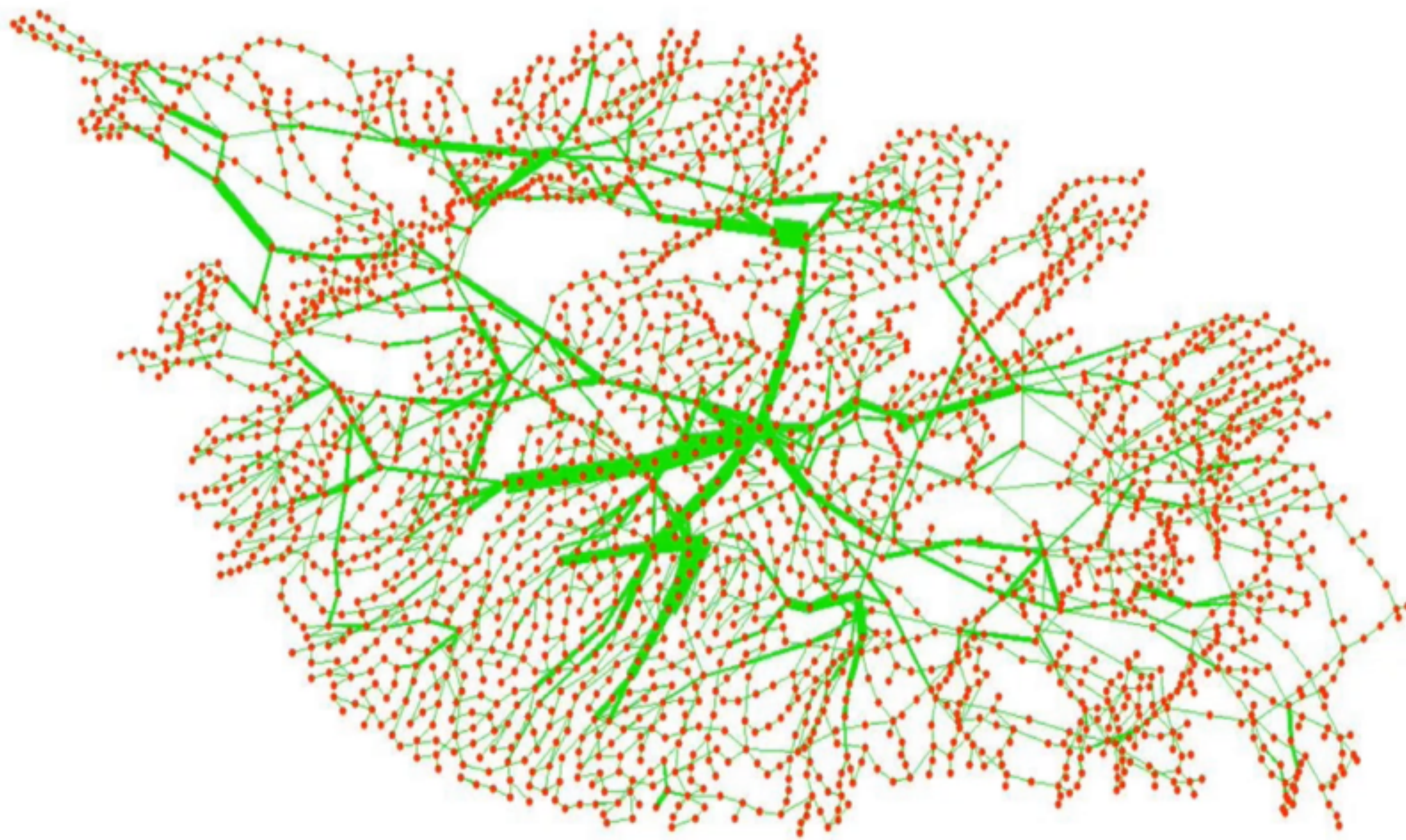


540 n-2 malignancies at 100%

378 n-2 malignancies at 115%

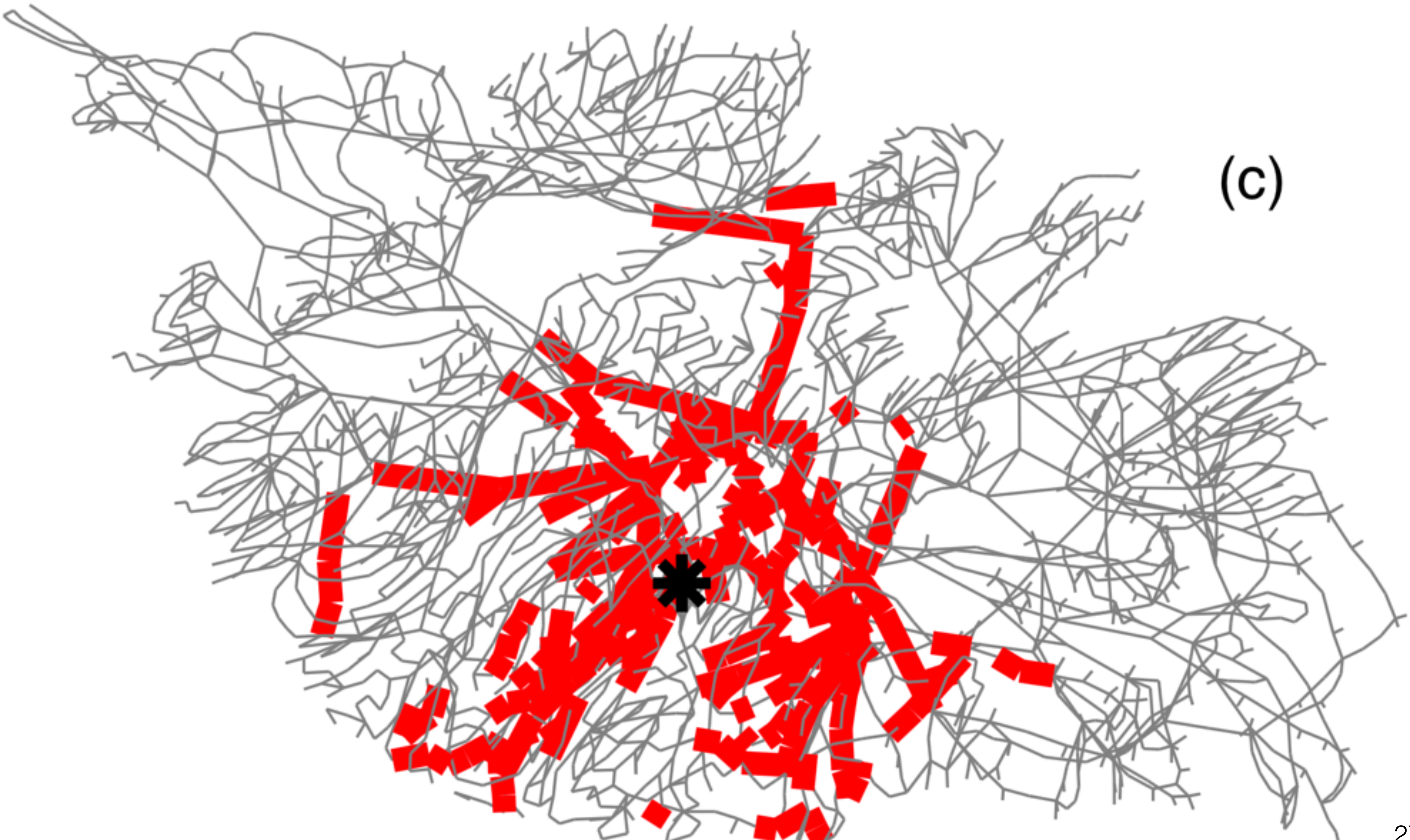


Which components negatively interact with a given component at different load levels?

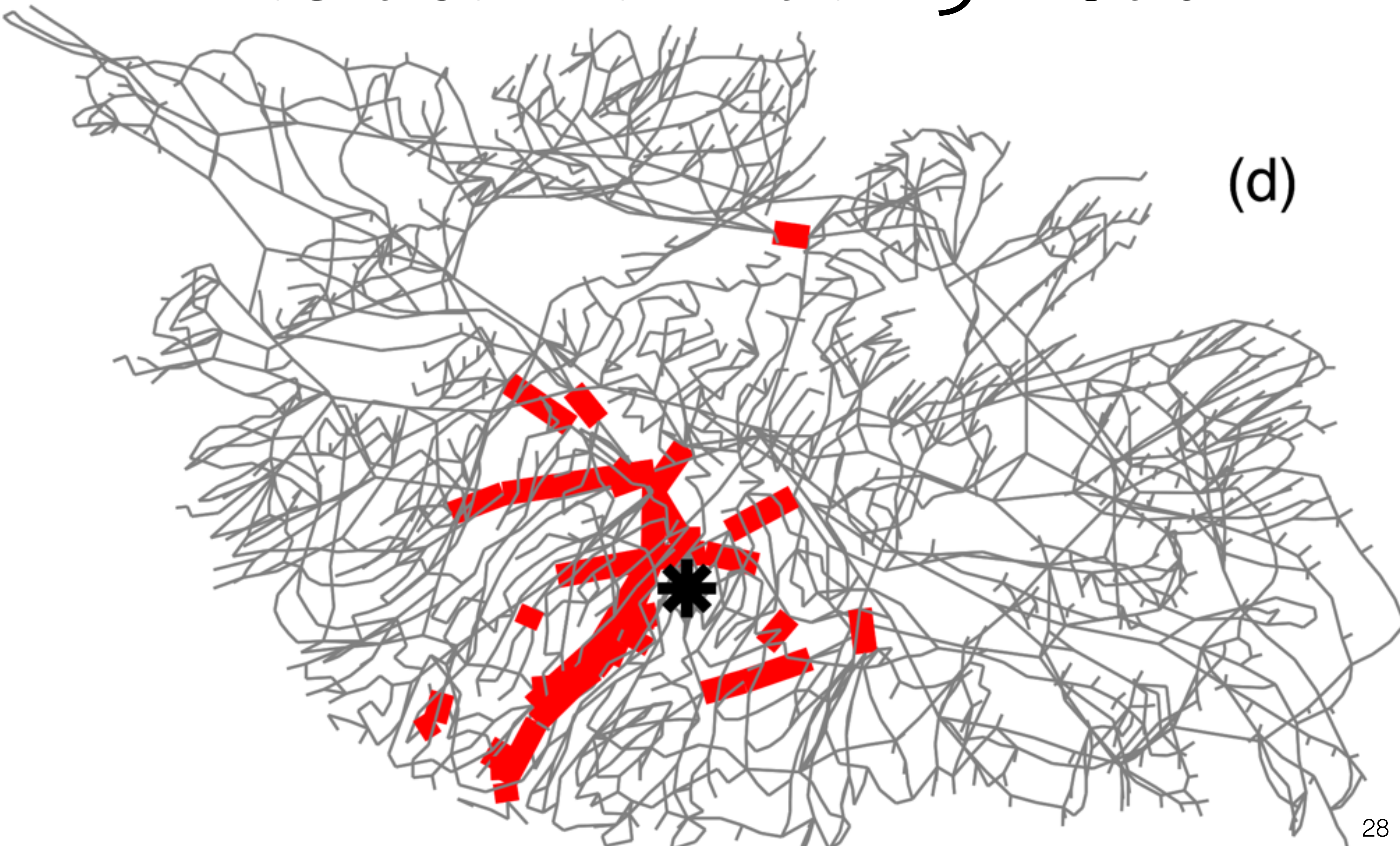


Branches that negatively interact with * at 90% load

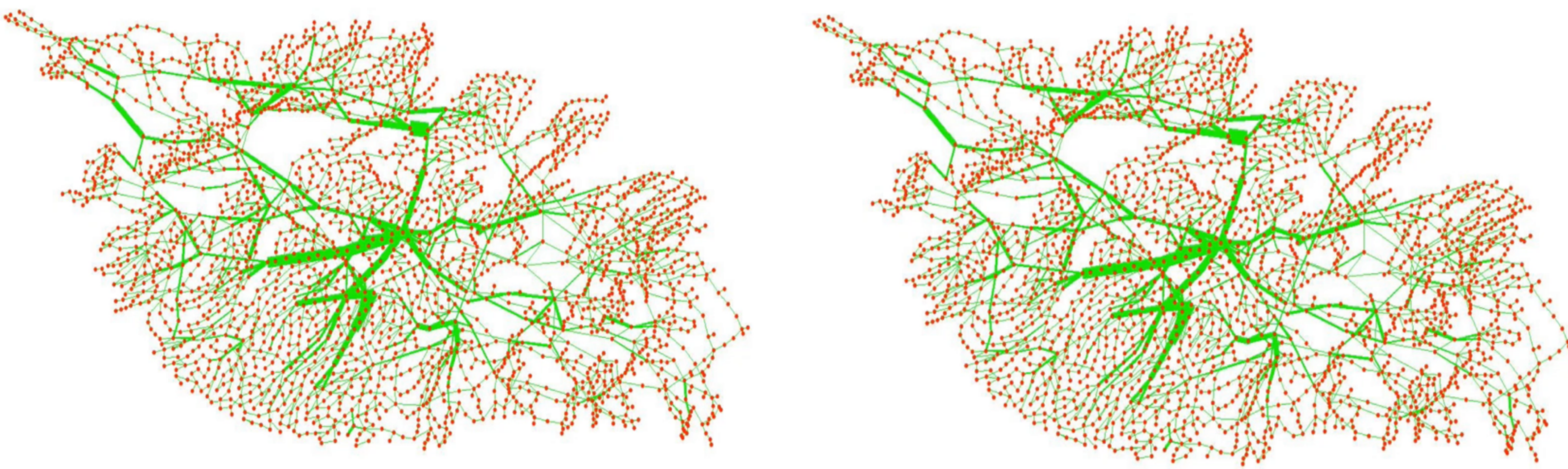
Branches that negatively interact with * at 100% load



Branches that negatively interact with * at 115% load



Returning to the Illustration



Case 1 (noon tomorrow)
High blackout risk

Case 2 (2 pm tomorrow)
Low blackout risk

We now have a way to describe the differences in risk between these two cases and explain why the two cases are different.

Conclusions

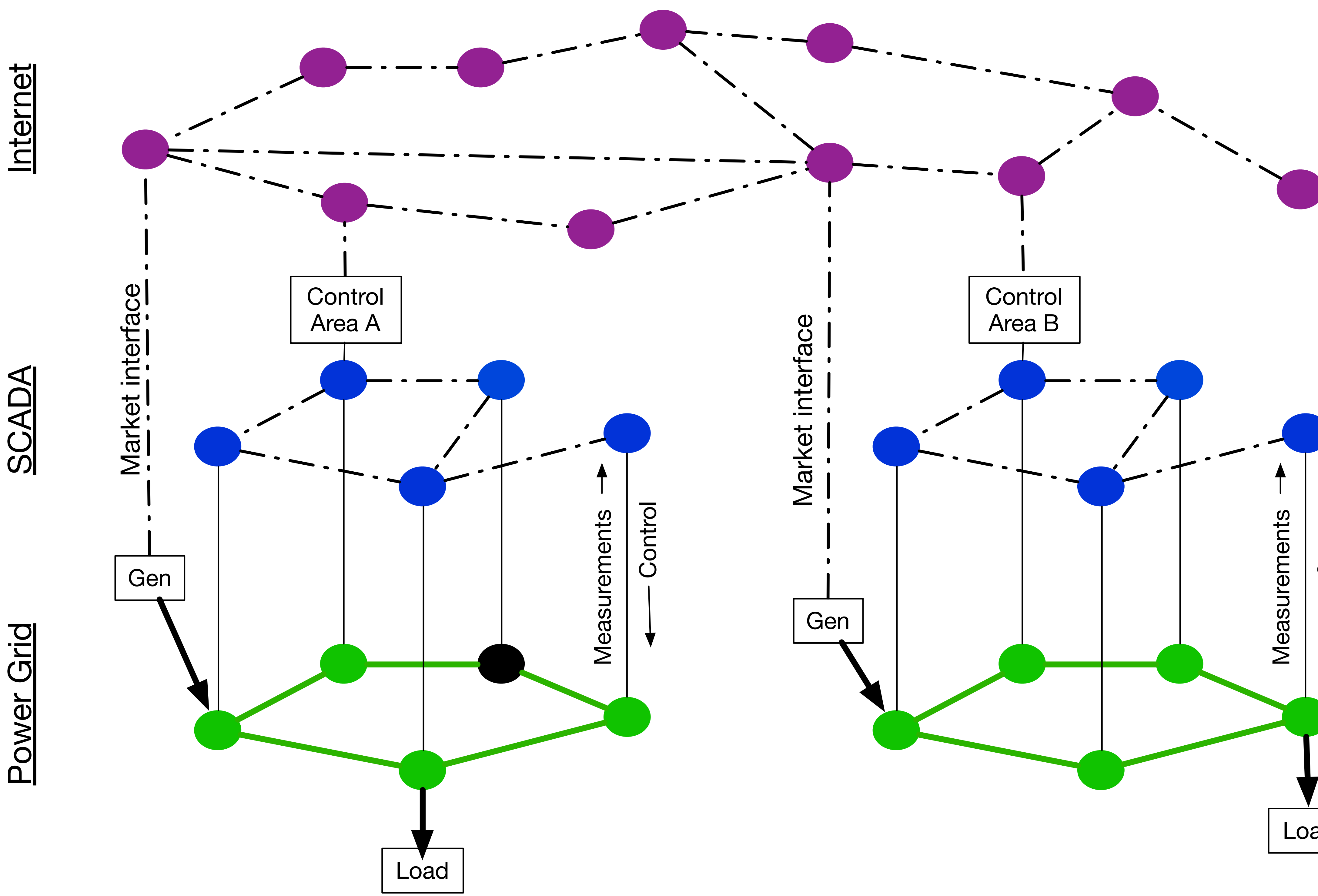
- It is possible to estimate cascading failure risk with a reasonable amount of computation (e.g., overnight given tomorrow's peak-load model).
  Random Chemistry approach is >100x faster than MC
  Does this hold up for correlated event probabilities?

- Doing so gives insight that can result in real risk reductions:
  More load is not always worse (8/14/2003, 9/8/2011)
  Adjusting the flow limits on critical lines
  Perhaps switching them out entirely?

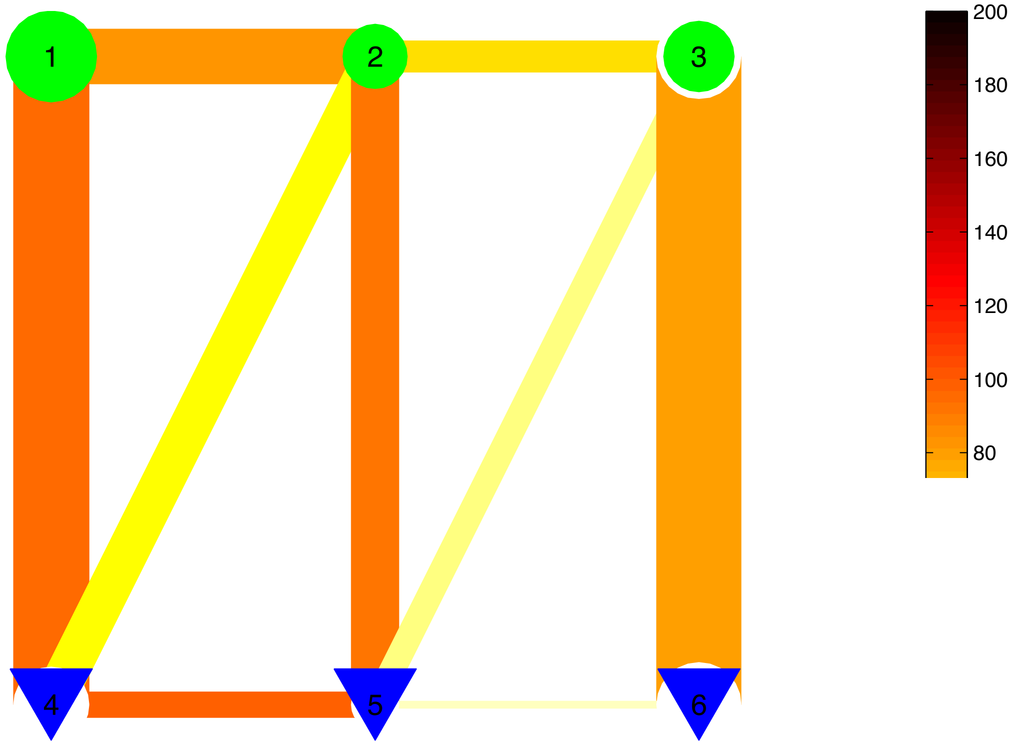
- Providing visual feedback to operators may produce new isight and ideas for risk reduction

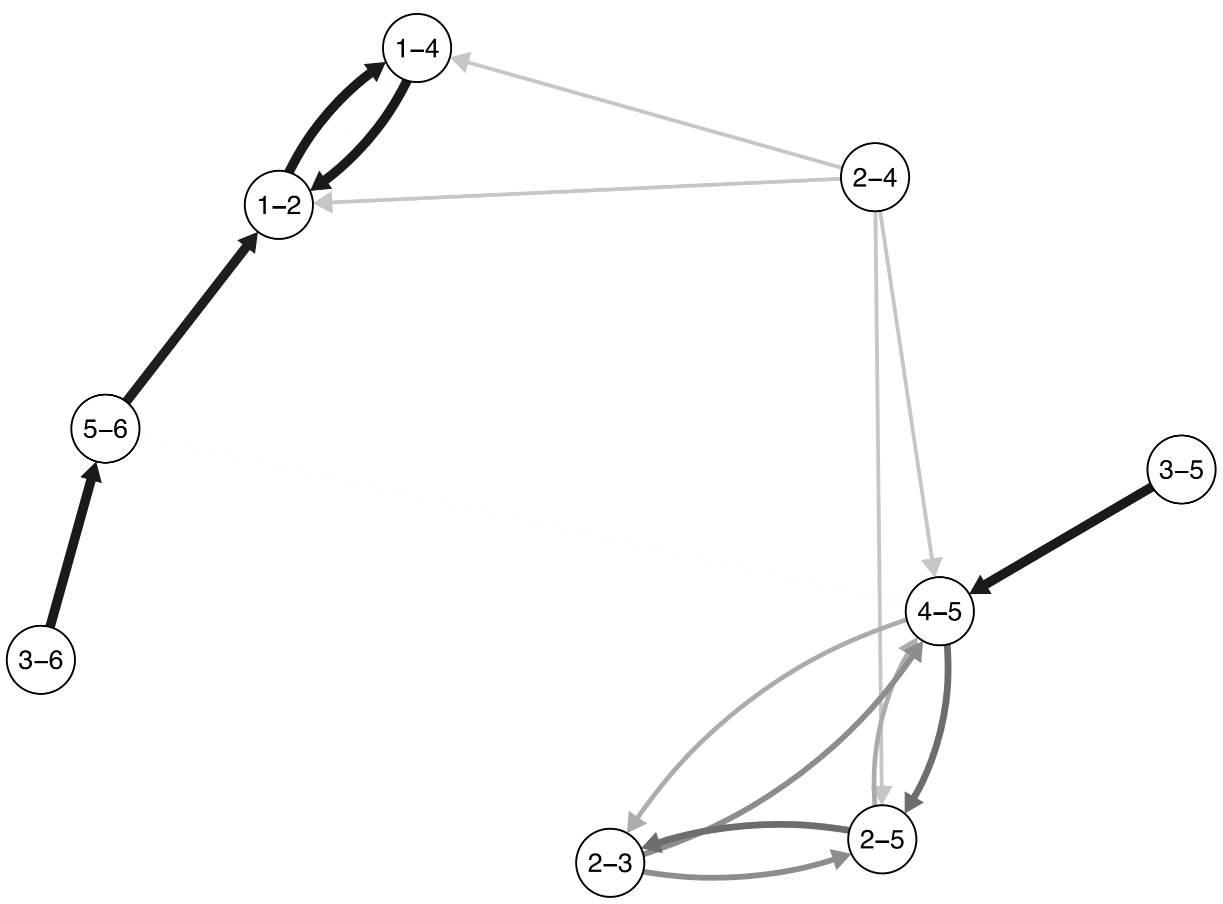
paul.hines@uvm.edu

Importantly, this method is completely model-agnostic. Describing risk in interdependent systems



Work in Progress: Influence Graphs





A larger influence graph

graph showing links with a weight of 1000 or greater

Beyond Contingency Analysis New Approaches to Cascading Failures Risk Analysis

For more information: Pooya Rezaei, Paul Hines and Margaret Eppstein, "Estimating Cascading Failure Risk with Random Chemistry," *IEEE Transactions on Power Systems* (in press) http://arxiv.org/abs/1405.4213

Credits
Good ideas: P. Rezaei, M. Eppstein
Funding: Dept. of Energy, Nat. Science Foundation
Errors: Paul Hines

NY city, Nov. 9, 1965

Modeling and Computation of Security-constrained Economic Dispatch with Multi-stage Rescheduling

Michael C. Ferris

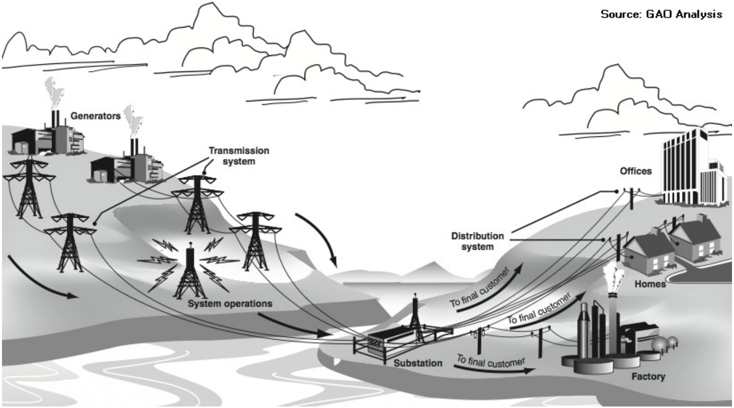
Joint work with: Yanchao Liu, Andy Philpott and Roger Wets

Supported by DOE

University of Wisconsin, Madison

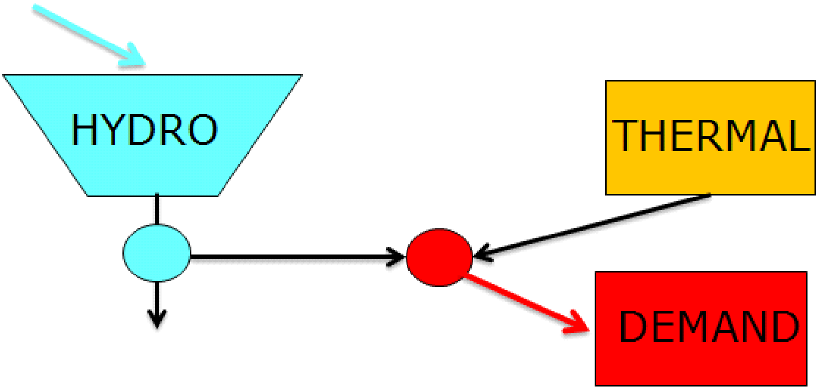
Grid Science Winter Conference, Santa Fe
January 15, 2015

Power generation, transmission and distribution



- Determine generators' output to reliably meet the load
  - $\sum$ Gen MW $= \sum$ Load MW, at all times.
  - Power flows cannot exceed lines' transfer capacity.

Hydro-Thermal System (Philpott/F./Wets)



Simple electricity "system optimization" problem

$$\begin{aligned}
\text{SO:} \quad \max_{d_k,u_i,v_j,x_i \geq 0} \quad & \sum_{k \in \mathcal{K}} W_k(d_k) - \sum_{j \in \mathcal{T}} C_j(v_j) + \sum_{i \in \mathcal{H}} V_i(x_i) \\
\text{s.t.} \quad & \sum_{i \in \mathcal{H}} U_i\left(u_i\right) + \sum_{j \in \mathcal{T}} v_j \geq \sum_{k \in \mathcal{K}} d_k, \\
& x_i = x_i^0 - u_i + h_i^1, \quad i \in \mathcal{H}
\end{aligned}$$

- u_i water release of hydro reservoir $i \in \mathcal{H}$
- v_j thermal generation of plant $j \in \mathcal{T}$
- x_i water level in reservoir $i \in \mathcal{H}$
- prod fn U_i (strictly concave) converts water release to energy
- $C_j(v_j)$ denote the cost of generation by thermal plant
- $V_i(x_i)$ future value of terminating with storage x (assumed separable)
- $W_k(d_k)$ utility of consumption d_k

SO equivalent to CE

$$
\begin{aligned}
\text{Consumers } k \in \mathcal{K} \text{ solve CP}(k)\text{:} \quad & \max_{d_k \geq 0} && W_k(d_k) - p^T d_k \\
\text{Thermal plants } j \in \mathcal{T} \text{ solve TP}(j)\text{:} \quad & \max_{v_j \geq 0} && p^T v_j - C_j(v_j) \\
\text{Hydro plants } i \in \mathcal{H} \text{ solve HP}(i)\text{:} \quad & \max_{u_i, x_i \geq 0} && p^T U_i(u_i) + V_i(x_i) \\
& \text{s.t.} && x_i = x_i^0 - u_i + h_i^1
\end{aligned}
$$

Perfectly competitive (Walrasian) equilibrium is a MOPEC

$$
\begin{aligned}
\text{CE:} \qquad & d_k \in \arg\max \text{CP}(k), && k \in \mathcal{K}, \\
& v_j \in \arg\max \text{TP}(j), && j \in \mathcal{T}, \\
& u_i, x_i \in \arg\max \text{HP}(i), && i \in \mathcal{H}, \\
& 0 \leq p \perp \sum_{i \in \mathcal{H}} U_i(u_i) + \sum_{j \in \mathcal{T}} v_j \geq \sum_{k \in \mathcal{K}} d_k.
\end{aligned}
$$

Nash Equilibria (as a MOPEC)

- Nash Games: x^* is a Nash Equilibrium if

$$x_i^* \in \arg\min_{x_i \in X_i} \ell_i(x_i, x_{-i}^*, p), \forall i \in \mathcal{I}$$

  x_{-i} are the decisions of other players.

- Prices p given exogenously, or via complementarity:

$$0 \leq H(x,p) \perp p \geq 0$$

- empinfo: equilibrium
  min loss(i) x(i) cons(i)
  vi H p
- Applications: Discrete-Time Finite-State Stochastic Games. Specifically, the Ericson & Pakes (1995) model of dynamic competition in an oligopolistic industry.

Key point: models generated correctly solve quickly

Here S is mesh spacing parameter

S	Var	rows	non-zero	dense(%)	Steps	RT (m:s)
20	2400	2568	31536	0.48	5	0 : 03
50	15000	15408	195816	0.08	5	0 : 19
100	60000	60808	781616	0.02	5	1 : 16
200	240000	241608	3123216	0.01	5	5 : 12

Convergence for $S = 200$ (with new basis extensions in PATH)

Iteration	Residual
0	1.56(+4)
1	1.06(+1)
2	1.34
3	2.04(−2)
4	1.74(−5)
5	2.97(−11)

Agents have stochastic recourse?

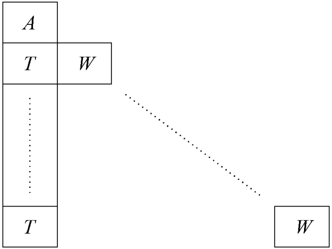
- Two stage stochastic programming, x^1 is here-and-now decision, recourse decisions x^2 depend on realization of a random variable
- ρ is a risk measure (e.g. expectation, CVaR)

$$\text{SP:}\;\max \quad c^T x^1 + \rho[q^T x^2]$$

$$\text{s.t.}\quad Ax^1 = b, \quad x^1 \geq 0,$$

$$T(\omega)x^1 + W(\omega)x^2(\omega) \geq d(\omega),$$

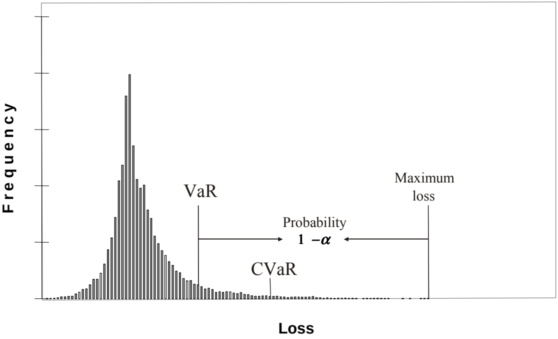
$$x^2(\omega) \geq 0, \forall \omega \in \Omega.$$



EMP/SP extensions to facilitate these models

Risk Measures

- Modern approach to modeling risk aversion uses concept of risk measures
- $\overline{CVaR}_\alpha$: mean of upper tail beyond α-quantile (e.g. $\alpha = 0.95$)



- mean-risk, mean deviations from quantiles, VaR, CVaR
- Much more in mathematical economics and finance literature
- Optimization approaches still valid, different objectives, varying convex/non-convex difficulty

Stochastic unit commitment: different risk measures

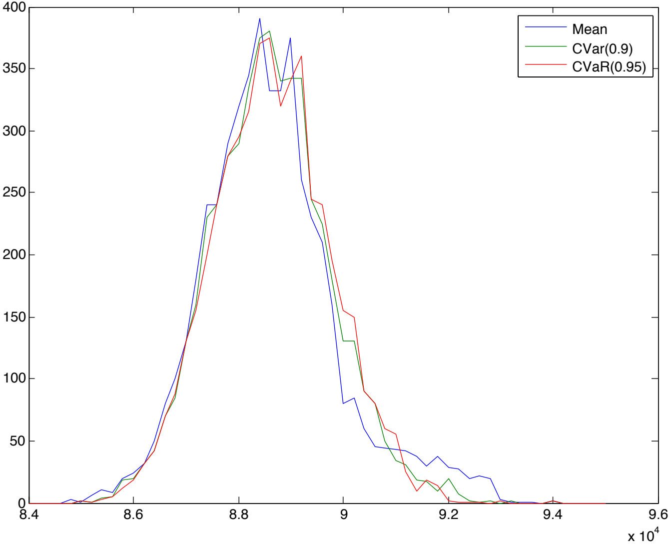


Figure : Frequency plot for cost for 5000 (out-of-sample) simulations

Equilibrium or optimization?

- Each agent has its own risk measure
- Is there a system risk measure?
- Is there a system optimization problem?

$$\min \sum_i C(x_i^1) + \rho_i \left(C(x_i^2(\omega)) \right) ????$$

- Can we modify (complete) system to have a social optimum by trading risk?
- How do we design these instruments? How many are needed? What is cost of deficiency?
- Can we solve efficiently / distributively?

Contracts in MOPEC (F./Wets)

- Competing agents (consumers, or generators in energy market)
- Each agent minimizes objective independently (cost)
- Market prices are function of all agents activities
- Additional twist: model must "hedge" against uncertainty
- Facilitated by allowing contracts bought now, for goods delivered later (e.g. Arrow-Debreu Securities)
- Conceptually allows to transfer goods from one period to another (provides wealth retention or pricing of ancilliary services in energy market)
- Can investigate new instruments to mitigate risk, or move to system optimal solutions from equilibrium (or market) solutions

Example as MOPEC: agents solve a Stochastic Program

Buy y_i contracts in period 1, to deliver $D(\omega)y_i$ in period 2, scenario ω
Each agent i:

$$\begin{aligned} \min \quad & C(x_i^1) + \rho_i \left(C(x_i^2(\omega)) \right) & \\ \text{s.t.} \quad & p^1 x_i^1 + v y_i \le p^1 e_i^1 & \text{(budget time 1)} \\ & p^2(\omega) x_i^2(\omega) \le p^2(\omega)(D(\omega) y_i + e_i^2(\omega)) & \text{(budget time 2)} \end{aligned}$$

$$0 \le v \perp -\sum_i y_i \ge 0 \qquad \text{(contract)}$$

$$0 \le p^1 \perp \sum_i \left(e_i^1 - x_i^1 \right) \ge 0 \qquad \text{(walras 1)}$$

$$0 \le p^2(\omega) \perp \sum_i \left(D(\omega) y_i + e_i^2(\omega) - x_i^2(\omega) \right) \ge 0 \qquad \text{(walras 2)}$$

Theory and Observations

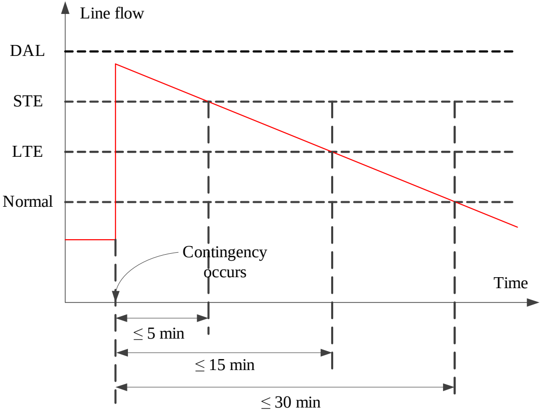
- agent problems are multistage stochastic optimization models
- perfectly competitive partial equilibrium still corresponds to a social optimum when all agents are risk neutral and share common knowledge of the probability distribution governing future inflows
- situation complicated when agents are risk averse
    - utilize stochastic process over scenario tree
    - under mild conditions a social optimum corresponds to a competitive market equilibrium if agents have time-consistent dynamic coherent risk measures and there are enough traded market instruments (over tree) to hedge inflow uncertainty
- Otherwise, must solve the stochastic equilibrium problem
- Solution possible via disaggregation only seems possible in special cases
    - When problem is block diagonally dominant (Wathen/F./Rutherford)
    - When overall (complementarity) problem is monotone
    - (Pang): when problem is a potential game
- Research challenge: develop reliable algorithms for large scale decomposition approaches to MOPEC

Security-constrained Economic Dispatch

- Base-case network topology g_0 and line flow x_0.
- If the k-th line fails, line flow jumps to x_k in new topology g_k.
- Ensure that x_k is within limit, for all k.
- SCED model:

$$
\begin{array}{llr}
\min\limits_{u,x_0,\ldots,x_k} & c^T u + \rho(u) & \triangleright \text{ Total cost} \\
\text{s.t.} & 0 \le u \le \bar{u} & \triangleright \text{ GEN capacity const.} \\
& g_0(x_0, u) = 0 & \triangleright \text{Base-case network eqn.} \\
& -\bar{x} \le x_0 \le \bar{x} & \triangleright \text{Base-case flow limit} \\
& g_k(x_k, u) = 0, \quad k = 1, \ldots, K & \triangleright \text{Ctgcy network eqn.} \\
& -\bar{x} \le x_k \le \bar{x}, \quad k = 1, \ldots, K & \triangleright \text{Ctgcy flow limit}
\end{array}
$$

Reality offers a sweeter deal...



Operating procedure (ISO-NE) requires post-contingency line loadings be:

- $\leq$ STE (short time emergency) rating in 5 minutes;
- $\leq$ LTE (long time emergency) rating in 15 minutes;
- $\leq$ Normal rating in 30 minutes.

What we will contribute

Research issues:

- Corrective actions are not modeled in ISO's dispatch software.
- Because it was "insolvable" due to its large size ($\geq$ 10GB LP).
  - "We looked into SCED with corrective actions before, and were hindered by the computational challenge." – Feng Zhao, senior analyst at ISO-NE, via private correspondence.

Our contributions:

- We **model** the *multi-period* corrective rescheduling in SCED; solutions much better quality
- **Enhance** the Benders' **algorithm** to solve the problem faster
- **Achieve** about 50$\times$ **speedup** compared to traditional approaches

Our model (K contingencies, T periods)

$$
\begin{array}{lll}
\min\limits_{x_0,\ldots,x_k,u_0,\ldots,u_k} & c^T u_0 & \\
\text{s.t.} & g_0(x_0, u_0) = 0 & \\
& h_0(x_0, u_0) \leq 0 & \\
& g_k(x_k^t, u_k^t) = 0 & k = 1, \ldots, K,\ t = 0, \ldots, T \\
& h_k(x_k^t, u_k^t) \leq 0 & k = 1, \ldots, K,\ t = 0, \ldots, T \\
& |u_k^t - u_k^{t-1}| \leq \Delta_t & k = 1, \ldots, K,\ t = 1, \ldots, T \\
& u_k^0 - u_0 = 0 & k = 1, \ldots, K
\end{array}
$$

- Subscript 0 indicates a quantity in the base-case network topology.
- This is a large-scale linear program.
- What special structure does it have?

Model structure

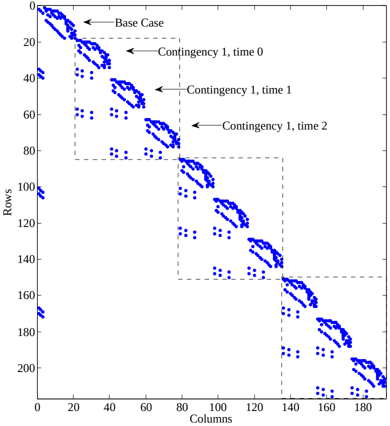


Figure : Sparsity structure of the Jacobian matrix of a 6-bus case, considering 3 contingencies and 3 post-contingency checkpoints.

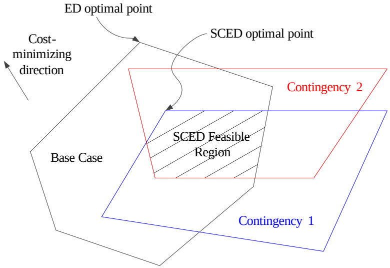


Figure : On the u_0 plane, the feasible region of a SCED is the intersection of K+1 polyhedra.

Current state of the art (unsatisfactory)

Table : CPLEX v.s. Vanilla Benders Algorithm

Case	Ctgcy	Big LP (time)		Vanilla Benders		
		Simplex	Barrier[1]	Iter	LPs	Time
118-bus	183	207.8	13.8	8	1464	123.5
2383-bus	20	175.0	205.5	52	1040	1281.2
2383-bus	50	1403.2	123.1	49	2450	2799.3
2383-bus	100	3621.8	240.6	32	3200	3688.6
2383-bus	400	-	2354.5	-	-	-

- Three time-periods: 5-min STE, 15-min LTE and 30-min Normal.
- Vanilla Benders' algorithm is inferior to the big LP formulation.
- Big LP cannot handle large instances.

[1] Barrier method without crossover. Crossover may take even more time.

How we enhanced the Benders' algorithm ...

1. Reduce the number of LPs
2. Solve subproblem LPs faster
3. Parallel computing
4. Add difficult contingencies to master model

Case	Ctgcy	Big LP (time)		**Enhanced Benders**		
		Simplex	Barrier	Iter	LPs	Time
118-bus	183	207.8	13.8	12	755	13.5
2383-bus	20	175.0	205.5	11	60	41.5
2383-bus	50	1403	123.1	11	135	46.5
2383-bus	100	3621	240.6	12	245	79.4
2383-bus	400	-	2354.5	13	879	197.8
2383 wp	2349			21	9529	515.7
2736 sp	2749			4	5500	220.9
2737 sop	2753			1	2753	100.5
2746 wop	2794			1	2794	118.5
2746 wp	2719			14	5558	333.5

Illustration

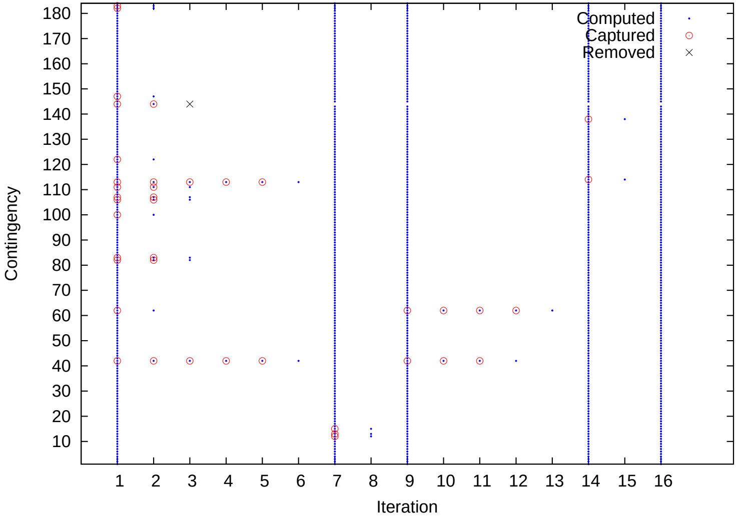


Figure : Benders' algorithm with reduced number of subproblem LPs, 118-bus case

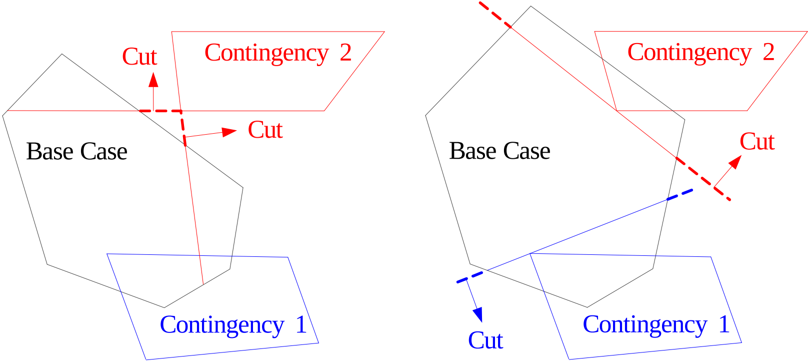
Computational Results

Case	Ctgcy	RedLP+Opt			Paraguss (8)			Fatmaster (5)		
		Iter	LPs	Time	Iter	LPs	Time	Iter	LPs	Time
118-bus	183	10	764	72.6	14	776	15.1	12	755	13.5
2383 wp	20	46	115	99.8	48	117	95.4	11	60	41.5
2383 wp	50	48	193	160.3	48	193	101.7	11	135	46.5
2383 wp	100	33	289	226.0	32	288	96.3	12	245	79.4
2383 wp	400	35	953	913.3	38	956	218.0	13	879	197.8

Case	Ctgcy	RedLP+Opt			Paraguss (40)			Fatmaster (5)		
		Iter	LPs	Time	Iter	LPs	Time	Iter	LPs	Time
2383wp	2349	106	12123	12165	104	9788	770	21	9529	516
2736sp	2749	45	5543	5836	44	5542	366	4	5500	221
2737sop	2753	1	2753	2801	1	2753	100	1	2753	101
2746wop	2794	1	2794	3046	1	2794	118	1	2794	119
2746wp	2719	262	8646	9738	278	8622	1428	14	5558	334

- Big LP for 2383-bus 2349-contingency case generates a 18GB LP. CPLEX could not solve it in 3 hours.
- Computer used for the lower table: Dell R710 (opt-a006) 2 3.46G Chips 12 Cores, 288G Memory.

Dealing with Infeasibility



(a) Contingency 2 is intrinsically infeasible. Either the corresponding subproblem is infeasible or its Benders' cuts will render the master problem infeasible.

(b) Each individual contingency is feasible, but they are not simultaneously feasible. Their Benders' cuts will render the master problem infeasible.

Figure : Two cases of infeasibility.

Identifying infeasible contingencies in Benders' algorithm

- If a subproblem is infeasible (in the first iteration), the corresponding contingency is intrinsically infeasible. Remove (tabu) it.
  - Typically line failure results in an islanded load node or sub-network.
- Master problem infeasible: solve a modified master model to find the "minimal" set of problematic contingencies using sparse optimization.

$$
\begin{aligned}
\min_{x_0,u_0} \quad & f_0(x_0,u_0) + \sum_{k\in K} M v_k \\
\text{s.t.} \quad & g_0(x_0,u_0) = 0, h_0(x_0,u_0) \le 0 \\
& \bar{w}_k^i + \bar{\lambda}_k^i (u_0 - \bar{u}_0^i) - v_k \le 0, v_k \ge 0 \qquad \forall (k,i) \in \text{CUT}
\end{aligned}
$$

  - Solution of this model indicates the violated cuts.
  - Tabu the contingency that has contributed one or more violated cuts.
- Start a pre-screening daemon in parallel when the Active List size is smaller than L^{fc}.
  - Tabu infeasible ones, and add feasible ones to the master problem.

Computational Results

Table : Solution for big cases on opt-a006, 80 threads, $L^{fc} = 5$

Case	Ctgcy	Iter	LPs	Time	To Master	Tabu
2383 wp	2896	15	7694	522.1	6	547
2736 sp	3269	4	6020	252.9	1	520
2737 sop	3269	4	6023	242.2	0	516
2746 wop	3307	4	6102	280.2	0	513
2746 wp	3279	8	6053	334.3	4	560
2383 wp	2353	16	7156	460.6	6	4
2736 sp	2749	4	5498	245.9	1	0
2737 sop	2753	1	2753	110.8	0	0
2746 wop	2794	1	2794	131.7	0	0
2746 wp	2719	14	5558	354.4	4	0

- Upper: all lines are in the Contingency List (N-1 security).
- Lower: all pre-screened lines are in the Contingency List.

SCED with SDP subproblems

- Economic dispatch is a short-term planning problem, so a “DC” model is OK.
- Contingency response is an operational problem, and should be studied on full AC network representation.
- But AC power flow gives a nonconvex problem, which cannot generate valid cuts from a Benders’ subproblem.

Idea

Relaxing the AC feasibility problem using semi-definite programming (SDP) to obtain a convex subproblem.

Goal

Producing a base-case dispatch solution such that contingencies are “really” controllable in the AC context.

SDP relaxation of AC feasibility problem

Model ACF-SDP:

$$\min_{W \succeq 0} \quad A_0 \bullet W$$

$$\text{s.t.} \quad \sum_{g \in \mathcal{G}_i} \underline{G}_g^{\text{real}} - D_i^{\text{real}} \leq A_{1i} \bullet W \leq \sum_{g \in \mathcal{G}_i} \bar{G}_g^{\text{real}} - D_i^{\text{real}} \qquad \forall i \in BUS$$

$$\sum_{g \in \mathcal{G}_i} \underline{G}_g^{\text{imag}} - D_i^{\text{imag}} \leq A_{2i} \bullet W \leq \sum_{g \in \mathcal{G}_i} \bar{G}_g^{\text{imag}} - D_i^{\text{imag}} \qquad \forall i \in BUS$$

$$-\bar{F}_{i,j} \leq A_{3ij} \bullet W \leq \bar{F}_{i,j} \qquad \forall (i,j) \in LINE$$

$$(\underline{V}_i)^2 \leq A_{4i} \bullet W \leq (\bar{V}_i)^2 \qquad \forall i \in BUS$$

$$\sum_{g \in \mathcal{G}_i} (G_g^0 - \Delta_g) \leq A_{5i} \bullet W \leq \sum_{g \in \mathcal{G}_i} (G_g^0 + \Delta_g) \qquad \forall i \in BUS$$

- It is a convex optimization problem and weak (strong) duality holds.
- It is a relaxation because the requirement that W has rank 1 is dropped.

Experiments

Case	Cont	Solution				Performance		
		Model	Tabu	Cost	Time	IF	FS	FT
14	20	LP	0	13253.3	4.2	12	12	0
		SDP	6	16065.8	18.4	6	0	0
		SDP0	6	16003.4	11.9	6	0	0
30	40	LP	0	582.0	4.0	1	1	0
		SDP	1	585.0	20.1	1	0	0
		SDP0	1	600.5	22.1	1	0	0
57	20	LP	0	12508.0	1.9	1	1	0
		SDP	1	12508.0	13.2	1	0	0
		SDP0	1	12560.0	50.9	1	0	0
118	15	LP	0	139716.8	54.0	16	16	0
		SDP	0	141372.2	2414.3	1	1	0
		SDP0	0	144220.1	11951.1	0	0	0

- SDP subproblem is "exact" in contingency response, no False Secure, no False Tabu.
- It takes longer time to solve (with room for improvement).

Summary

1. SCED is a million-dollar problem for system operators.
2. SCED with corrective actions can save money, but is hard to solve.
    - Too big for CPLEX
    - Original Benders' decomposition algorithm is slow.
3. Our algorithmic enhancements yield significant speedup.
4. Potential for practical deployment.
5. SDP extension allows for more accurate operational modeling.

Extension

1. Decomposition approach is useful in many applications.
2. Currently in collaboration with ISO-NE to deploy our algorithm.

Conclusions

- Optimization critical for understanding of power system markets
- Different behaviors are present in practice and modeled here
- Modern optimization within applications requires multiple model formats, computational tools and sophisticated solvers
- Policy implications addressable using MOPEC
- Stochastic MOPEC models capture behavioral effects (as an EMP)
- Extended Mathematical Programming available within the GAMS modeling system
- Modeling, optimization, statistics and computation embedded within the application domain is critical

What is EMP?

Annotates existing equations/variables/models for modeler to provide/define additional structure

- equilibrium
- vi (agents can solve min/max/vi)
- bilevel (reformulate as MPEC, or as SOCP)
- disjunction (or other constraint logic primitives)
- randvar
- dualvar (use multipliers from one agent as variables for another)
- extended nonlinear programs (library of plq functions)

Currently available within GAMS

Convex Energy Functions for Power Systems Analysis

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Los Alamos National Laboratory

LANL Grid Science Conference
January 15, 2015

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2. Energy Functions for Power Systems
3. Applications of Convex Energy Functions
4. Convexity of Energy Function
5. Conclusions and Future Work

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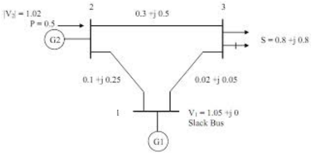
Power System Operations



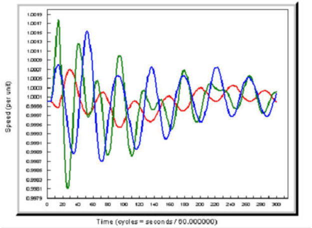
Power Flow Analysis

State Estimation

Power System Operations

Generator Control

Transient Stability Analysis



Power System Operations: Typical Assumptions

Traditional Assumptions and Approach

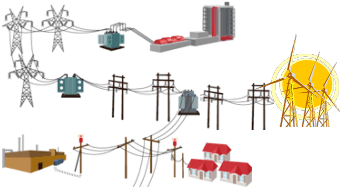
- Predictable Loads and Generation
- Power Flow Directions mostly known
- Linearized Analysis+Real-time simulations/monitoring
- Heuristic Approaches to Nonlinearities

Power Grids: The Future

The grid is changing:

1. large number of distributed power sources
2. increasing adoption of renewables

$\Rightarrow$ large-scale, complex, & heterogeneous networks with stochastic disturbances



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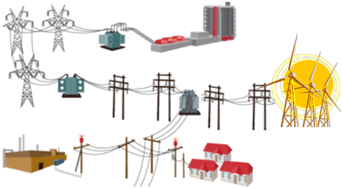


Implications

- Linearized Analysis (DC Power Flow) no longer sufficiently accurate
- Need efficient and reliable algorithms for "nonlinear" power systems analysis

Notation

Notation

Nodes=Buses i, Edges=Lines $(i,j) \in \mathcal{E}$

Voltage phasor $V_i \exp(\mathbf{i}\theta_i)$, $\mathcal{V} = \{V_i \exp(\mathbf{i}\theta_i)\}$

Complex Admittance $Y_{ij} = G_{ij} + \mathbf{i}B_{ij}$

Complex Power Injection $S_i = P_i + \mathbf{i}Q_i$

Complex Current Injection I_i, $\rho_i = \log(V_i), \rho_{ij} = \rho_i - \rho_j, \theta_{ij} = \theta_i - \theta_j$

Bus Types

Bus Type	Fixed Quantities	Variable Quantities
(P,V) buses (Generators)	$P_i, V_i = v_i$	P_i, θ_i
(P,Q) buses (Loads)	P_i, Q_i	V_i, θ_i
Slack Bus	$\theta_{\mathcal{S}}, \rho_{\mathcal{S}} = 0$	P_i, Q_i

Why is Power Systems Analysis Hard?

Linear Circuits Analysis

I given, find $\mathcal{V}$

$$I = Y\mathcal{V}$$

Linear Equation, Easy

Power Flow Analysis

S given, find v

$$S = \mathcal{V}.*\overline{I} = \mathcal{V}.*\left(\overline{Y\mathcal{V}}\right)$$

Multivariate Quadratic Equations: Hard!

Power Flow Equations in Polar Coordaintes

Power Flow Equations

$$P_i = \sum_j V_i V_j \left(B_{ij} \sin\left(\theta_i - \theta_j\right) + G_{ij} \cos\left(\theta_i - \theta_j\right)\right) \quad \forall i$$

$$Q_i = \sum_j V_i V_j \left(G_{ij} \sin\left(\theta_i - \theta_j\right) - B_{ij} \cos\left(\theta_i - \theta_j\right)\right) \quad \forall i \in \mathrm{pq}$$

$$V_i = v_i \quad \forall i \in \mathrm{pv}$$

Traditional Solution Methods

Multivariate nonlinear equations

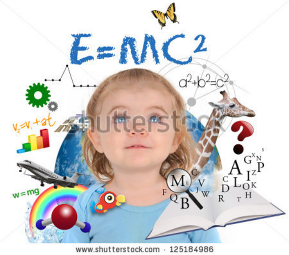
Solved via Iterative Linearization: Newton-Raphson

Works well under "nice conditions"

What if solver fails? No solution?

Our Solution: Use the physics!

1. Use energy function as analysis tool
2. Variational formulation of power flow equations
3. Computational tractability via convexity



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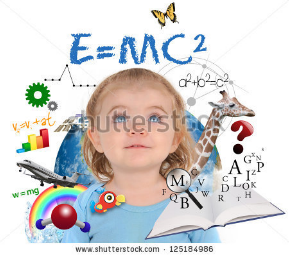

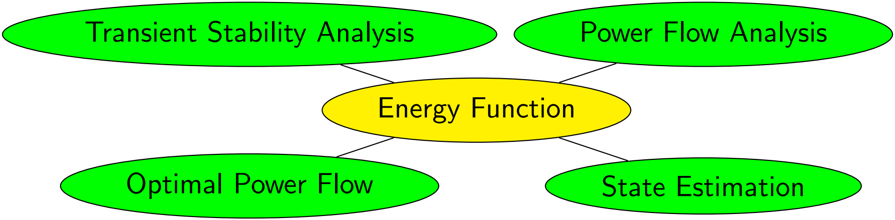

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Variational Formulation for Resistive DC Networks

Loss Minimization in Resistive Network

$$\underset{\{I_{ij}:(i,j)\in\mathcal{E}\}}{\text{Minimize}} \sum_{(i,j)\in\mathcal{E}} \frac{1}{2} I_{ij}^2 R_{ij} \text{ (Resistive Losses)}$$

$$\text{Subject to } I_i = \sum_{j:(i,j)\in\mathcal{E}} I_{ij} - \sum_{j:(j,i)\in\mathcal{E}} I_{ji} \qquad \text{(Conservation of Current)}$$

Solution of Loss Minimization

$$L(I,V) = \sum_{(i,j)\in\mathcal{E}} I_{ij}^2 R_{ij} + (V_j - V_i) I_{ij} - \sum_i I_i V_i$$

$$\frac{\partial L}{\partial I_{ij}} = 0 \equiv I_{ij} = \frac{V_i - V_j}{R_{ij}} \equiv \text{ Ohm's Law!}$$

Variational Formulation for Lossless AC Networks

Reactive Loss Minimization in a Lossless AC Network

$$\underset{\{I_{ij}:(i,j)\in\mathcal{E}\}}{\text{Minimize}} \sum_{(i,j)\in\mathcal{E}} B_{ij} \int_{-\frac{\pi}{2}}^{\frac{f_{ij}}{B_{ij}}} \arcsin(y)\,\mathrm{d}\,y \qquad \text{(Reactive Losses?)}$$

$$\text{Subject to } P_i = \sum_{j:(i,j)\in\mathcal{E}} f_{ij} - \sum_{j:(j,i)\in\mathcal{E}} f_{ji} \qquad \text{(Conservation of Active Power)}$$

$$|f_{ij}| \leq B_{ij}$$

Solution of Reactive Loss Minimization

$$L(f,\theta) = \sum_{(i,j)\in\mathcal{E}} B_{ij} \int_{-\frac{\pi}{2}}^{\frac{f_{ij}}{B_{ij}}} \arcsin(y)\,\mathrm{d}\,y + (\theta_j - \theta_i)\, f_{ij}$$

$$\frac{\partial L}{\partial f_{ij}} = 0 \equiv f_{ij} = B_{ij} \sin(\theta_i - \theta_j) \equiv \text{ Active Power}$$

[Bent et al., 2013][Boyd and Vandenberghe, 2009]

Variational Formulation for Lossless AC Networks

Dual Form

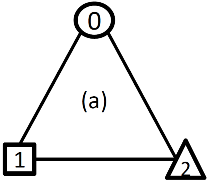
$$\underset{\theta}{\text{Minimize}} \sum_{(i,j)\in\mathcal{E}} -B_{ij}\cos\left(\theta_i-\theta_j\right) - \sum_i P_i\theta_i$$

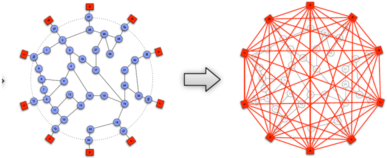
$$\text{Subject to } |\theta_i-\theta_j| \leq \frac{\pi}{2}$$

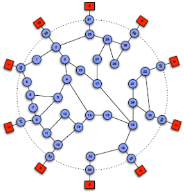
[Bergen and Hill, 1981]

Energy Functions for Power Systems - History

1958	2/3-bus case	Aylett
1960-1980	Kron Reduction Ignore transfer conductances	Various authors Summarized in Pai, 1981
1981-1990	Structure preserving models	Bergen/Hill Varaiya et al Van Cutsem et al







Energy Function for Lossless Power Systems

Main Assumption

Transmission lines purely inductive $G_{ij} = 0$

Energy Function for Power Systems

$$E\left(\rho,\theta\right) = -\sum_{i} P_i\theta_i - \sum_{i\in\mathrm{pq}} Q_i\rho_i - \frac{1}{2}\sum_{j,k} B_{jk}\exp\left(\rho_i+\rho_j\right)\cos\left(\theta_i-\theta_j\right)$$

[Cutsem and Ribbens-Pavella, 1985][Narasimhamurthi and Musavi, 1984]

Energy Function Properties

Stationary Points $\equiv$ Power Flow Equations

$$\frac{\partial E(\rho,\theta)}{\partial \theta_i} = 0 \equiv P_i = \sum_{j \neq i} B_{ij} \exp(\rho_i + \rho_j) \sin(\theta_i - \theta_j)$$

$$\frac{\partial E(\rho,\theta)}{\partial \rho_i} = 0 \equiv Q_i = \sum_{j \neq i} B_{ij} \left(\exp(\rho_i + \rho_j) \cos(\theta_i - \theta_j) - \exp(2\rho_i)\right)$$

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Convexification Roadmap

Determine Region of Convexity $\mathcal{C}$ of $E(\rho, \theta)$

$\downarrow$

Operational Constraints $\subset \mathcal{C}$

$\downarrow$

Variationally enforce power flow constraints in convex way

Convexity Region and Power Flow Solutions

Why seek solutions in Convexity Region?

- Solutions guaranteed to be locally stable with respect to Swing Equation Dynamics.
- Boundary of convexity region, Hessian singular $\implies$ Jacobian of power flow equations singular
- For tree networks, PF solvable if and only if PF solvable in Convexity Region
- Related to "Principal Singular Surfaces", "Stable Region" [C.J.Tavora and O.J.M.Smith, 1972] [Araposthatis et al., 1981] (all (P, V) buses)

Convex Power Flow Solvers

Energy function $E(\rho, \theta)$ convex over domain $\mathcal{C}$

$$(\rho^*, \theta^*) = \underset{\rho,\theta\in\mathcal{C}}{\operatorname{argmin}}\, E(\rho, \theta) \tag{1}$$

Power Flow Certificate

$(\rho^*, \theta^*) \in \operatorname{int}(\mathcal{C}) \implies (\rho^*, \theta^*)$ is PF soln
$(\rho^*, \theta^*) \notin \operatorname{int}(\mathcal{C}) \implies$ No PF soln in $\operatorname{int}(\mathcal{C})$

Open Questions

When can $(\rho^*, \theta^*) \in \operatorname{int}(\mathcal{C})$ be guaranteed? Trees?
"Critical Slowdown" of power flow solvers near collapse avoided?

Optimal Power Flow

$$\underset{P,Q,\rho,\theta}{\text{Minimize}} \quad c(P,Q) \quad \text{(Strictly convex generation cost)} \tag{2}$$

$$\text{Subject to} \nabla_\theta E(\rho,\theta;P,Q)=0, \nabla_\rho E(\rho,\theta;P,Q)=0 \quad \text{Nonconvex PF Eqn}$$

$$(\rho,\theta)\in S \quad \text{Operational Constraints, Convex}$$

Energy function $E(\rho,\theta;P,Q)$ strictly convex over domain $\mathcal{C}$, $S\subset\mathcal{C}$

Convex-Concave Saddle Point OPF

$$(P^*,Q^*,\rho^*,\theta^*)=\underset{P,Q}{\text{argmax}}\,\underset{\rho,\theta\in S}{\text{argmin}}\,\lambda E(\rho,\theta;P,Q)-c(P,Q)$$

Convex Optimal Power Flow Solvers

Optimal Power Flow Certificate ($\lambda \to 0$)

$(\rho^*, \theta^*) \in \operatorname{int}(S) \implies (P^*, Q^*, \rho^*, \theta^*)$ is soln to (2)

(2) has soln with $(\rho^*, \theta^*) \in \operatorname{int}(S) \implies$ soln optimal for Saddle Point OPF

Open Questions

What operational constraints can be encoded in S? Apparent power, voltage magnitude bounds ...

When is S subset of $\mathcal{C}$?

Relationship to SDP/SOCP relaxations of OPF?

A general strategy for classes of QCQPs? Polar vs Cartesian

Topology Estimation

Sparse Topology Estimation from Phasor Measurements

Measurements $\{(\rho^i, \theta^i)\}_{i=1}^k$

Unknown B_{ij}

$$\min \sum_i E\left(\rho^i, \theta^i; B\right) - \min_{(\rho,\theta)} E\left(\rho, \theta; B\right) + \lambda \left\|B\right\|_1$$

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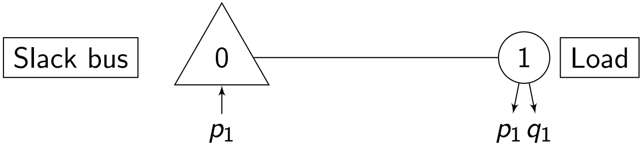
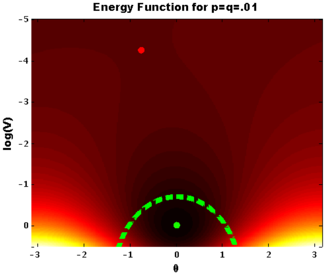
Intuition from 2-bus case

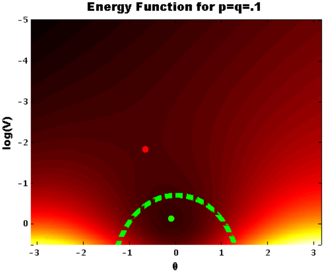


Energy Function for 2-bus case

$$E(\rho_1, \theta_1) = b\left(\frac{1}{2}\exp(2\rho_1) - v_0 \exp(\rho_1)\cos(\theta_1)\right) - p_1\theta_1 - q_1\rho_1$$

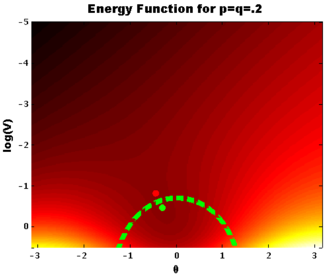

Energy Function for p=q=.01
log(V)
θ
-5
-4
-3
-2
-1
0
-3
-2
-1
0
1
2
3

(a) $p = q = .01$

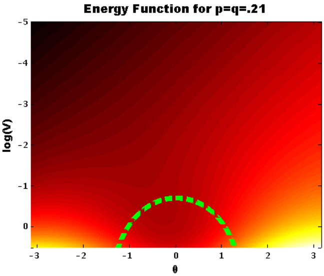

Energy Function for p=q=.1
log(V)
θ
-5
-4
-3
-2
-1
0
-3
-2
-1
0
1
2
3

(b) $p = q = .1$

Intuition from 2-bus case



(a) $p = q = .2$



(b) $p = q = .25$

Convexity of 2-bus case

Hessian of Energy Function

$$\nabla^2 E(\theta_1, \rho_1) = b \begin{pmatrix} 2\exp(2\rho_1) - v_0 \exp(\rho_1)\cos(\theta_1) & v_0 \exp(\rho_1)\sin(\theta_1) \\ v_0 \exp(\rho_1)\sin(\theta_1) & v_0 \exp(\rho_1)\cos(\theta_1) \end{pmatrix}$$

This matrix is positive semidefinite if and only if

$$\cos(\theta_1) \geq \frac{v_0 \exp(-\rho_1)}{2} = \frac{1}{2}\frac{V_0}{V_1}$$

Condition eliminates low-voltage solution

No (P, Q) nodes connected

Special topology: No transmission lines connecting (P, Q) nodes.

Condition for Convexity

$$\cos(\theta_i - \theta_j) \geq \frac{v_i \exp(-\rho_j)}{2} = \frac{\exp(\rho_i - \rho_j)}{2} \quad \forall (i, j) \in E, i \in \mathrm{pv}, j \in \mathrm{pq}$$

Let $v_i = 1\text{pu}$ at all $i \in \mathrm{pv}$.

$$|\theta_i - \theta_j| \leq 45\,\text{deg}, V_j \geq \frac{1}{\sqrt{2}} \approx .7 \text{ suffices for convexity}$$

General Topologies

Condition for Convexity: Nonlinear Convex Matrix Inequality

$$M(\rho,\theta) \succeq 0, M \in \mathcal{S}^{|\mathrm{pq}|}$$

$$[M(\rho,\theta)]_{ii} = 2\sum_{j\neq i} B_{ij} - \sum_{j\neq i} \frac{B_{ij} v_j \exp(\rho_j - \rho_i)}{\cos(\theta_{ij})}$$

$$[M(\rho,\theta)]_{ij} = -\frac{B_{ij}}{\cos(\theta_{ij})} \forall (i,j) \in \mathcal{E}, i,j \in \mathrm{pq}$$

Conservatism in Convexity Region Estimate

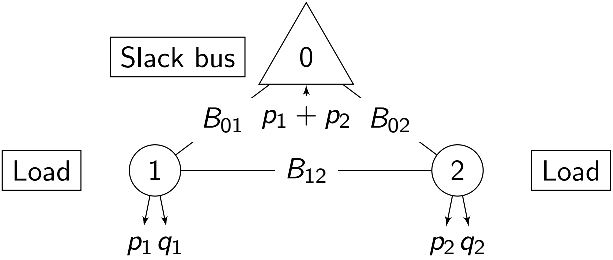
Condition for Convexity: Sufficient, but Necessary?

- For tree networks, yes
- In general, unknown: Initial numerical tests show necessity for small networks
- Holds for all test systems available in MATPOWER
- "Almost all" power flow solutions within convexity domain

Open Questions

- Relationship to existence of power flow solutions: Answered for trees
- Closing the gap for non-tree networks

Estimated vs Actual Regions of Convexity: 3-bus network

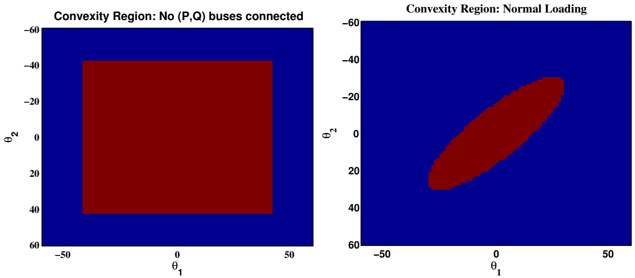


Reduced Energy Function

Solve for ρ as a function of θ

Plug into energy function $E(\rho(\theta), \theta)$

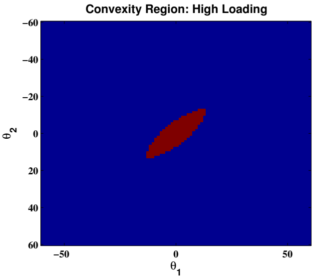
Estimated vs Actual Regions of Convexity: 3-bus network



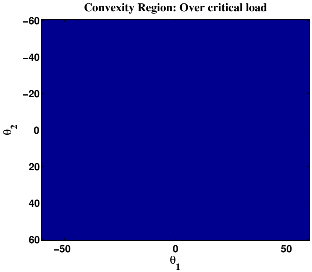
(a) No (P, Q) connected (b) Base Load

Figure: Theoretical Convexity Region=Numerical Convexity Region

Estimated vs Actual Regions of Convexity: 3-bus network



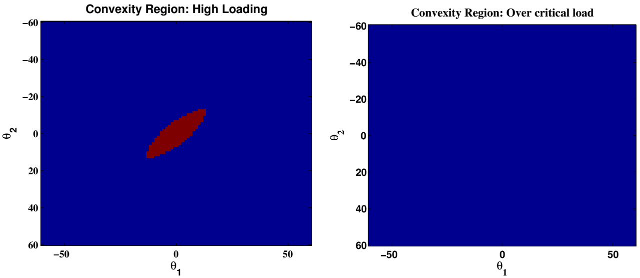
(a) 10xBase Load



(b) Critical Overload

Figure: Theoretical Convexity Region=Numerical Convexity Region

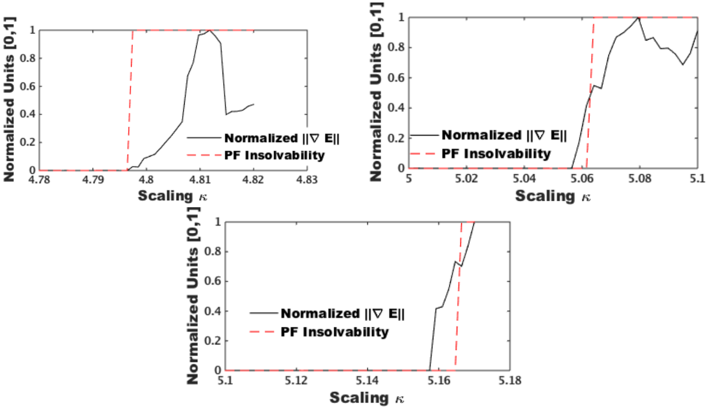
Estimated vs Actual Regions of Convexity: 3-bus network



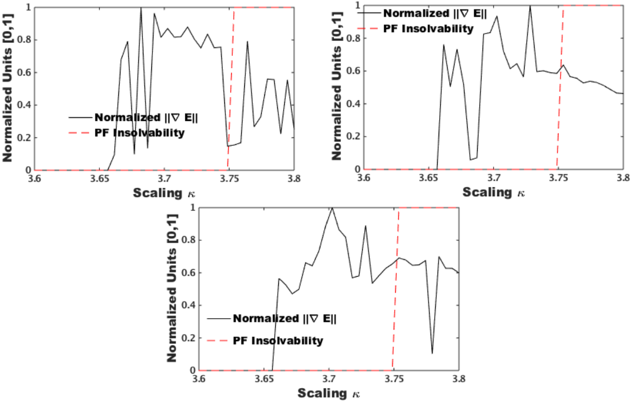
(a) 10xBase Load (b) Critical Overload

Figure: Theoretical Convexity Region=Numerical Convexity Region

Scale injections $\kappa P, \delta\kappa Q$. Detect Insolvability using SDP relaxation [Molzahn et al., 2012]

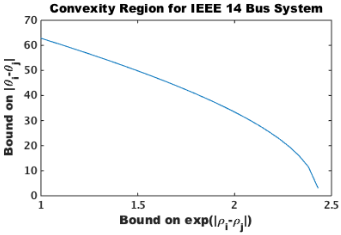


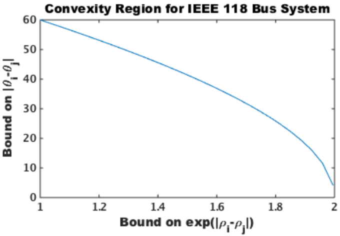
Scale injections $\kappa P, \delta\kappa Q$. Detect Insolvability using SDP relaxation [Molzahn et al., 2012]



Convexity Region Operational Constraints

Fix bound on $\max\left(\frac{V_i}{V_j}, \frac{V_j}{V_i}\right) = \exp\left(|\rho_i - \rho_j|\right) \forall (i,j) \in \mathcal{E}$.
Find maximum δ such that $|\theta_i - \theta_j| \leq \delta \forall (i,j) \in \mathcal{E} \implies (\rho, \theta) \in \mathcal{C}$.





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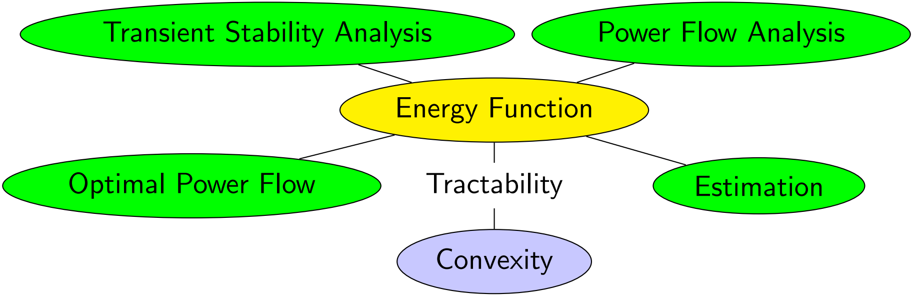
Summary

Summary

- Power of Energy Function as an Analysis Tool
- Several Applications in Different Power System Problem Domains
- "Nice" power flow solutions easy to find
- Convexity Analysis $\Longrightarrow$ Computational Tractability



Ongoing and Future Work

Ongoing Work

- Extensions to Lossy case: Fixed $\frac{B}{G}$ ratio already works
- Networks with small $\frac{B}{G}$ ratios - Initialized to region of convergence of Newton's method
- Algorithmic developments and testing on IEEE benchmarks/real systems
- Relationship to Exactness of Convex Relaxations

Future Work

- Scaling up algorithms - ADMM, Cutting plane etc.
- Other Infrastructure Networks: Gas, Transportation etc.?
- Variational modeling principles

Variational Modeling for Convexity

Nonconvex Formulation

Control Variables: u , Dependent Physical Variables: x

$$\underset{u,x}{\text{Minimize}} \quad \underbrace{f(u)}_{\text{Convex Control Cost}}$$

$$\text{Subject to} \quad \underbrace{h(u,x)=0}_{\text{Physics}}, \quad \underbrace{x \in S}_{\text{Safety Constraints}}$$

Convexity via Variational Prinicple

Variational Principle: $h(u,x)=0 \equiv \nabla_x E(u,x)=0$

$$\underset{u,x}{\text{Minimize}} \quad f(u)+\lambda E(u,x) \quad (\lambda << 1)$$

Subject to $x \in S$

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- Ian Hiskens, Scott Backhaus for initial discussions leading to this work
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- Enrique Mallada, Dan Molzahn for useful comments
- Misha Chertkov/Florian Dorfler for images used in slides

References/Questions

- Older version under review at ACC
- Journal version under development, posted to ArXiv soon.
- http://www.its.caltech.edu/∼dvij
- dvij@cs.washington.edu

Questions?

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IOWA STATE UNIVERSITY

Damping interarea oscillations with generator redispatch using synchrophasors

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is gratefully acknowledged

a general theme: PMUs + models gives actionable information

January 2015

Damping electromechanical modes of oscillation

Generator redispatch is an open loop control that works by exploiting nonlinearity: the change in the Jacobian when the operating equilibrium is changed by the redispatch. Contrast with closed loop controls directly affecting Jacobian entries.

We have derived a new formula for eigenvalue sensitivity with respect to generator redispatch.

The formula largely depends on power system quantities, such as power flow and mode shape, that can be measured.

Previous approaches that use generator redispatch to damp oscillations

1. Heuristics in terms of mode shapes [Fischer-Erlich].

2. Exact formulas for damping sensitivity from a dynamic power grid model.
   The formulas depend on both left and right eigenvectors or their derivatives.

3. Numerical eigenvalue sensitivities by repetitive computation of eigenvalues of a power grid dynamic model.

There are problems getting online dynamic grid models, especially for loads.

Model assumptions

We make usual assumptions for energy function analysis:

1. AC power flow.
2. Lossless transmission lines.
3. Generators:
   - Have constant voltage magnitude.
   - Their overall dynamics is given by the swing equation.
4. Loads that allow:
   - Active power to depend on frequency.
   - Reactive power to depend on voltage magnitude.

Eigenvalue sensitivity: New formula

Generator redispatch dP causes changes $d\theta$ in angles across the lines and changes dV^{ln} in load bus voltages:

$$dP \quad \Rightarrow \quad d\theta \text{ and } dV^{\mathrm{ln}}$$

Then changes $d\theta$, dV^{ln} cause changes $d\lambda$ in the eigenvalue:

$$d\theta \text{ and } dV^{\mathrm{ln}} \quad \Rightarrow \quad d\lambda \quad \text{(our new formula)}$$

New formula: dλ

$$d\lambda = \frac{\begin{pmatrix} \text{mode shape or right eigenvector of } \lambda\text{: } x, \\ \text{changes in angles across the lines: } d\theta, \\ \text{changes in load voltage magnitudes: } dV^{\mathrm{ln}}, \\ \text{active power flow through the lines: } p, \\ \text{part of reactive power flow through the lines: } q, \\ \text{net reactive power injection at load buses: } Q, \end{pmatrix}}{\begin{pmatrix} \text{eigenvalue } \lambda,\ \text{mode shape } x \\ \text{generator inertias } M,\ \text{bus dampings } D \end{pmatrix}}$$

$$= \frac{\left(x, d\theta, dV^{\mathrm{ln}}, p, q, Q\right)}{(\lambda, x, M, D)} = \frac{\left(x, d\theta, dV^{\mathrm{ln}}, p, q, Q\right)}{(\alpha)}$$

To rank generator redispatches we only need to know the phase of α for that mode.

New formula: $d\lambda$

The sensitivity for a nonresonant eigenvalue λ of the system is given by

$$d\lambda = -\frac{1}{\alpha}\left\{\sum_{k=1}^{\ell}\left\{[(x'_{\nu_k})^2 - (x'_{\theta_k})^2]p_k - 2x'_{\theta_k}x'_{\nu_k}q_k\right\}d\theta_k \right.$$
$$\left. + \sum_{i=m+1}^{n}\left[\sum_{k=1}^{\ell}|A_{ik}|(C_{q_k}q_k + C_{p_k}p_k) + C_{Q_i}Q_i\right]dV_i^{\ln}\right\},$$

where $\alpha = 2\lambda x^T Mx + x^T Dx$,
and C_{q_k}, C_{p_k}, C_{Q_i} are functions of x'.

Key ideas and tricks to derive the formula

1. Classical assumptions:
    - Lossless lines.
    - No dependence of load real power on voltage magnitude.

   that yield potential energy function R:

$$R = -\sum_{\substack{i,j \\ i\neq j, i\sim j}} b_{ij} V_i V_j \cos(\delta_i - \delta_j) - \sum_{i=1}^{n} (P_i \delta_i + \tfrac{1}{2} b_{ii} V_i^2 + Q_i \ln V_i)$$

   and a symmetric network Laplacian L.

2. Quadratic form of eigenvalue problem [Mallada-Tang]

$$Q(\lambda) = M\lambda^2 + D\lambda + L.$$

   Q is a symmetric complex matrix.

3. New idea of working with complex $x^T Qx$ (*not* $\bar{x}^T Qx$)
4. "Line" angle coordinates θ [Bergen-Hill] and new line voltage coordinates ν

$$\theta_k = \begin{cases} \delta_i - \delta_j & \text{if bus } i \text{ is sending end of line } k, \\ \delta_j - \delta_i & \text{if bus } i \text{ is receiving end of line } k, \end{cases}$$

$$\nu_k = \ln(V_i V_j).$$

These new coordinates greatly simplify the derivation.

Special cases

- **Mode with zero damping**. $d\lambda$ becomes purely imaginary.
- **Voltage magnitude constant**. General formula simplifies to

$$d\lambda = \sum_{k=1}^{\ell} \frac{(x'_{\theta_k})^2 p_k}{2\lambda x^T M x + x^T D x} d\theta_k \tag{1}$$

The terms of summation (1) that contribute more are those in which the product $(x'_{\theta_k})^2 p_k$ is large.

x'_{θ_k} = *change* in right eigenvector x angle across line k
p_k = real power flow through line k

Computing damping ratio after redispatch is done

- The new formula for $d\lambda$ may be used to compute the damping ratio of the interarea mode λ after redispatch is done

$$\zeta = -\frac{\mathrm{Re}\{\lambda + d\lambda\}}{|\lambda + d\lambda|} = -\frac{\sigma + d\sigma}{\sqrt{(\sigma + d\sigma)^2 + (\omega + d\omega)^2}}.$$

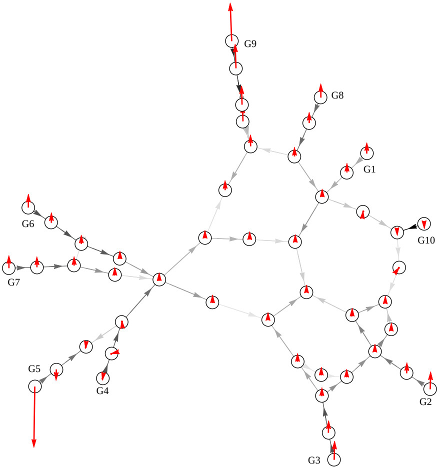
Example: New England 10-machine system

- Interarea modes at the Base Case

Mode No.	Mode λ (1/s)	f (Hz)	Damping Ratio (%)
1	-0.0403 + j3.4135	0.5433	1.1816
2	**-0.0188 + j4.7631**	**0.7581**	**0.3955**
3	-0.0249 + j5.4994	0.8753	0.4528
4	-0.0558 + j6.0159	0.9575	0.9275

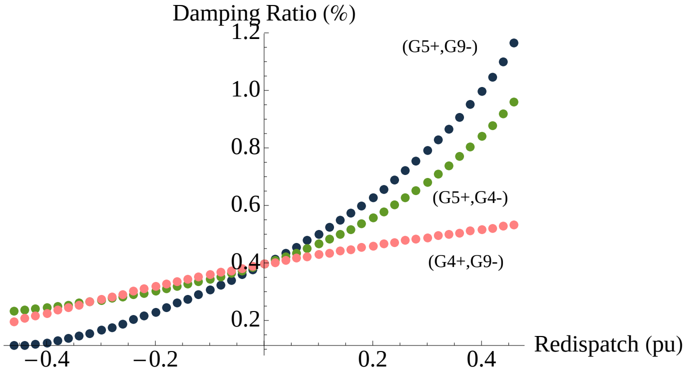
- We will look at damping mode 2 with generator redispatch

Example: New England 10-machine system



- Arrows in gray scale show the magnitude and direction of the power flow at the base case.
- Red arrows show the oscillation mode shape for λ_2.

Damping ratio of Mode 2 of New England 10-machine system



- Gradient of damping ratio at base case from formula indicates the effectiveness of larger redispatches
- Of the 45 possible pairs, pair (G5+,G9-) has the largest increase in damping ratio.

Comments on redispatch for increasing λ_2 damping ratio

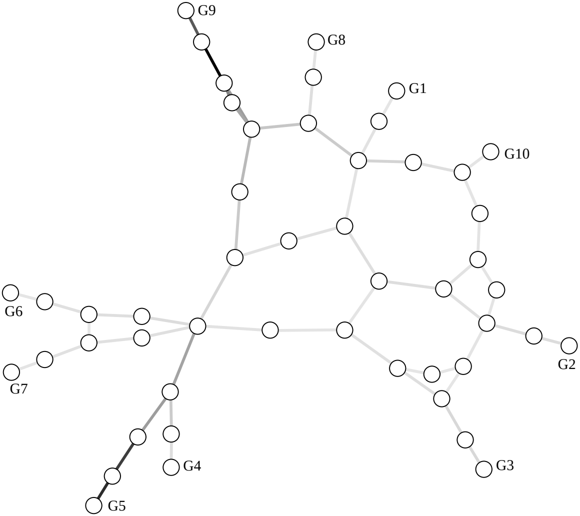
- The pairs with the largest increase in the damping ratio are the ones in which G5+ is involved, that is, G5 with an increase in its generation.
- Why G5 is playing a key role? ... get some insights from the components of the new formula.

Getting insights from Re$\{d\lambda\}$

- For this redispatch, change in $d\theta$ is larger than change in $dV^{\ln}$, so look at $d\theta$ components of formula.
- The pairs with the largest increase in damping ratio are also the ones with the largest increase in the damping of the interarea mode λ_2.
  So take the real part of the formula $d\lambda$:

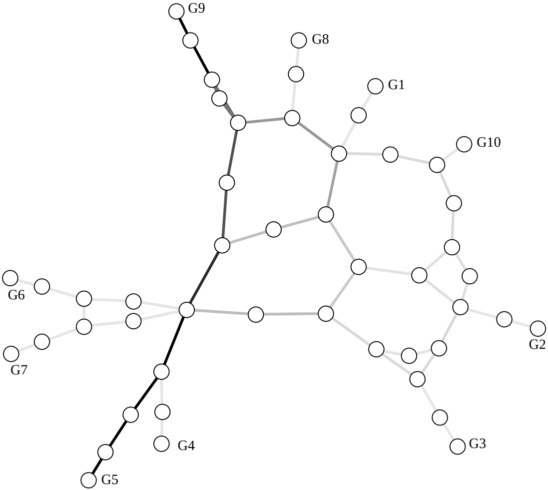
$$\begin{aligned} \mathrm{Re}\{d\lambda\} &= \sum_{i=1}^{\ell} \mathrm{Re}\{C_{\theta_k}\} d\theta_k + \sum_{i=1}^{n} \mathrm{Re}\{C_{V_i}\} dV_i^{\ln} \\ &= \mathrm{Re}\{C_\theta\} \cdot d\theta + \mathrm{Re}\{C_V\} \cdot dV^{\ln}, \end{aligned} \tag{2}$$

$\mathrm{Re}\{d\lambda\} = \mathrm{Re}\{C_\theta\} \cdot d\theta + \mathrm{Re}\{C_V\} \cdot dV^{\ln}$



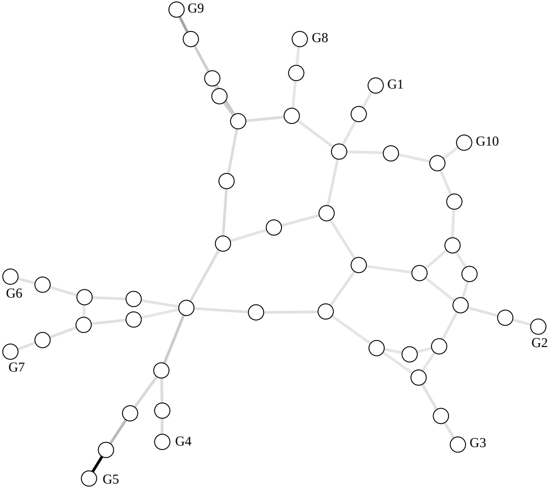
- The gray scale in lines shows $|\mathrm{Re}\{C_\theta\}|$ for λ_2.

$$\mathrm{Re}\{d\lambda\} = \mathrm{Re}\{C_\theta\} \cdot d\theta + \mathrm{Re}\{C_V\} \cdot dV^{\ln}$$



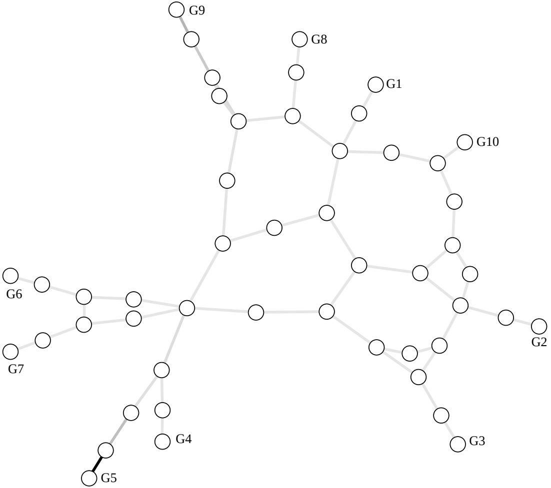
- The gray scale in lines show the changes in power dp, due to redispatch in pair (G5+,G9-).

$$\mathrm{Re}\{d\lambda\} = \mathrm{Re}\{C_\theta\} \cdot d\theta + \mathrm{Re}\{C_V\} \cdot dV^{\ln}$$



- The gray scale in lines show the changes in angles $d\theta$ across the lines, due to redispatch in (G5+,G9-).

$$\mathrm{Re}\{d\lambda\} = \mathrm{Re}\{C_\theta\} \cdot d\theta + \mathrm{Re}\{C_V\} \cdot dV^{\ln}$$



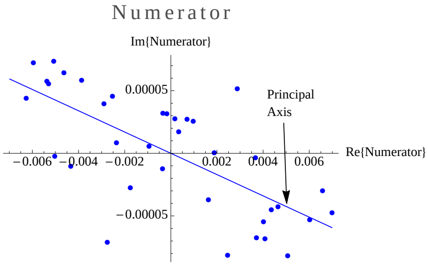
- The gray scale in lines shows $|\mathrm{Re}\{C_\theta\} \cdot d\theta|$ for redispatch (G5+,G9-).

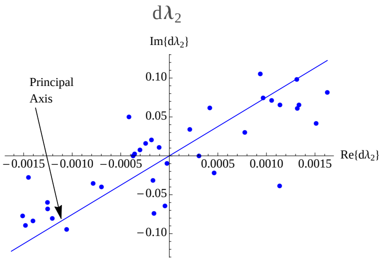
Obtaining the formula denominator's phase ($\angle\alpha$) from measurements

$$d\lambda = \frac{\text{Numerator}}{\alpha} \quad \Rightarrow \quad \angle\alpha = \angle\text{Numerator} - \angle d\lambda$$

- There are always small random load variations around an operating point.
- For such small random load variations:
  - Samples of $d\lambda$ can be obtained from PMUs.
  - Samples of $d\theta$ and $dV^{\ln}$ can be gotten from the load flow equations with simulated random load variations, then samples of the Formula's numerator can be computed.
  - The $d\lambda$ samples and the numerator samples can be analyzed with Principal Component Analysis, then the phase of α can be obtained from the Principal Axes of the samples.

Samples' plots for random loads variations generated with the software Mathematica





- Plots show the samples of 50 points after trimming by 30%.
- Principal Axes are computed and shown as lines

$$\begin{aligned}\angle\alpha &= \angle\text{Numerator} - \angle d\lambda \\ &= 179.51^\circ - 89.24^\circ = 90.27^\circ\end{aligned}$$

Conclusions

- Using a judicious combination of new and old methods, we can derive a new formula for the sensitivity of oscillatory eigenvalues λ with respect to generator redispatch.
- The formula depends on:
  1. The mode shape of λ.
  2. The eigenvalue λ of interest.
  3. The power flow through every line.

     These power system quantities can, at least in principle, be observed from measurements.
  4. The assumed equivalent generator dynamics only appear as a factor common to all redispatches.
- For purely imaginary modes the change in λ becomes purely imaginary.

Conclusions and Ongoing Work

- We have an approach to ranking the generator pairs for redispatch to damp the oscillations where the dynamics is largely determined from PMUs.
- We are exploring the insights and applications of the formula.
- We are refining the combination of synchrophasor measurements and calculations.
  Goal: Dynamics from PMUs and statics from the state estimator. Then results largely independent of poorly known dynamic models.

S. Mendoza-Armenta, I. Dobson, A formula for damping interarea oscillations with generator redispatch, IREP Symposium - Bulk Power System Dynamics and Control - IX Crete, August 2013.
http://arxiv.org/pdf/1306.3590v2.pdf

Detecting Topological Inefficiency

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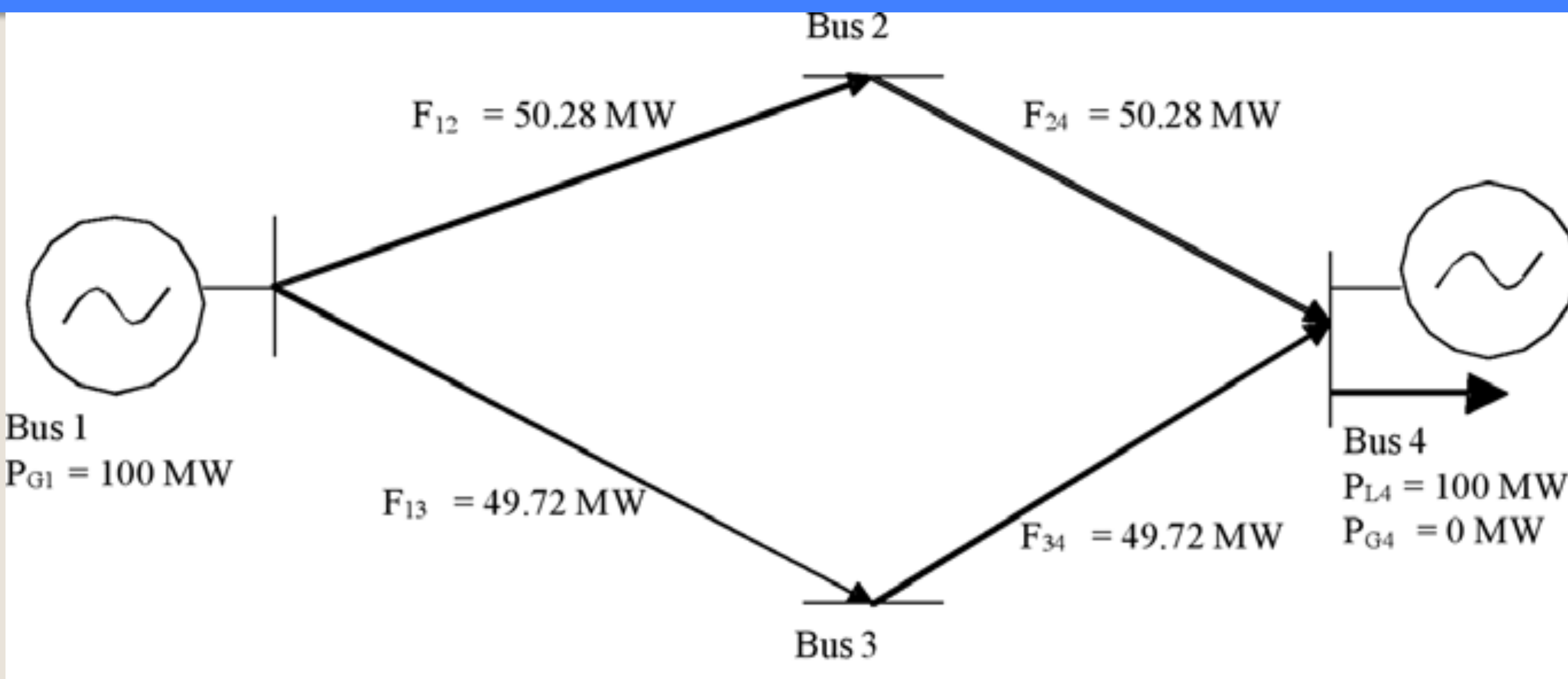
Credit and no blame: Luis Ayala (Penn State), Clayton Barrows (NREL), Russell Bent (LANL), Temitope Phillips (Chevron)

LANL Winter Grid Conference

16 January 2015

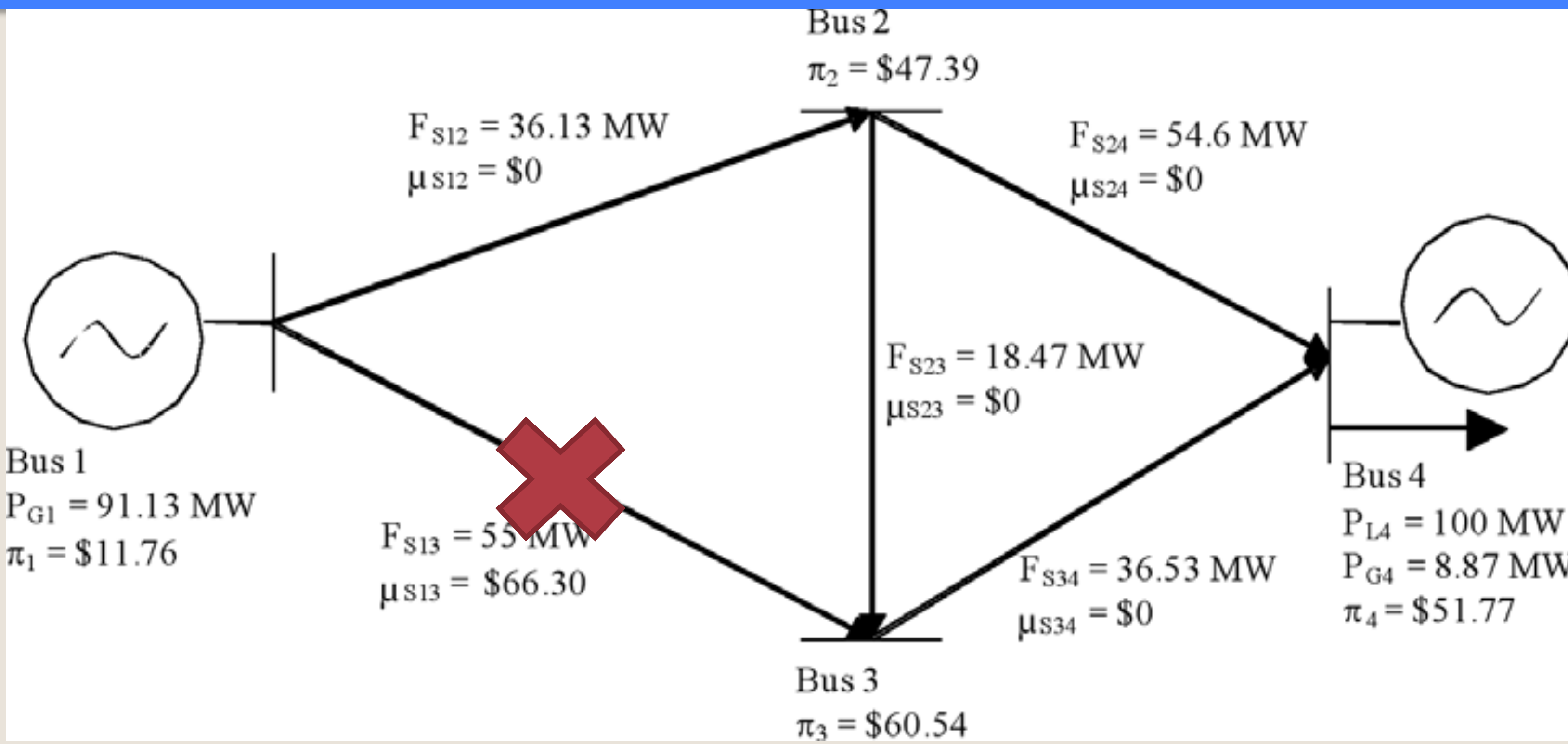
Motivating (i.e. Unrealistic) Toy Example



Rules of the toy example:

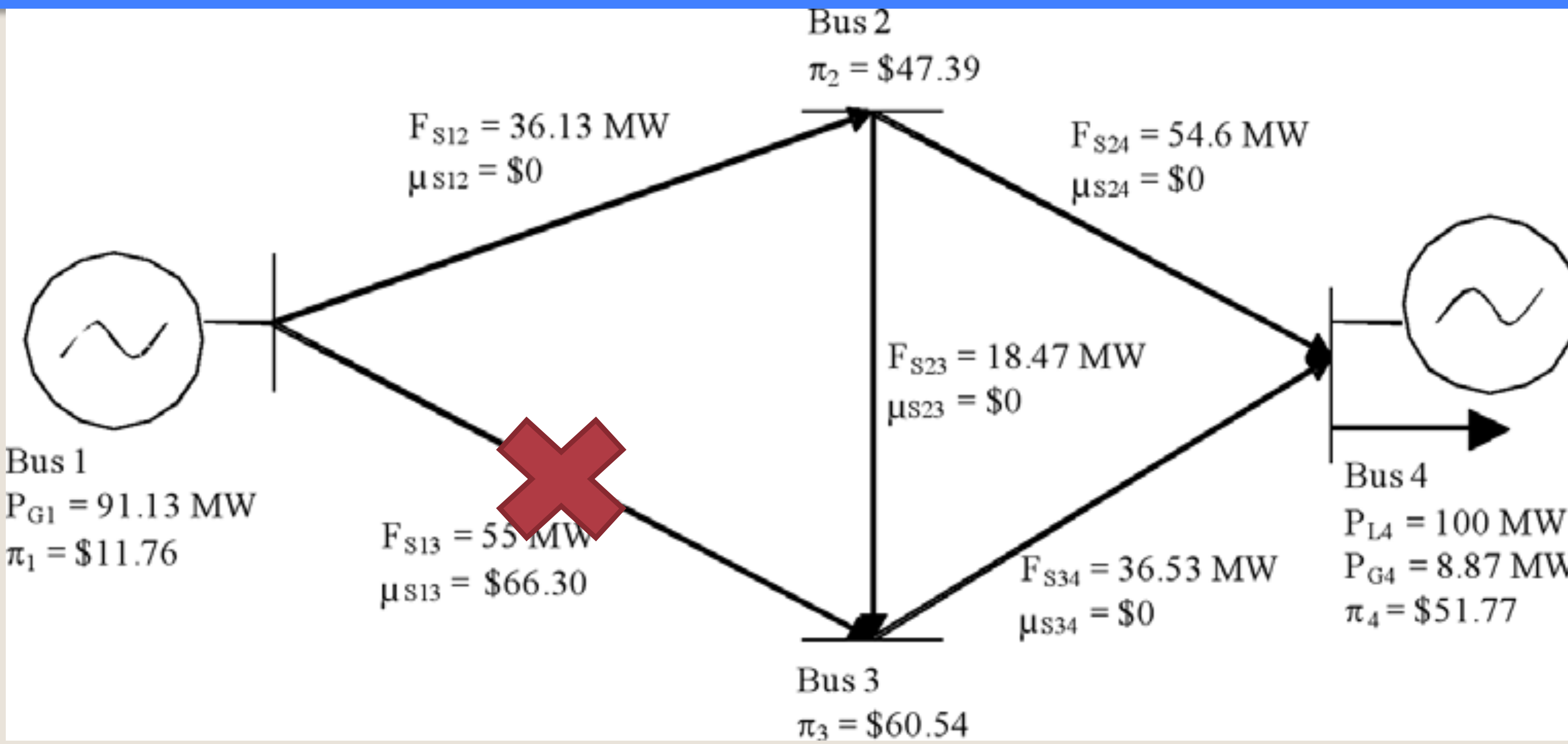
1. Cheap generation at node 1; expensive generation and customers (100 MW) at node 4;
2. All lines can carry the same fixed load (55 MW);
3. Parallel edges have the same resistance.

Building More is Not Always Better



- The link between buses 2 and 3 overloads line (1,3)
- Congestion -> Out of merit dispatch -> Higher system cost

Building More is Not Always Better



This is a cutesy example of "Braess' Paradox" in an electric transmission circuit.

(Reviewer #3: is this really a Paradox, or just Kirchhoff's Laws coming back to bite us?)

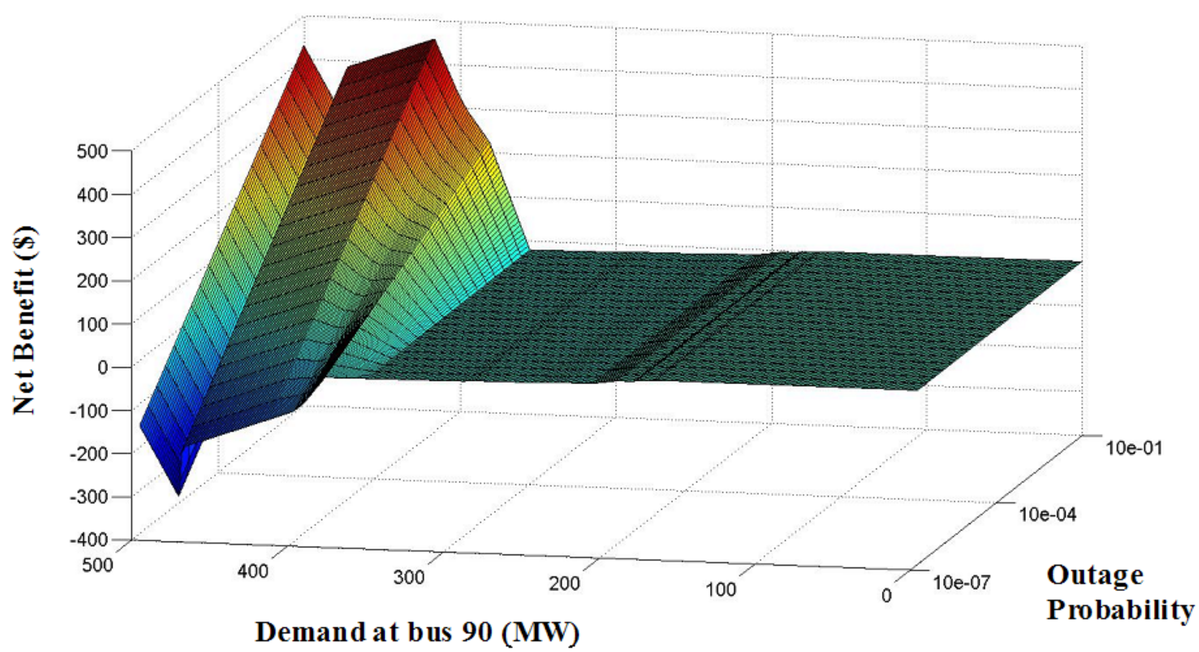
All Braess, All the Time

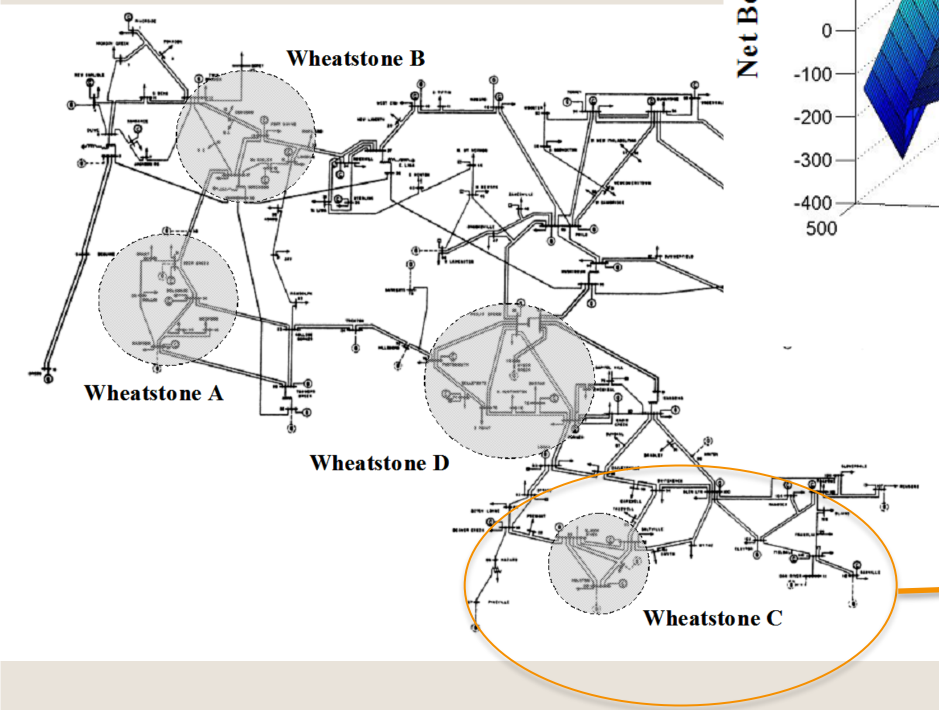
Braess (1968): Traffic paradoxes

Every system that could possibly exhibit behavior remotely Braess-like:

- Computer networks (Korilis, Lazar, Orda; 1997, 1999);
- General pipes (Calvert and Keady, 1991);
- Springs (Penchina and Penchina, 2003);
- Semiconductors (Pala, et al., 2012);
- Biological Cell Networks
- Crowd Control (Hughes, 2003);
- Basketball Teams (Simmons, 1999);
- Multi-agent Systems (Wolpert, 2002);
- Newcomb's Problem (Irvine, 1998);
- May be self-resolving (Nagurney, 2012);

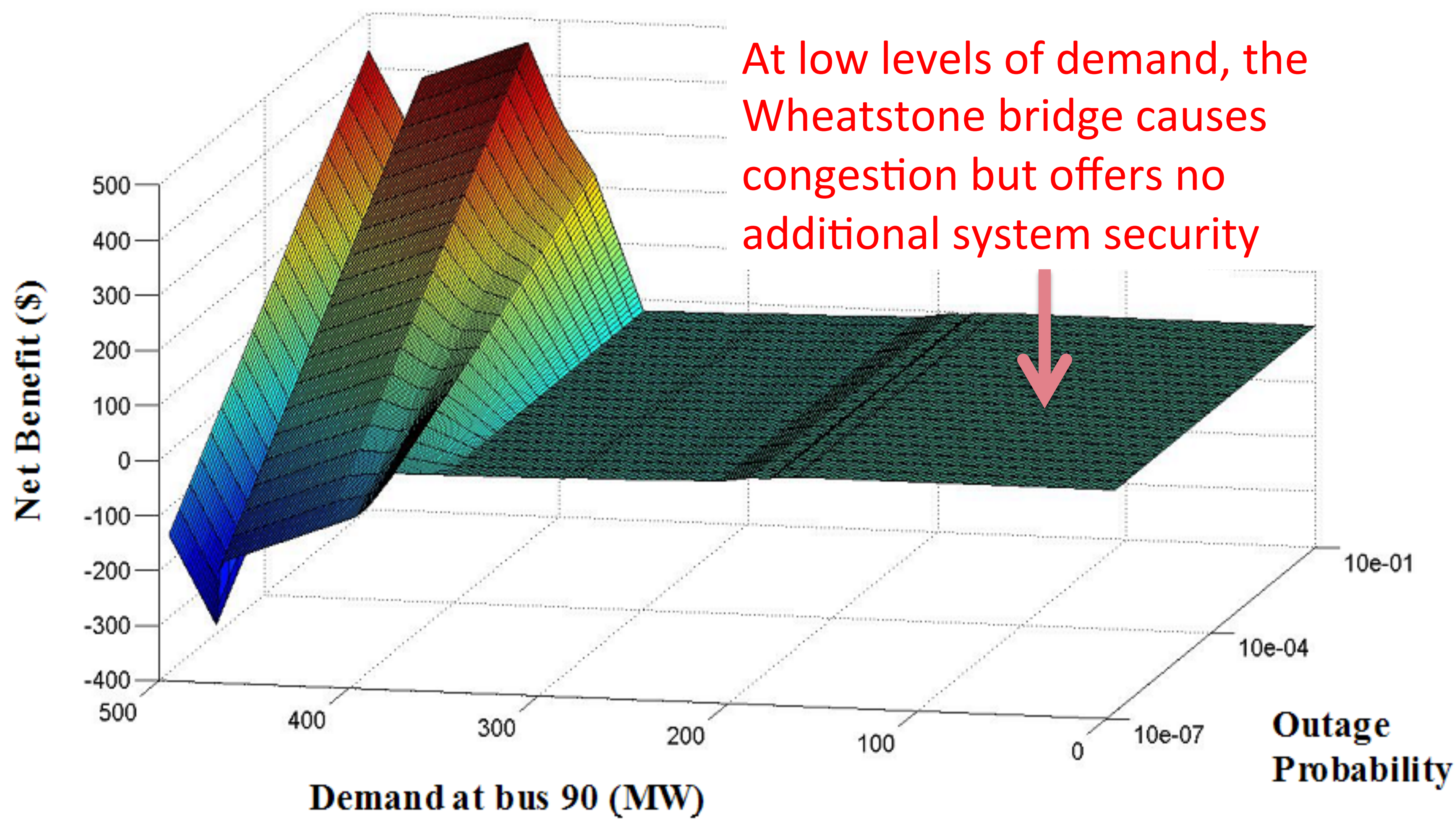
More Realistic: IEEE 118 Bus





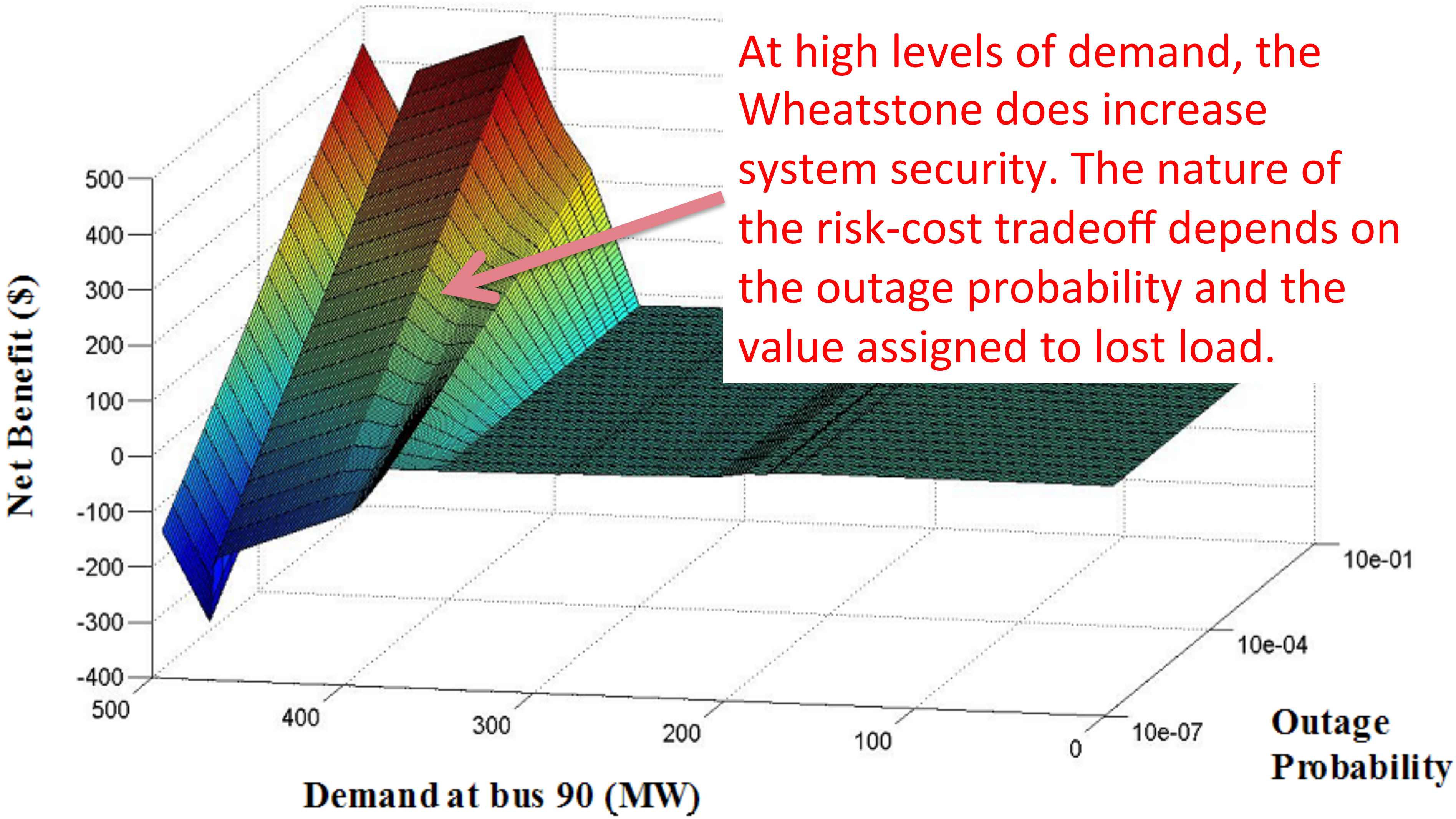
Source: Blumsack, Lave, Ilic, *Energy Journal* 2007.

The Risk-Cost Nature of Transmission Security



Source: Blumsack, Lave, Ilic, *Energy Journal* 2007.

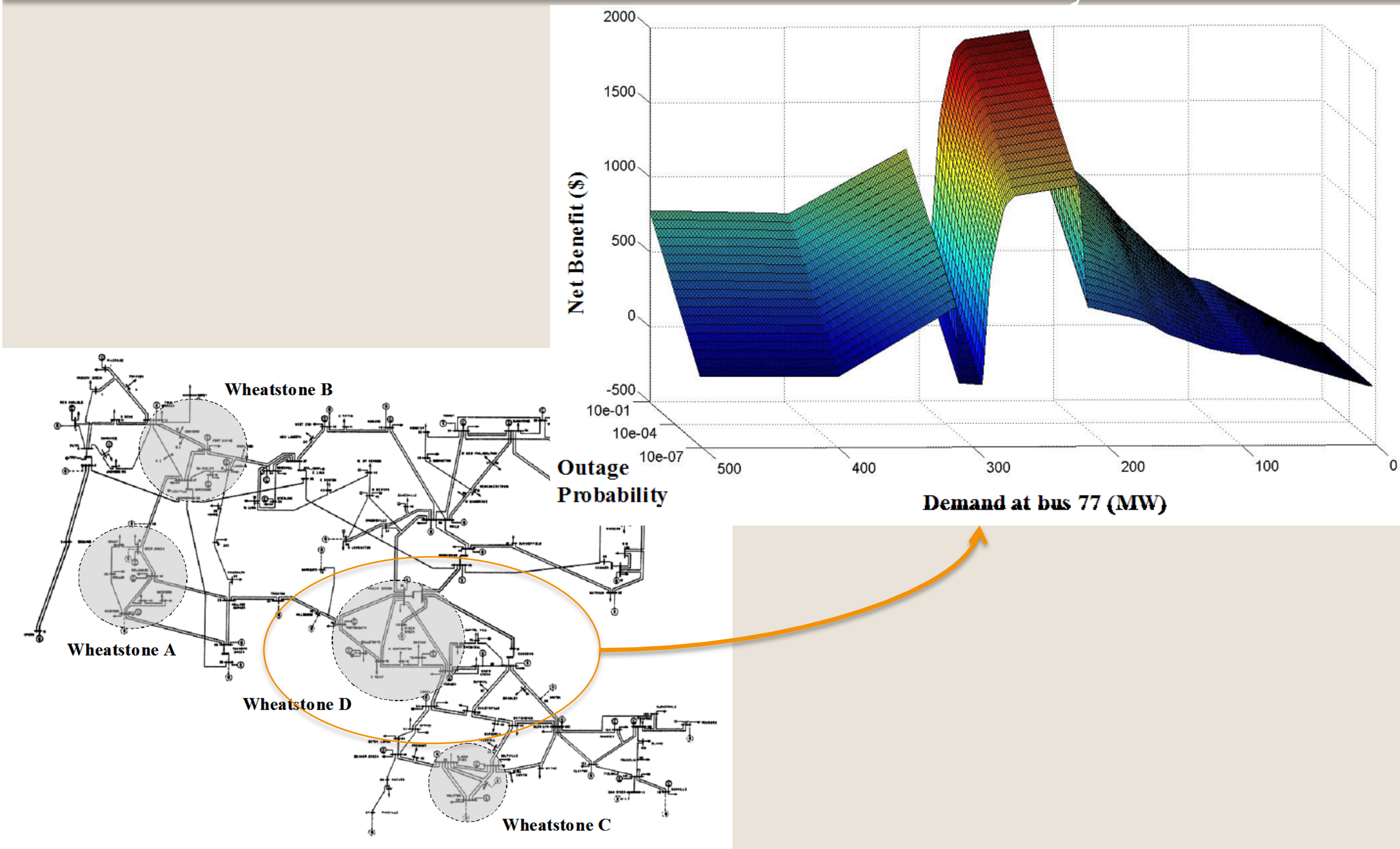
The Risk-Cost Nature of Transmission Security



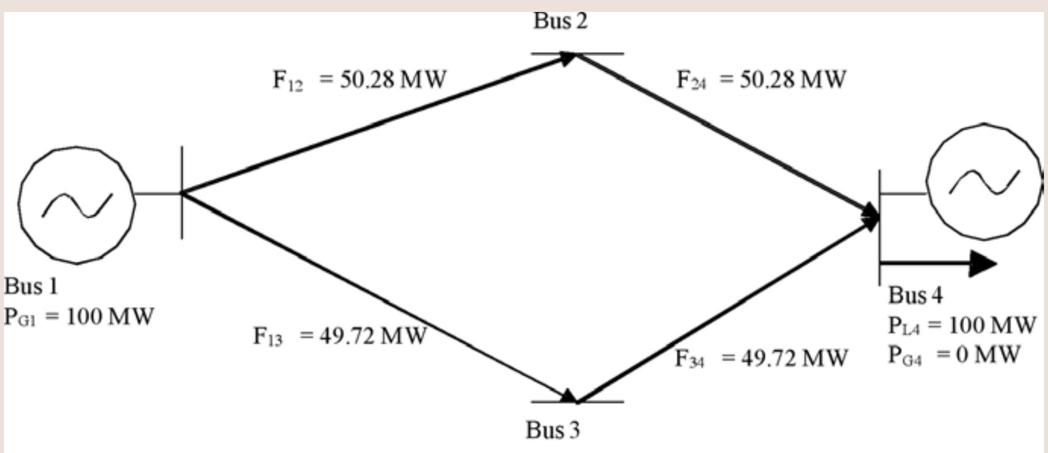
At high levels of demand, the Wheatstone does increase system security. The nature of the risk-cost tradeoff depends on the outage probability and the value assigned to lost load.

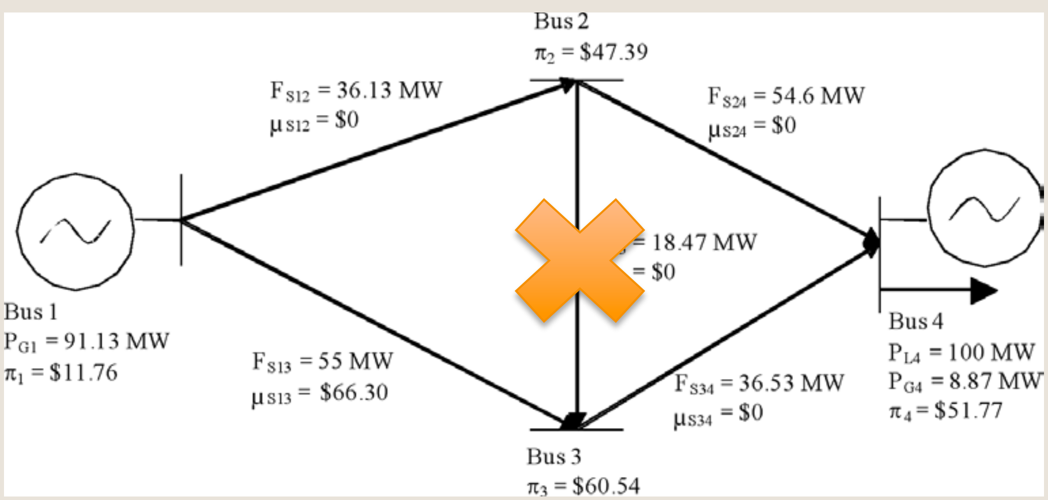
Source: Blumsack, Lave, Ilic, *Energy Journal* 2007.

The Risk-~~Cost~~ Benefit Nature of Transmission Security?



Discrete (Optimal) Topology Control





- Opening redundant circuits for economic reasons, unless a failure occurs elsewhere in the system.
- Some security cost, but hopefully not too large if done smartly.

Achieving Optimal Topology Control

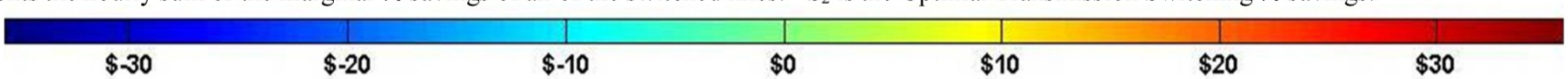
- Discrete topology control is a hard optimization problem. So we could find clever new ways to solve large MILPs.

- Use off-line screening to identify areas of the network that are more likely to exhibit Braess type behavior (or to exhibit risk-cost security properties).

Who Needs a Big Optimization Problem?

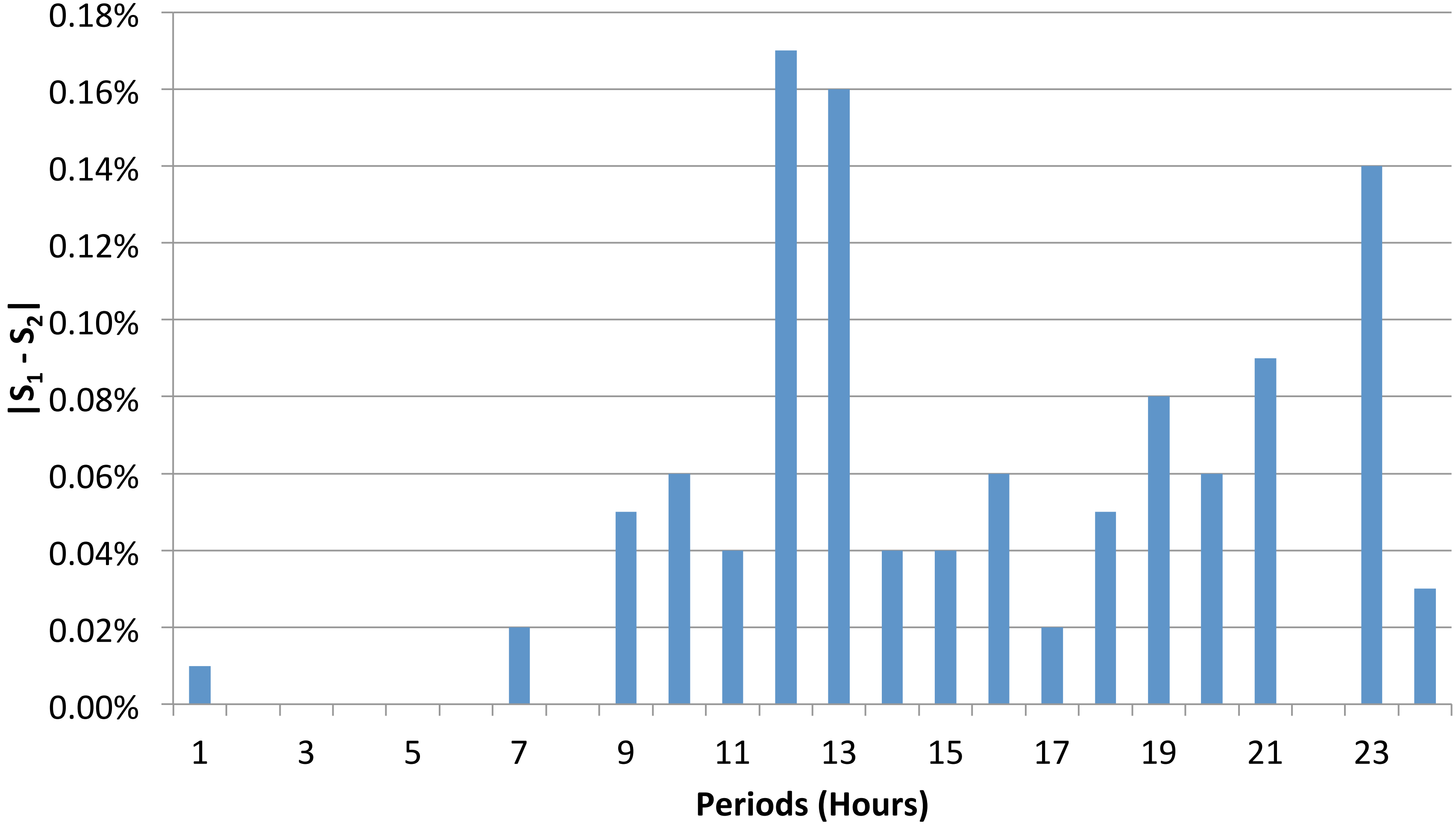
Lines	Periods (Hours) 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
109-111	-.001	---	---	---	---	---	-.001	---	.193	.420	.463	.470	.463	.470	.470	.445	.436	.436	.387	.372	.372	---	---	---
112-113	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	.011
113-215	-.027	.000	.000	.000	.000	.000	-.027	---	---	---	-.251	---	---	---	-.252	---	---	---	---	---	---	---	-.335	---
201-202	.000	---	---	---	---	---	.000	---	---	---	---	---	---	---	-.002	---	---	---	---	---	---	---	---	-.006
209-211	---	---	---	---	---	---	.035	---	---	---	.374	---	---	---	---	---	---	---	---	---	---	---	.199	---
215-216	---	---	---	---	---	---	---	---	---	---	---	.015	.010	---	.015	-.002	---	---	---	---	---	---	---	---
215-221	---	---	---	---	---	---	---	---	---	---	---	-.024	-.024	---	---	---	---	---	---	---	---	---	---	---
217-218	---	.000	.000	.000	.000	.000	---	.017	.024	.033	.032	---	---	.032	---	---	---	---	---	.035	.035	.034	---	.016
218-221	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	.003	---	---	---	---	---	---	---	---
218-221	---	---	---	---	---	---	.000	---	---	---	---	---	---	---	.003	---	.003	---	.003	---	---	---	---	---
219-220	---	---	---	---	---	---	---	---	.025	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---
219-220	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	.050	---	---	---	---
220-223	---	---	---	---	---	---	---	---	.018	---	---	.027	.028	.027	---	.029	.030	---	---	---	---	---	---	---
220-223	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	.034	---	---	---	---
309-311	.020	---	.000	---	---	---	.020	.175	.342	.429	.450	.444	.450	.444	.444	.447	.439	.439	.412	.402	.402	---	---	.106
310-311	---	.000	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---
318-321	.000	---	---	---	---	---	---	---	-.001	---	---	-.003	-.003	-.003	-.003	-.003	---	---	---	---	---	---	---	---
318-321	---	---	---	---	---	---	.000	-.001	---	---	---	-.003	-.003	-.003	---	-.003	---	---	-.003	-.003	---	---	---	---
320-323	-.001	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	.002
320-323	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	.017	.018	---	---	---	---	---
# Sw'd[1]	6	3	3	2	2	2	7	3	6	3	5	7	7	6	7	7	4	3	5	6	3	1	2	5
Cost $k[2]	7.27	7.26	7.25	7.25	7.25	7.25	7.27	7.34	7.44	7.54	7.59	7.60	7.59	7.60	7.60	7.56	7.55	7.55	7.51	7.50	7.50	7.51	7.44	7.31
$[S_1]$[3]	-0.01	0.00	0.00	0.00	0.00	0.00	0.03	0.19	0.60	0.88	1.07	0.93	0.92	0.97	0.68	0.92	0.91	0.89	0.82	0.89	0.81	0.03	-0.14	0.13
$[S_2]$[4]	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.19	0.55	0.82	1.03	0.76	0.76	0.93	0.72	0.86	0.89	0.84	0.74	0.83	0.72	0.03	0.00	0.10

[1]#switched lines produced by Optimal Transmission Switching. [2]Total system cost of the un-switched system in thousands of dollars.
[3] S_1 represents the hourly sum of the marginal % savings of all of the switched lines. [4] S_2 is the Optimal Transmission Switching % savings.



Source: Barrows and Blumsack. *IEEE Trans. Power Sys.*. 2012

How About Little Optimization Problems?



Source: Barrows, Blumsack and Bent. *Proc. HICSS-46*, 2013

Who's on Braess?

Screening for Braess' Paradox

- Toy examples
  - Four-bus power network
  - Four-node gas pipeline network
- Larger networks
  - Electrical networks: clustering and sensitivity based screens
  - Gas networks: spanning trees

Four Interesting Observations

1. Detecting Braess' Paradox efficiently is impossible (Roughgarden, 2004);
2. For networks obeying Kirchhoff's Laws, Braess' Paradox can only be observed in Wheatstone Bridge sub-structures (Milchtaich, 2005);
3. For Hazen-Williams networks, the two-terminal Wheatstone Bridge is the simplest structure to exhibit Braess' Paradox (Calvert and Keady; 1991);
4. Every network can be decomposed into series-parallel and Wheatstone Bridge subgraphs (Duffin, 1965).

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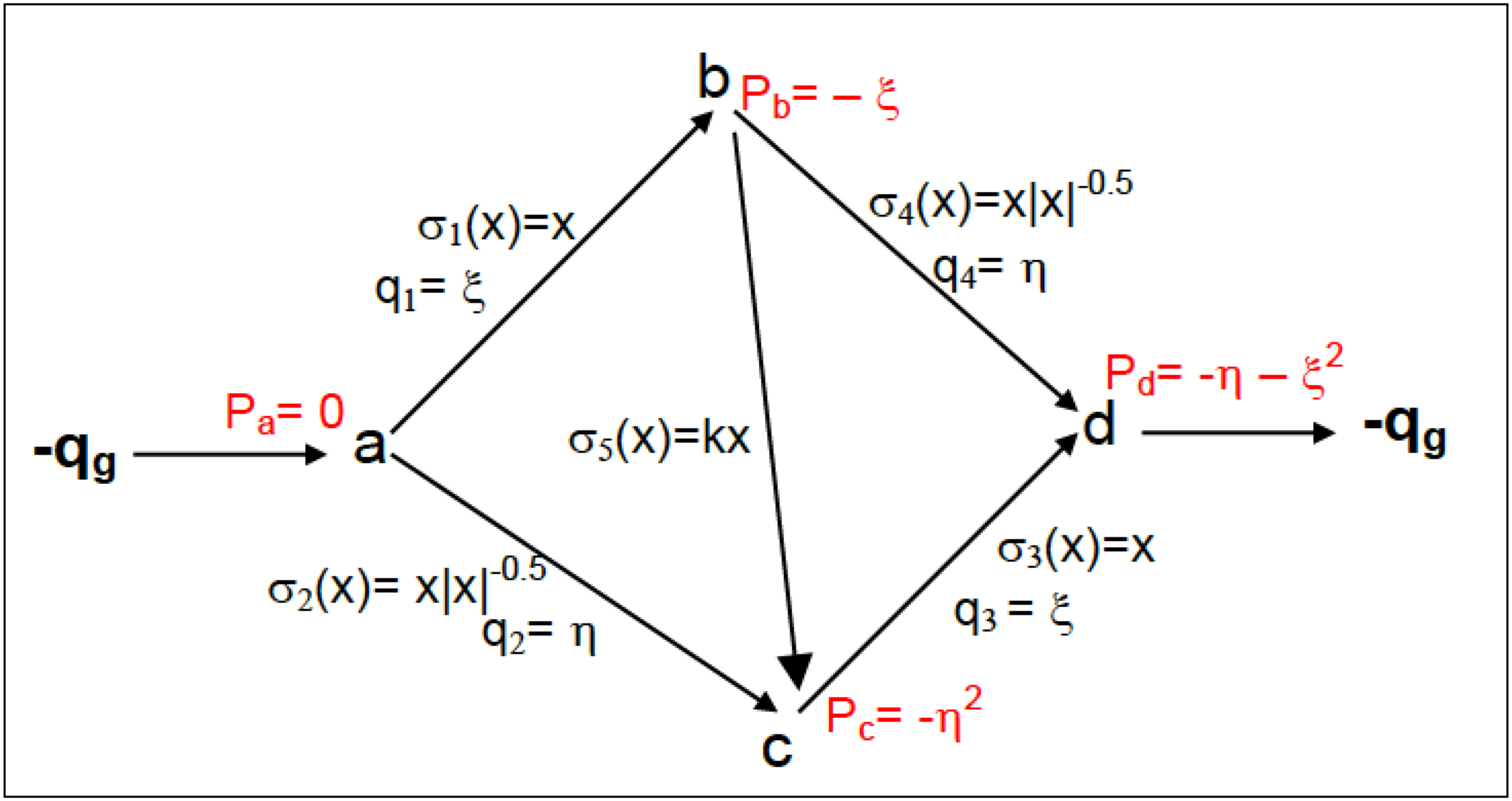
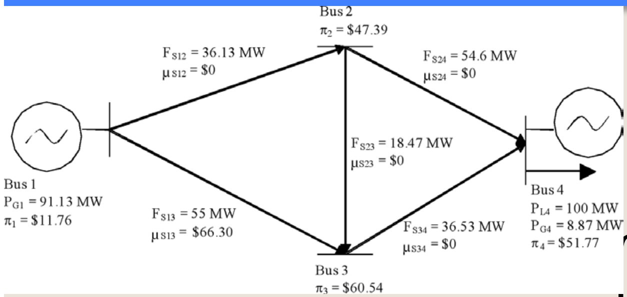
Calvert-Keady Framework



Detecting Braess: Toy Power Network



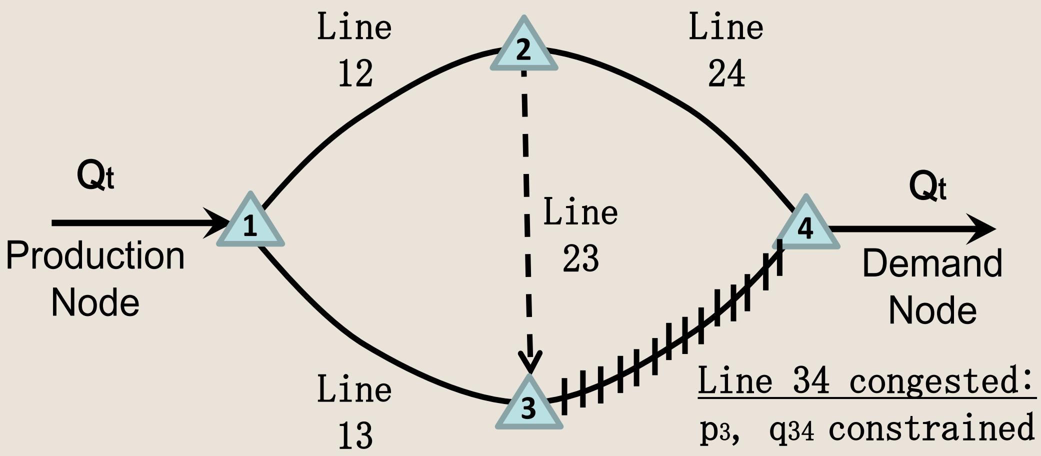
$$F_l^{new} = \begin{cases} F_l^{old} + b_k^{-1}\mathbf{A}'_l\,\mathbf{B}^{-1}\mathbf{A}_k B_l \delta_k^{old}, & l \neq k \\ \left(F_l^{old} - \Delta B_l \delta_l^{old}\right)(1 - b_l^{-1}\mathbf{A}'_l\,\mathbf{B}^{-1}\mathbf{A}_l), & l = k, \end{cases}$$

$$F_l^{old} + b_k^{-1}\mathbf{A}'_l\,\mathbf{B}^{-1}\mathbf{A}_k B_l \delta_k^{old} \geq F_l^{\max}$$

$$\Rightarrow \Delta B_k^{-1} \geq \frac{\mathbf{A}'_l\,\mathbf{B}^{-1}\mathbf{A}_k B_l \delta_k^{old}}{F_l^{\max} - F_l^{old}} - \mathbf{A}_k'\mathbf{B}^{-1}\mathbf{A}_k .$$

Source: Blumsack and Ilic. *Proc. PES*. 2008.

Detecting Braess: Toy Gas Network



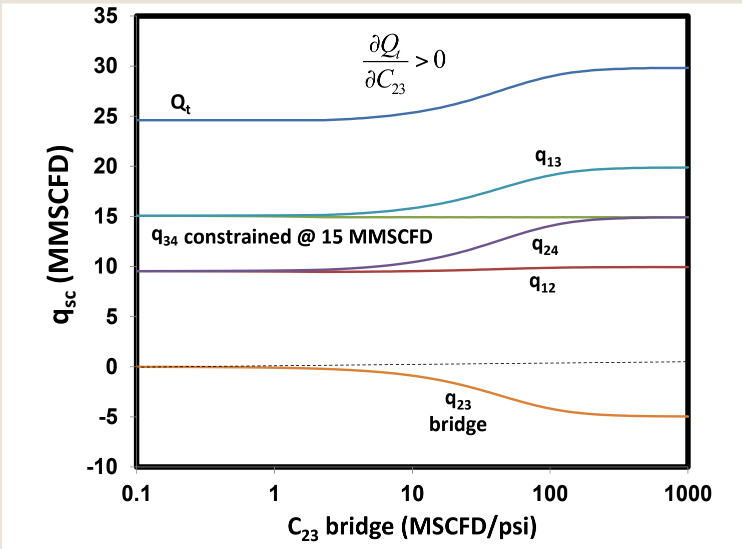
Linear conductivity analog (Ayala and Leong, 2012)

$$q_{ij} = L_{ij} \cdot (p_i - p_j)$$

$$L_{ij} = T_{ij} \cdot C_{ij}$$

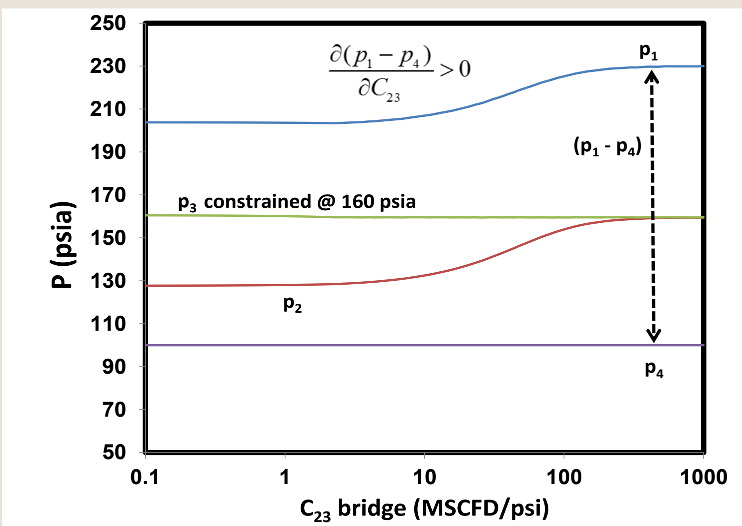
$$T_{ij} = \sqrt{1 + \frac{2}{r_{ij} - 1}}$$

Detecting Braess: Toy Gas Network



Pipe transportation capacity versus Wheatstone Bridge pipe conductivity (C_{23}):

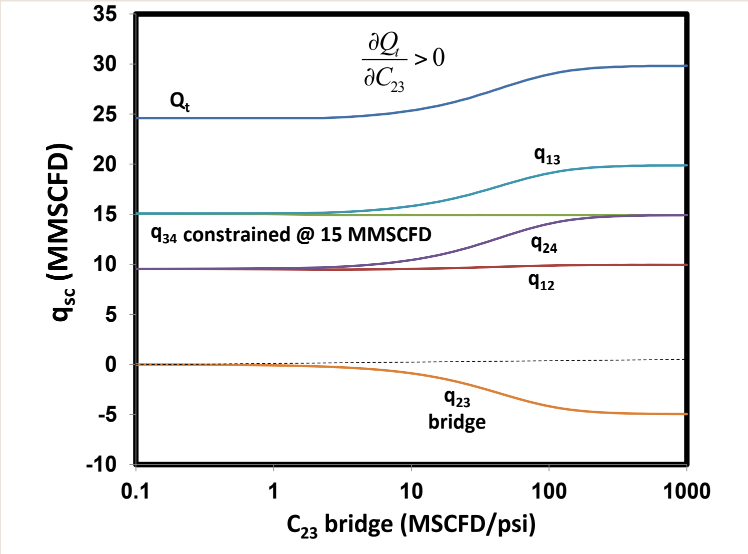
$$\partial Q_t / \partial C_{23} > 0$$

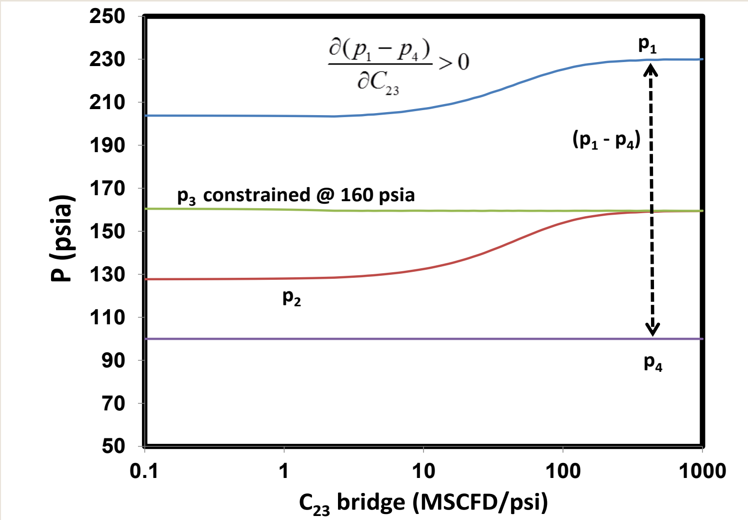


Network pressure loss versus Wheatstone Bridge pipe conductivity (C_{23}):

$$\left(\frac{\partial (p_1 - p_4)}{\partial C_{23}} \right) > 0$$

Detecting Braess: Toy Gas Network





Network condition:

$$T_{12}T_{34} \cdot C_{12}C_{34} - T_{24}T_{13} \cdot C_{24}C_{13} = 0$$

Equivalently (since parallel pipe pressure ratios are the same):

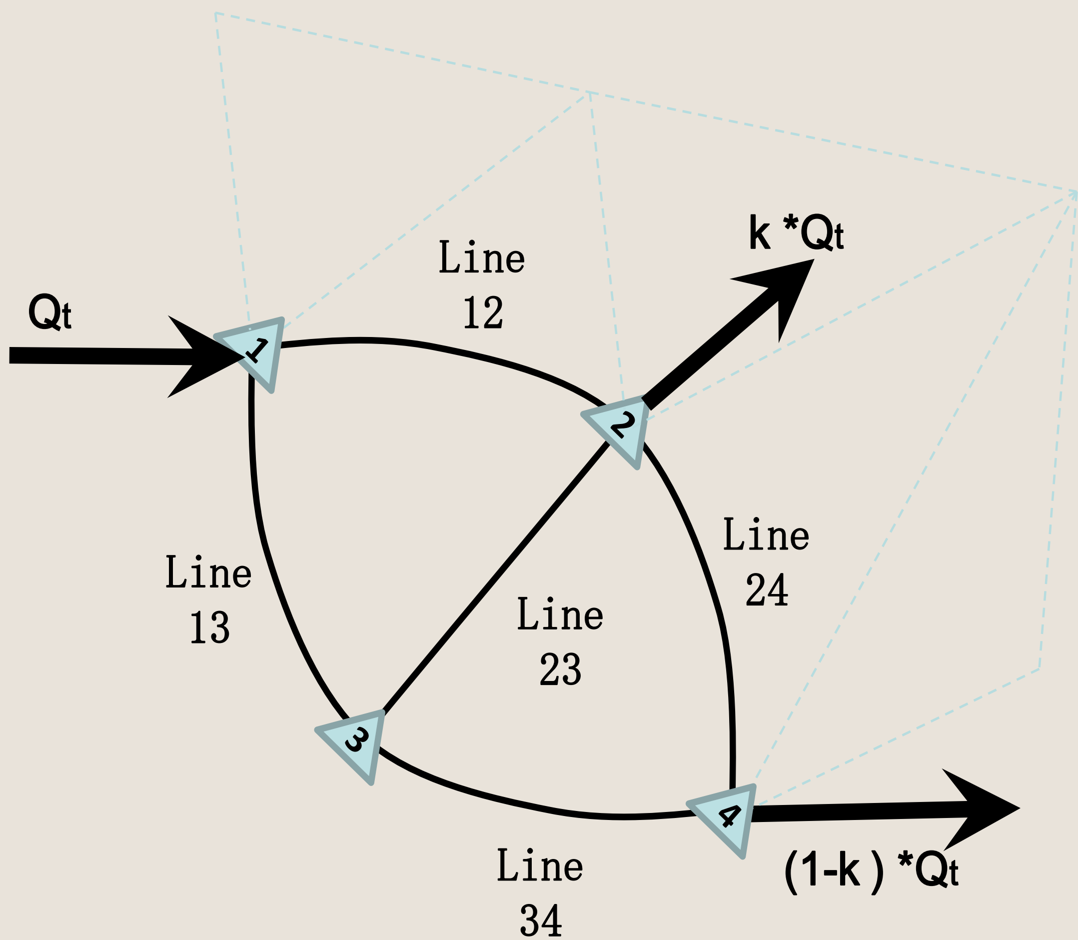
$$C_{12}C_{34} - C_{24}C_{13} = 0$$

$$C_{12}C_{34} > C_{24}C_{13} \text{ leads to } \left(\partial Q_t / \partial C_{23}\right) < 0$$

$$C_{24}C_{13} > C_{12}C_{34} \text{ leads to } \left(\partial Q_t / \partial C_{23}\right) > 0$$

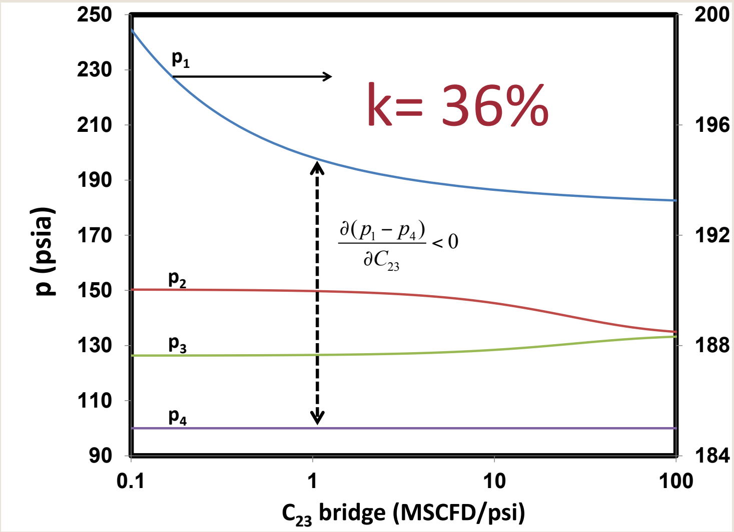
Source: Ayala and Blumsack, *Oil and Gas Facilities* 2012.

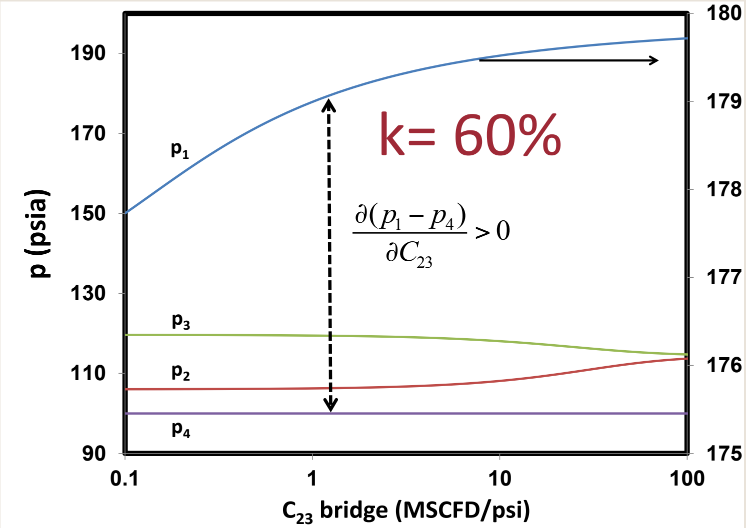
The Complexity of Braess' Paradox in Pipeline Networks



The existence of a Wheatstone Bridge topology within a larger network may induce larger pressure drops even without causing "congestion" in the pipeline system.

The Complexity of Braess' Paradox in Pipeline Networks





If the fraction of fluid take-off to one point versus another (the parameter *k*) exceeds a critical threshold:

$$k_c = \frac{L_{12}L_{34} - L_{23}L_{13}}{L_{34}(L_{12} + L_{13})}$$

$$= \frac{T_{12}T_{34} \cdot C_{12}C_{34} - T_{23}T_{13}C_{23}C_{13}}{T_{34}C_{34}(T_{12}C_{12} + T_{13}C_{13})}$$

Screening for Braess' Paradox in Large Networks via Clustering

Factoid of the day:

The clustering coefficient of the Wheatstone network is 5/6.

Second factoid of the day:

No other four-node network (with minimum geodesic path length equal to two) has the same clustering coefficient.

Clustering-Based Algorithm

Step 1: Using the node-edge adjacency matrix, reduce all simple series and parallel connections. (This step may need to be iterated.)

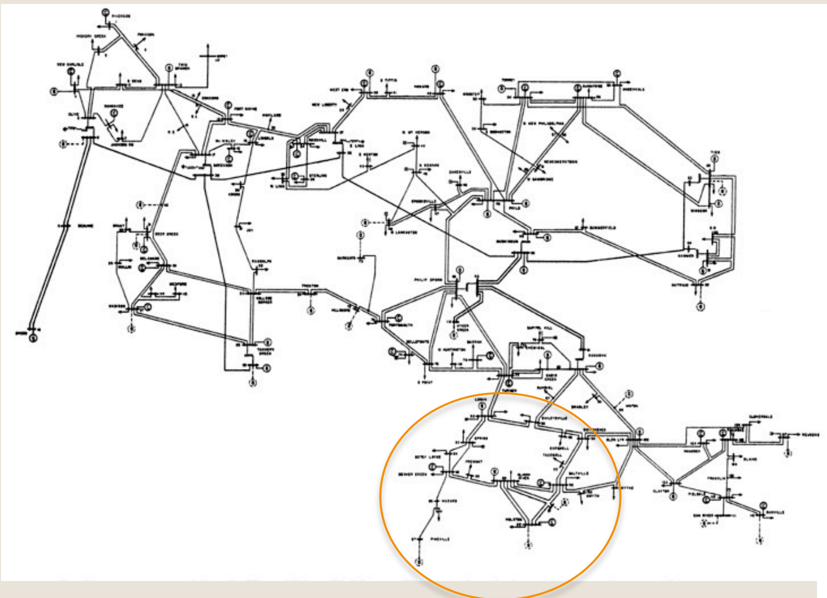
Step 2: Define R_1 as the set of all node pairs with geodesic path length two, and R_2 as a subset of R_1 such that there are two such geodesic paths.

Step 3: Calculate $WS = T \cap D \cap R_2 \cap R_3$

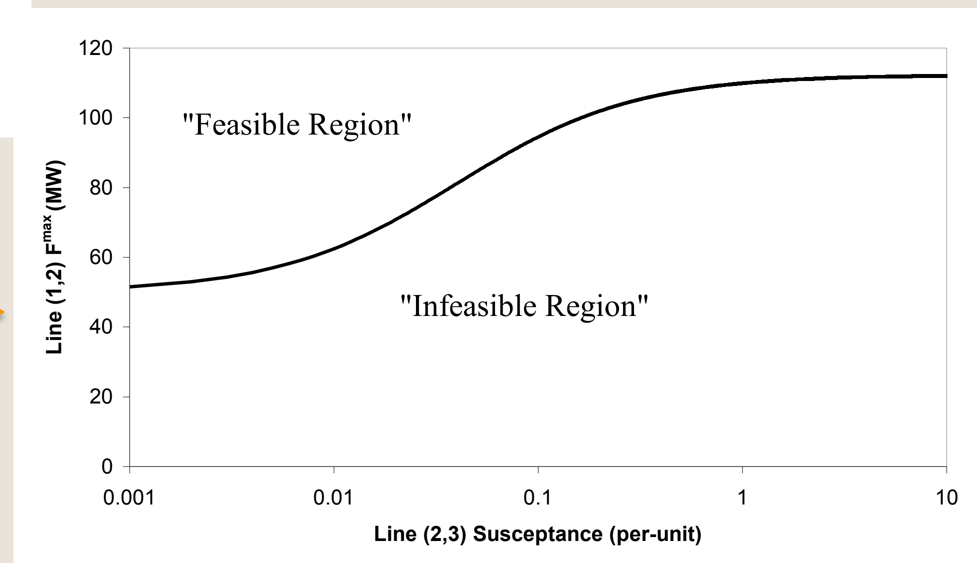
Step 4: For all node pairs in WS, construct the adjacency matrix consisting of the node pairs and all neighboring nodes.

Step 5: Calculate the clustering coefficient for each subgraph in Step 4. Those with a clustering coefficient equal to 5/6 are Wheatstone Networks

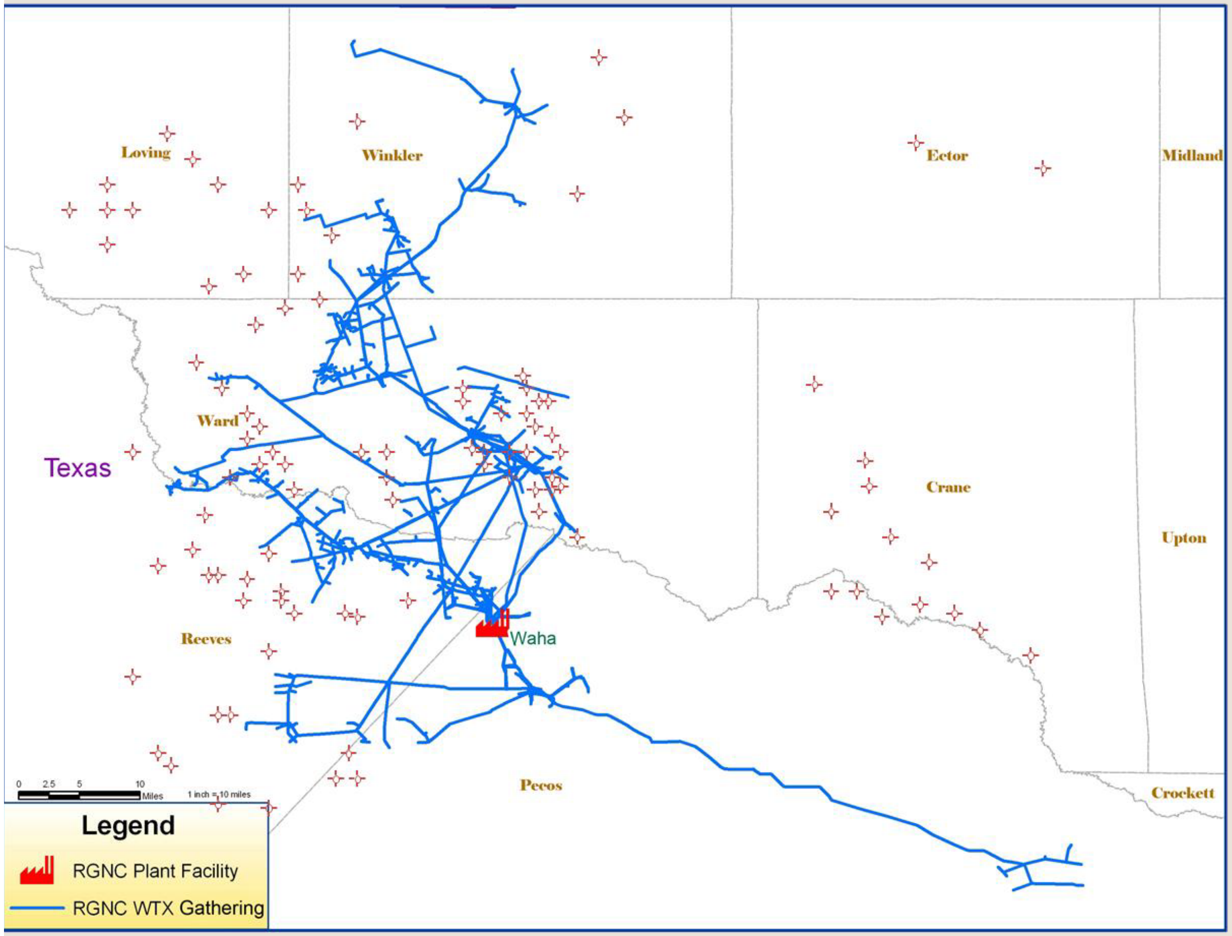
Implementation on 118 Bus Network



Clustering-based algorithm plus some network equivalencing produces screening curves like the one below.



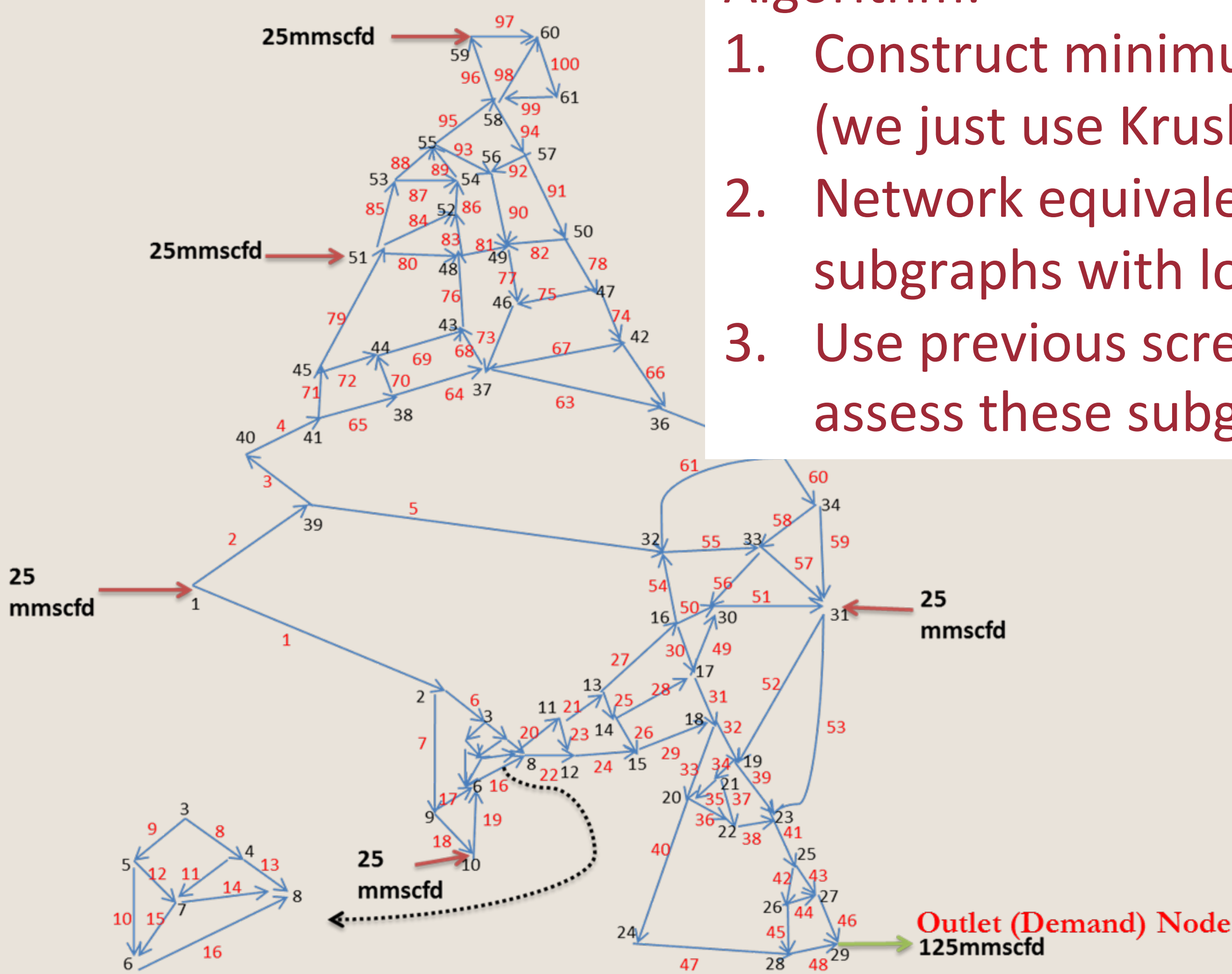
Another Approach: Spanning Trees



Waha gathering system:

- 61 nodes (some of which have compressors, others just have valves)
- 100 edges
- 5 supply nodes, one consumption node

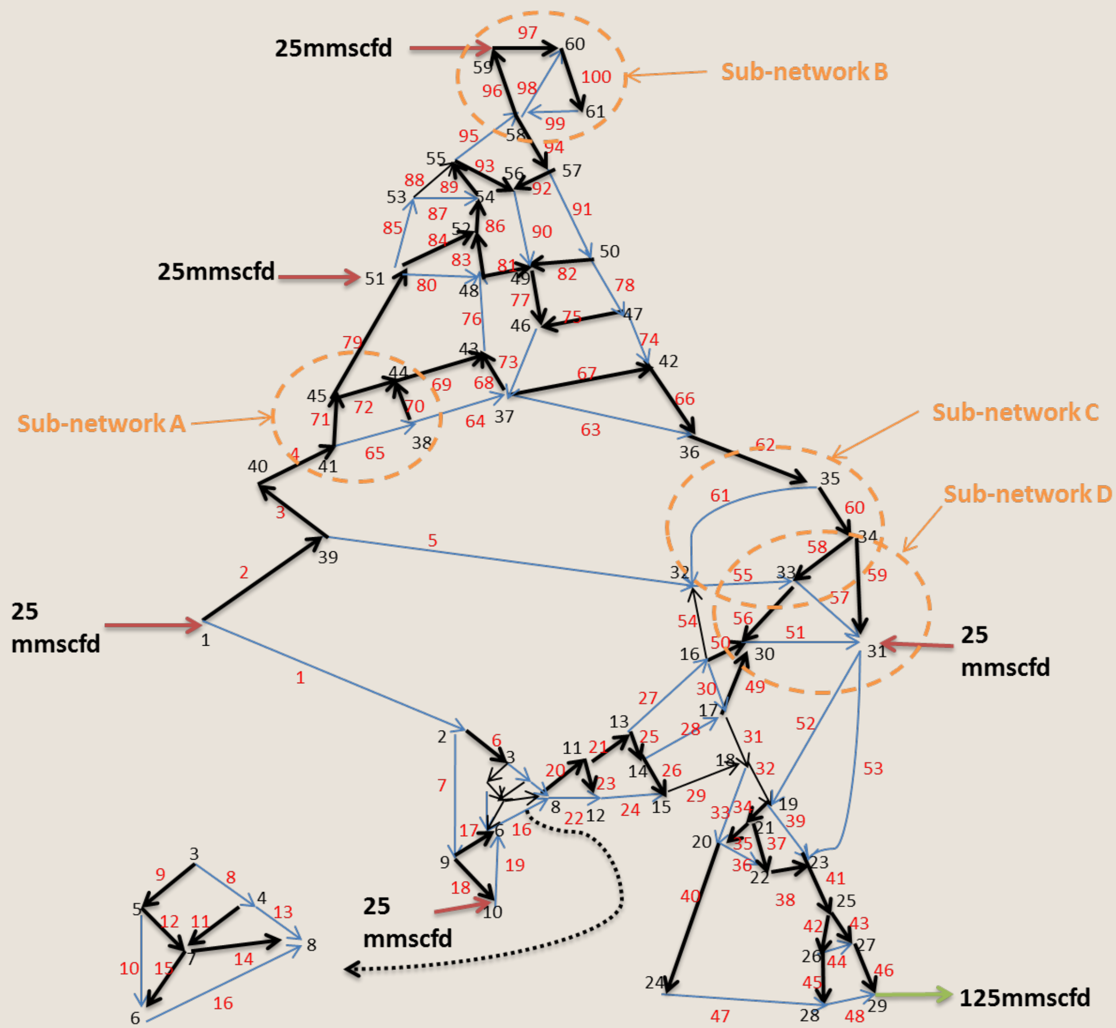
Another Approach: Spanning Trees



Algorithm:

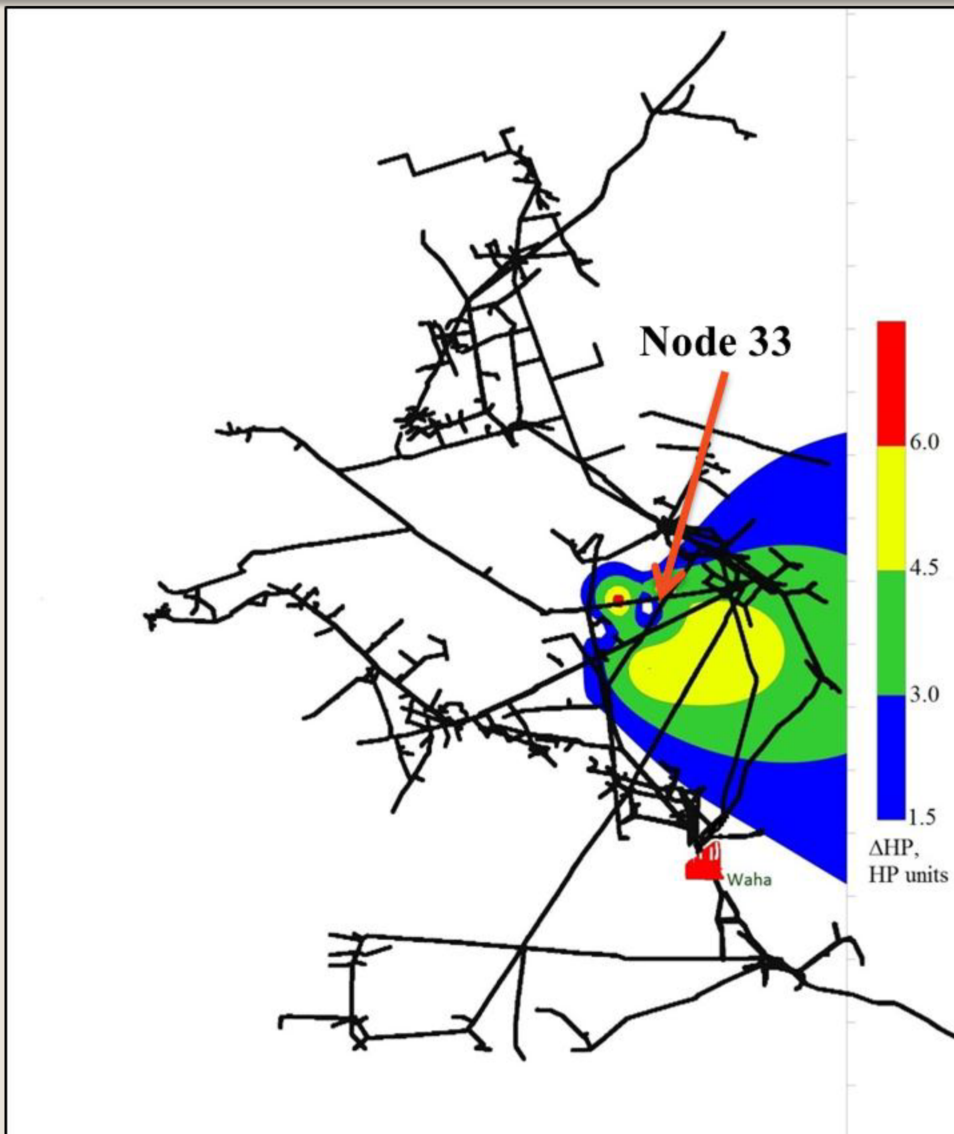
1. Construct minimum spanning tree (we just use Kruskal's method)
2. Network equivalencing to isolate subgraphs with loops
3. Use previous screening tools to assess these subgraphs

Another Approach: Spanning Trees



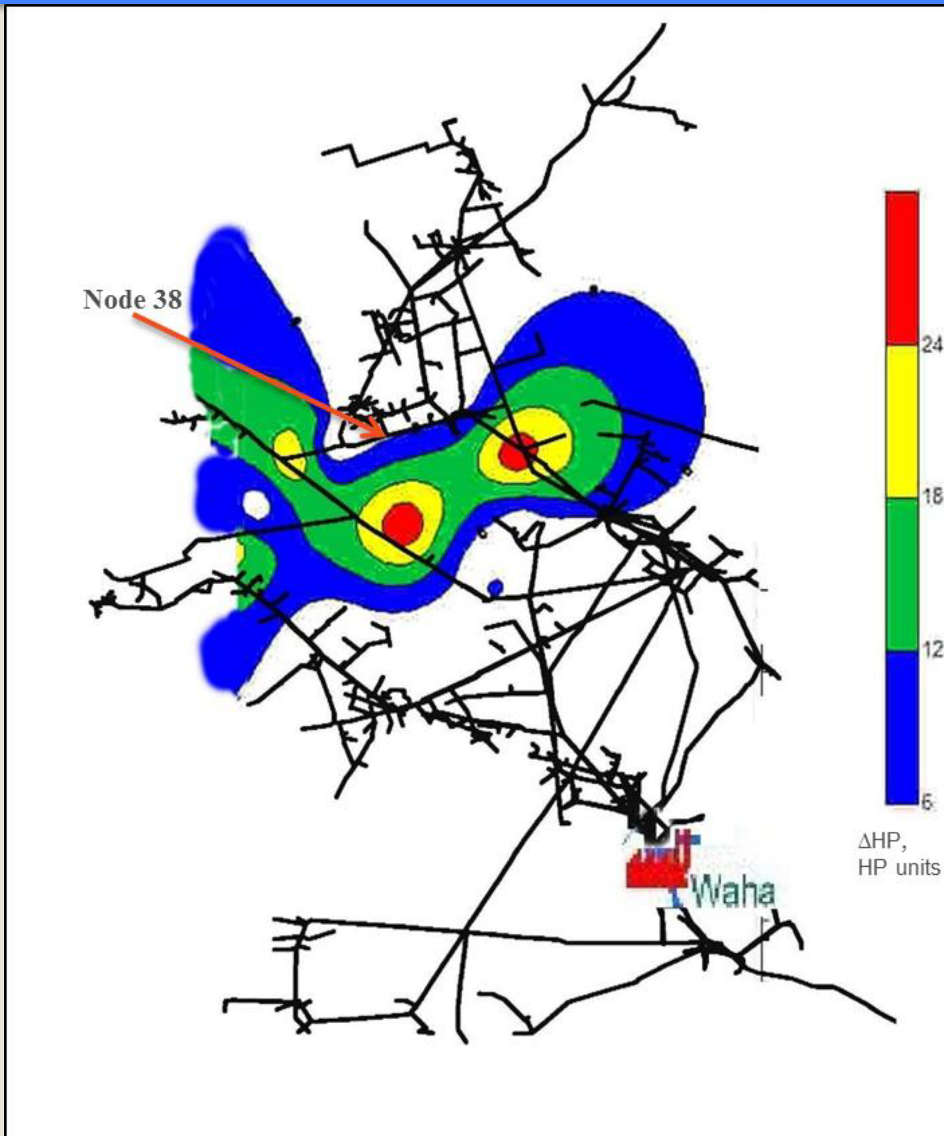
- Elements in the spanning tree are shown in bold;
- Subgraphs of interest are highlighted (there are more possible subgraphs to consider…possibly unwieldy….

The Reach of Topological Inefficiency



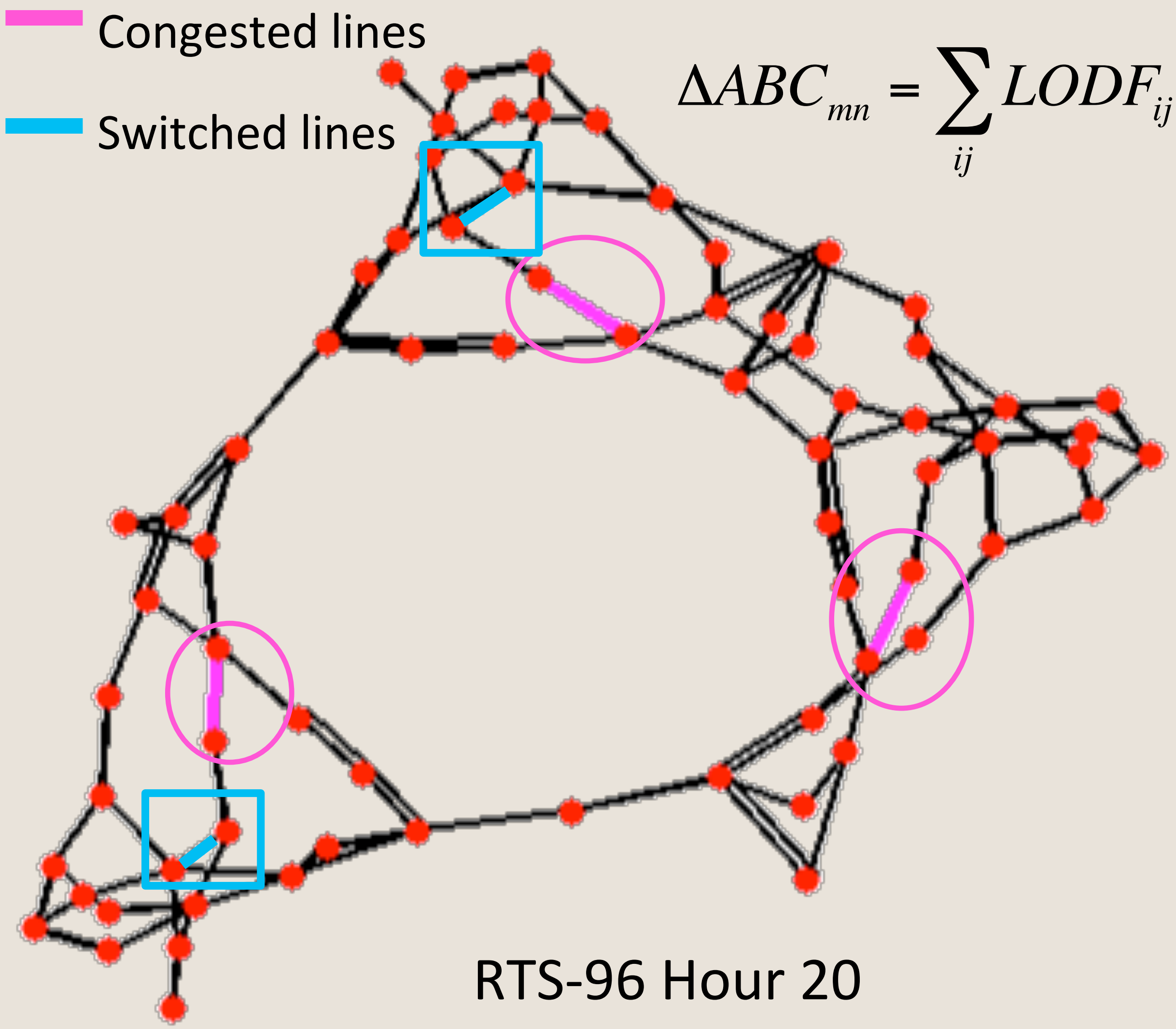
- Pressure is held constant at node 33.
- Multiple topological inefficiencies contribute to increased horsepower requirements

The Reach of Topological Inefficiency



- Here, pressure is held constant at node 38.
- Note that holding pressure constant at the demand node does not itself induce any paradoxical behavior (this is probably a fluke though we aren't sure).

So What?

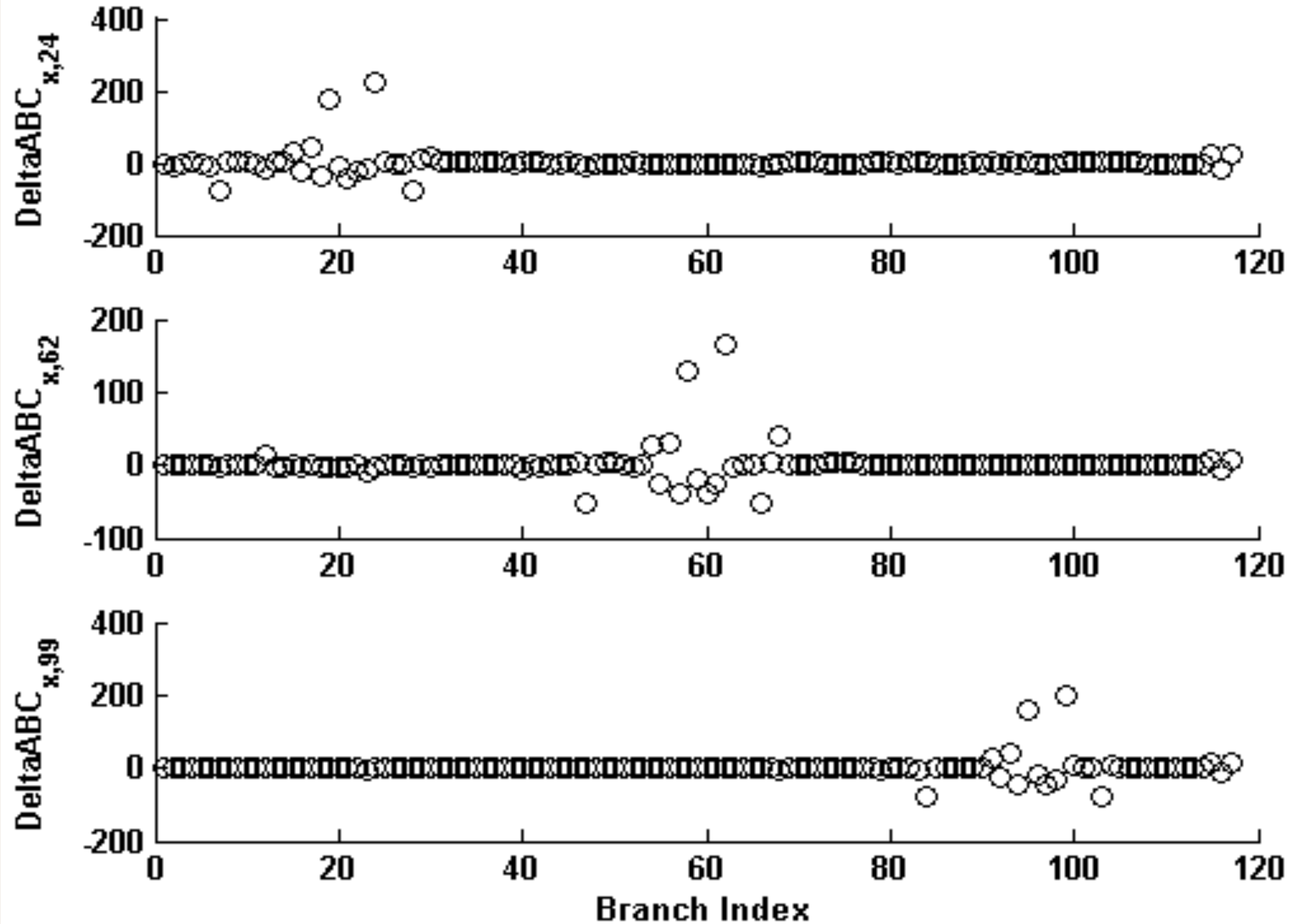


$$\Delta ABC_{mn} = \sum_{ij} LODF_{ij,mn} \times \left(P^*_{mn}\,\mu_{ij} - P_{mn}\mu_{ij}\right)$$

Measures re-allocation of flow from overloaded to under-utilized branches

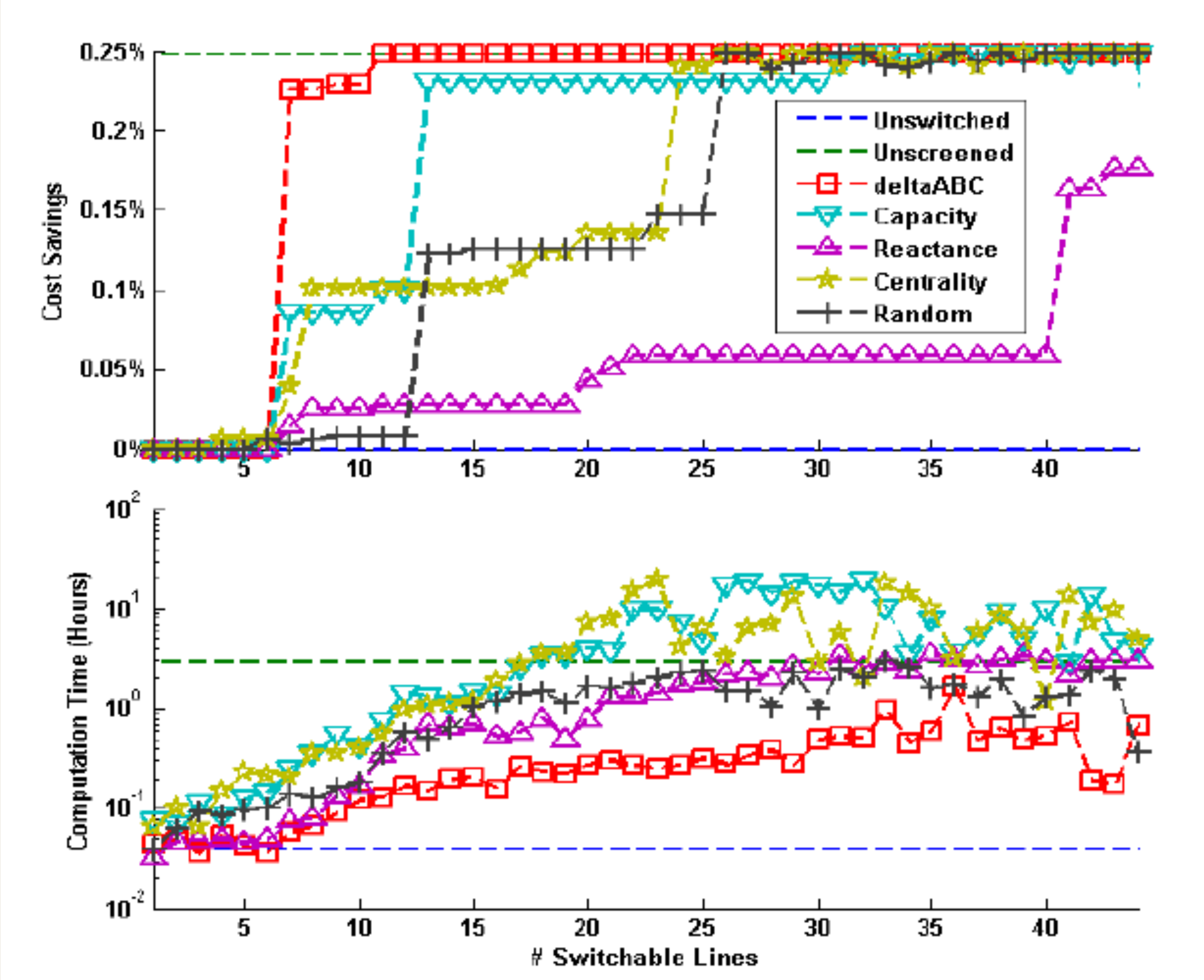
RTS-96 Hour 20

Screen for RTS-96 System



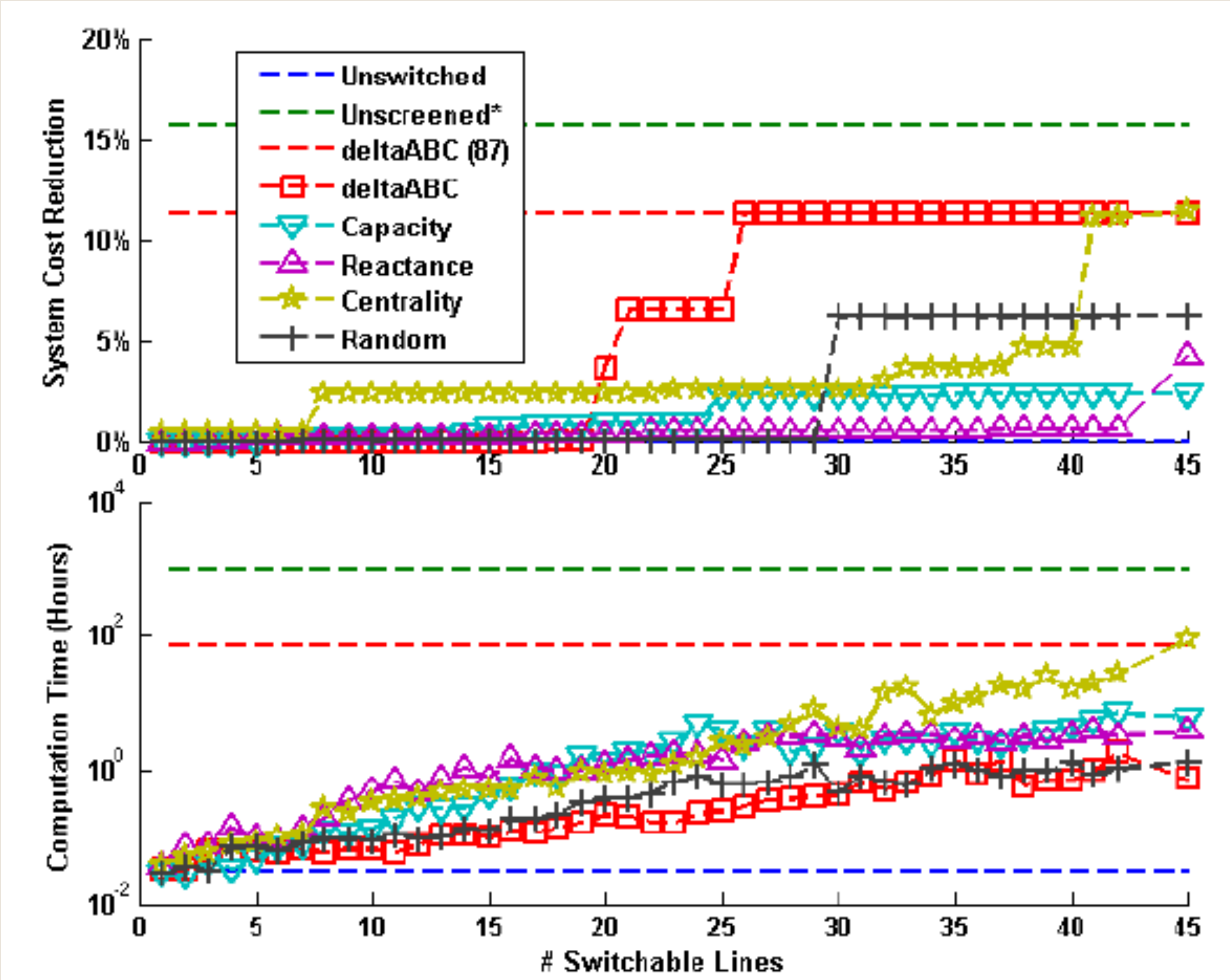
Source: Barrows, Blumsack and Bent. *Proc. HICSS-46*. 2013

RTS-96, Hour 14



Source: Barrows, Blumsack and Bent. *Proc. HICSS-46*, 2013

IEEE 118 Bus System



Source: Barrows, Blumsack and Bent. *Proc. HICSS-46*, 2013

Prospects for Topology Control

- Discrete topology control is a hard optimization problem. But there are probably clever ways to shrink the size of the problem.
- Subgraph screening is one possible way, but algorithms (mine, anyway) need improvement.
  - Some are fast but run the risk of false negatives (and false positives? we don't really know)
  - Others work well…but just aren't that efficient.

Thank You!

Seth Blumsack

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New developments on solving AC-OPF on sparse networks

Daniel Bienstock and Gonzalo Muñoz, Columbia University

January 2015

Optimal power flow problem in rectangular coordinates, simplest form

Variables:

- Complex voltages $\boldsymbol{e_k + j\,f_k}$, power flows $\boldsymbol{P_{km}, Q_{km}}$, auxiliary variables

Notation: For a bus $\boldsymbol{k}$, $\boldsymbol{\delta(k)}$ = set of lines incident with $\boldsymbol{k}$; $\boldsymbol{V}$ = set of buses

Basic problem

$$\mathbf{min} \quad \sum_{k \in V} C_k$$

$$\text{s.t.} \quad \forall km: \quad P_{km} = \boldsymbol{g_{km}}(e_k^2 + f_k^2) - \boldsymbol{g_{km}}(e_k e_m + f_k f_m) + \boldsymbol{b_{km}}(e_k f_m - f_k e_m) \tag{1a}$$

$$\forall km: \quad Q_{km} = -\boldsymbol{b_{km}}(e_k^2 + f_k^2) + \boldsymbol{b_{km}}(e_k e_m + f_k f_m) + \boldsymbol{g_{km}}(e_k f_m - f_k e_m) \tag{1b}$$

$$\forall km: \quad |P_{km}|^2 + |Q_{km}|^2 \leq \boldsymbol{U_{km}} \tag{1c}$$

$$\forall k: \quad \boldsymbol{P_k^{\min}} \leq \sum_{km \in \delta(k)} P_{km} \leq \boldsymbol{P_k^{\max}} \tag{1d}$$

$$\forall k: \quad \boldsymbol{Q_k^{\min}} \leq \sum_{km \in \delta(k)} Q_{km} \leq \boldsymbol{Q_k^{\max}} \tag{1e}$$

$$\forall k: \quad \boldsymbol{V_k^{\min}} \leq e_k^2 + f_k^2 \leq \boldsymbol{V_k^{\max}}, \tag{1f}$$

$$\forall k: \quad C_k = \boldsymbol{F_k}\left(\sum_{km \in \delta(k)} P_{km}\right). \tag{1g}$$

Here, $\boldsymbol{F_k}$ is a quadratic function for each k.

Optimal power flow problem in rectangular coordinates, simplest form

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Notation: For a bus $\boldsymbol{k}$, $\boldsymbol{\delta(k)}$ = set of lines incident with $\boldsymbol{k}$; $\boldsymbol{V}$ = set of buses

Basic problem

$$
\begin{aligned}
&\textbf{min} && \sum_{k \in V} C_k \\
\text{s.t.}\quad &\forall km: && P_{km} = \boldsymbol{g_{km}}(e_k^2 + f_k^2) - \boldsymbol{g_{km}}(e_k e_m + f_k f_m) + \boldsymbol{b_{km}}(e_k f_m - f_k e_m) && \text{(2a)} \\
&\forall km: && Q_{km} = -\boldsymbol{b_{km}}(e_k^2 + f_k^2) + \boldsymbol{b_{km}}(e_k e_m + f_k f_m) + \boldsymbol{g_{km}}(e_k f_m - f_k e_m) && \text{(2b)} \\
&\forall km: && |P_{km}|^2 + |Q_{km}|^2 \leq \boldsymbol{U_{km}} && \text{(2c)} \\
&\forall k: && \boldsymbol{P_k^{\min}} \leq \sum_{km \in \delta(k)} P_{km} \leq \boldsymbol{P_k^{\max}} && \text{(2d)} \\
&\forall k: && \boldsymbol{Q_k^{\min}} \leq \sum_{km \in \delta(k)} Q_{km} \leq \boldsymbol{Q_k^{\max}} && \text{(2e)} \\
&\forall k: && \boldsymbol{V_k^{\min}} \leq e_k^2 + f_k^2 \leq \boldsymbol{V_k^{\max}}, && \text{(2f)} \\
&\forall k: && C_k = \boldsymbol{G_k}\left(\sum_{km \in \delta(k)} Q_{km} \right). && \text{(2g)}
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$$

Here, $\boldsymbol{G_k}$ is a quadratic function for each k.

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$$\forall km: \quad |P_{km}|^2 + |Q_{km}|^2 \leq \boldsymbol{U_{km}} \tag{3c}$$

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Here, $\boldsymbol{F_k, G_k}$ are quadratic functions for each k. **Many** possibilities, all structurally similar.

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These are QCQPs, quadratically constrained quadratic programs, with an underlying graph structure.

QCQPs

$$\begin{aligned} \mathbf{min} \quad & x^T M^0 x \ + \ 2c_0^T x \ + \ d_0 && && (6a) \\ \text{s.t.} \quad \forall km: \quad & x^T M^i x \ + \ 2c_i^T x \ + \ d_i \quad \geq \quad 0, && 1 \leq i \leq m, && (6b) \\ & x \in \mathbb{R}^n. && && (6c) \end{aligned}$$

Each matrix M^i symmetric.
This description includes linear inequalities, bounds on individual variables, quadratic/linear equations.

QCQPs

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Reformulation

observation: $\boldsymbol{x^T M^i x \ + \ 2c_i^T x = (1 \ x^T) \begin{pmatrix} 0 & c_i^T \\ c_i & M^i \end{pmatrix} \begin{pmatrix} 1 \\ x \end{pmatrix}} \ = \ \boldsymbol{(1 \ x^T) \tilde{M}^i \begin{pmatrix} 1 \\ x \end{pmatrix}}$

definition: for matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\quad \boldsymbol{A \bullet B \ \doteq \ \sum_{i,j} a_{ij} b_{ij}}$

so for vector $\boldsymbol{y}$ and matrix $\boldsymbol{A}$, $\quad \boldsymbol{y^T A y \ = \ A \bullet y y^T}$

So **QCQP** can be rewritten as:

$$\begin{aligned} \boldsymbol{Q^*} \ \doteq \ \textbf{min} \quad & \tilde{M}^0 \bullet X \ + \ d_0 && (8a)\\ \text{s.t.} \quad \forall km: \quad & M^i \bullet X \ + \ d_i \quad \geq \quad 0, \qquad 1 \leq i \leq m, && (8b)\\ & X \in \mathbb{R}^{(n+1)\times(n+1)}, \quad X \succeq 0, \quad \textbf{of rank 1}. && (8c) \end{aligned}$$

The **semidefinite relaxation** of this problem is:

$$\begin{aligned} \boldsymbol{\tilde{Q}} \ \doteq \ \textbf{min} \quad & \tilde{M}^0 \bullet X \ + \ d_0 && (9a)\\ \text{s.t.} \quad \forall km: \quad & M^i \bullet X \ + \ d_i \quad \geq \quad 0, \qquad 1 \leq i \leq m, && (9b)\\ & X \in \mathbb{R}^{(n+1)\times(n+1)}, \quad X \succeq 0. && (9c) \end{aligned}$$

$$\boldsymbol{\tilde{Q} \ \leq \ Q^*}$$

The critical observation

- Lavaei and Low, 2011: the SDP relaxation of AC-OPF **frequently** is very tight
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- There is **no** exact algorithm for SDP
- Lavaei, Low, Hiskens-Molzahn:
  when the underlying network has **low tree-width**, the SDP relaxation can be solved much faster
  why: standard SDP solvers can leverage low tree-width
- What exactly is tree-width?

Tree-width

Let $\boldsymbol{G}$ be an undirected graph with vertices $\boldsymbol{V(G)}$ and edges $\boldsymbol{E(G)}$.

A tree-decomposition of $\boldsymbol{G}$ is a pair $\boldsymbol{(T, Q)}$ where:

- $\boldsymbol{T}$ is a tree. **Not** a subtree of $\boldsymbol{G}$, just a tree

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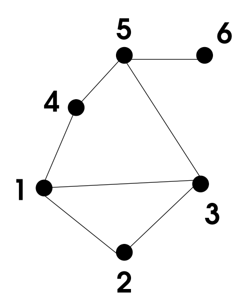
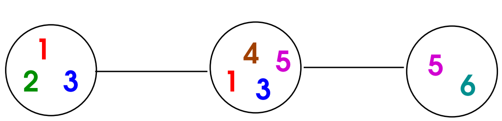
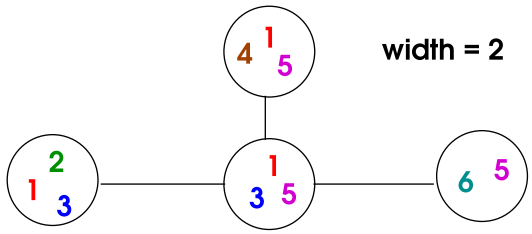
- The **width** of $\boldsymbol{(T, Q)}$ is $\boldsymbol{\max_{t \in T} |Q_t| - 1}$.







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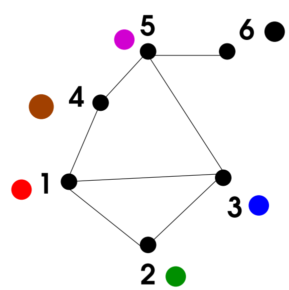
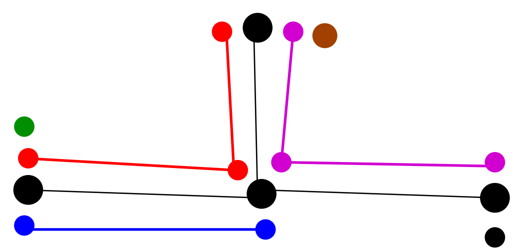




$\longrightarrow$ two subtrees T_u, T_v may overlap even if $\{u, v\}$ is **not** an edge of G

History

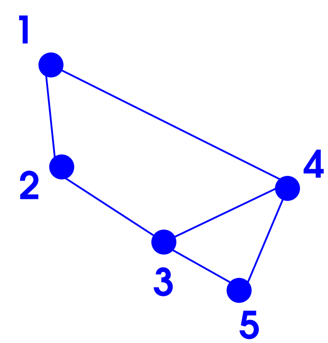
Fulkerson and Gross (1965), binary packing integer programs

$$\begin{aligned} \mathrm{IP} = \max \quad & c^T x && (10a)\\ \text{s.t.} \quad & Ax \le b, && (10b)\\ & x \in \{0,1\}^n && (10c) \end{aligned}$$

Here, $\boldsymbol{A}$ is has $\mathbf{0, 1}$-valued entries. **Idea:** use the structure of $\boldsymbol{A}$.
The **intersection graph of A**, $\boldsymbol{G_A}$, has:

- A vertex for each column of A.
- An edge between two columns $\boldsymbol{j, k}$ if there is a row $\boldsymbol{i}$ with $\boldsymbol{a_{ij} \neq 0,\ a_{ik} \neq 0}$.

$$\begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{bmatrix} 1 & & & 1 & \\ 1 & & & & \\ 1 & 1 & & & \\ & 1 & 1 & & \\ & & 1 & 1 & 1 \end{bmatrix} \end{array}$$



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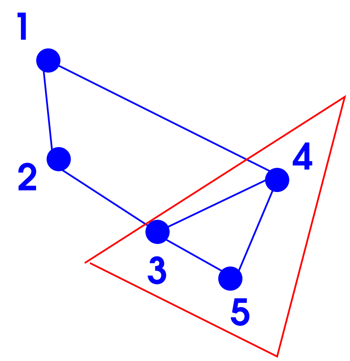

Each row of A induces a clique of $\boldsymbol{G_A}$.

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Theorem. If $\boldsymbol{G_A}$ is an **interval graph**, then

$$\begin{aligned} \text{IP} = \text{LP} = \max \quad & c^T x && \text{(13a)} \\ \text{s.t.} \quad & Ax \leq b, && \text{(13b)} \\ & x \in [0,1]^n. && \text{(13c)} \end{aligned}$$

(so IP = value of its continuous relaxation).

A graph $\boldsymbol{G = (V, E)}$ is an interval graph, if there is a **path** $\boldsymbol{P}$, and a family of subpaths $\boldsymbol{P_v}$ (one for each $\boldsymbol{v \in V}$), such that

- For each **pair of vertices** $\boldsymbol{u}$ and $\boldsymbol{v}$ of $\boldsymbol{G}$, we have $\boldsymbol{\{u, v\} \in E}$ **whenever** $\boldsymbol{P_u}$ and $\boldsymbol{P_v}$ intersect.
- The largest clique size of $\boldsymbol{G}$ is $\boldsymbol{\max_{p \in P} |\{v \in V \,:\, p \in P_v\}|}$.
  (The maximum number of subpaths that simultaneously overlap anywere on $\boldsymbol{P}$)

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Definition: (Gavril, 1974) A graph $\boldsymbol{G = (V, E)}$ is **chordal**, if there exists

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(equivalent: a graph is chordal iff every cycle of length > 3 has a chord).

Contrast with tree-decompositions

A tree-decomposition of G is a pair (T, Q) where:

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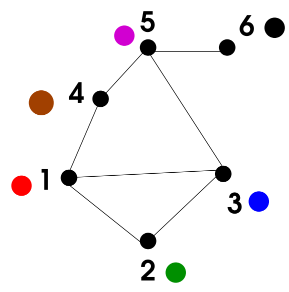

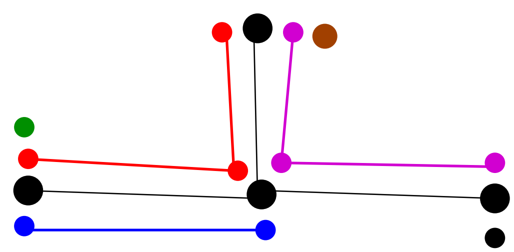

$\longrightarrow$ two subtrees T_u, T_v may overlap even if $\{u, v\}$ is **not** an edge of G

So: A graph G has a tree-decomposition of width w iff there is a **chordal supergraph** of G of clique number $w + 1$.

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Theorem. If $\boldsymbol{G_A}$ is **chordal**, then

$$\begin{aligned} \text{IP} = \text{LP} = \max \quad & c^T x && (16a)\\ \text{s.t.} \quad & Ax \le b, && (16b)\\ & x \in [0,1]^n. && (16c) \end{aligned}$$

(so IP = value of its continuous relaxation).

Chordal graphs are "nice." In fact, they are **perfect**.

Why small tree-width helps

Cholesky factorization of:

$$A = \begin{pmatrix} * & * & * & & & & & \\ * & * & & * & & & & \\ * & & * & * & * & & * & \\ & * & * & * & & * & & * \\ & & * & & * & * & & \\ & & & * & * & * & & \\ & & * & & & & * & * \\ & & & * & & & * & * \end{pmatrix}$$

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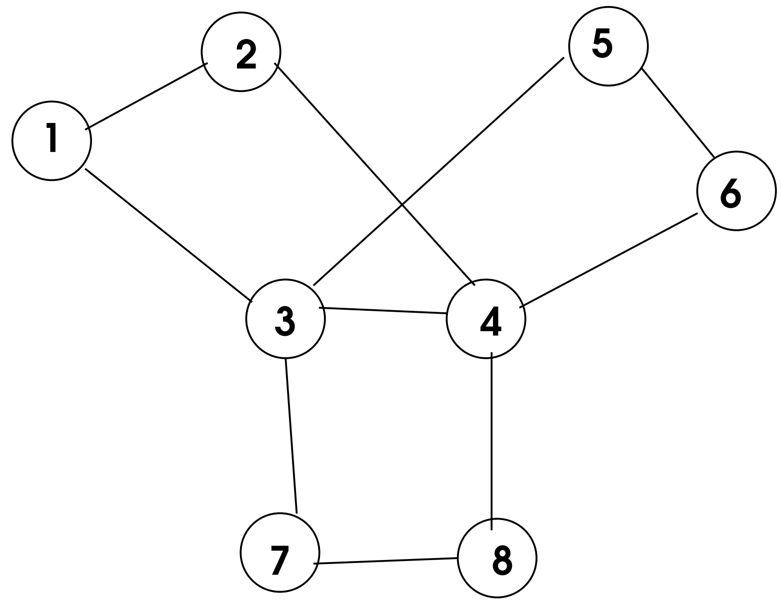
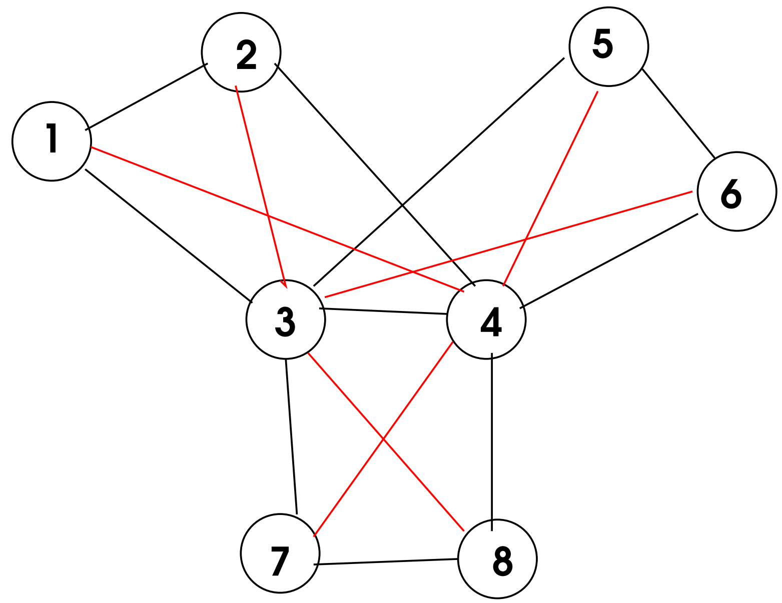


Chordal supergraph:



Pivoting order: 1, 2, 5, 6, 7, 8, 3, 4

Graph Minors Project: Robertson and Seymour, 1983 - 2004

$\longrightarrow$ Tree-width as a measure of the complexity of a graph

CAUTION

CAUTION

sparsity $\neq$ **small tree-width**

CAUTION

sparsity $\neq$ small tree-width

$\exists$ graphs of max deg 3 and arbitrarily high tree-width

Graph Minors Project: Robertson and Seymour, 1983 - 2004

→ Tree-width as a measure of the complexity of a graph

- Algorithms community: small tree-width makes hard problems easy (late 1980s)

- Many NP-hard problems can be solved in polynomial time on graphs with small tree-width:
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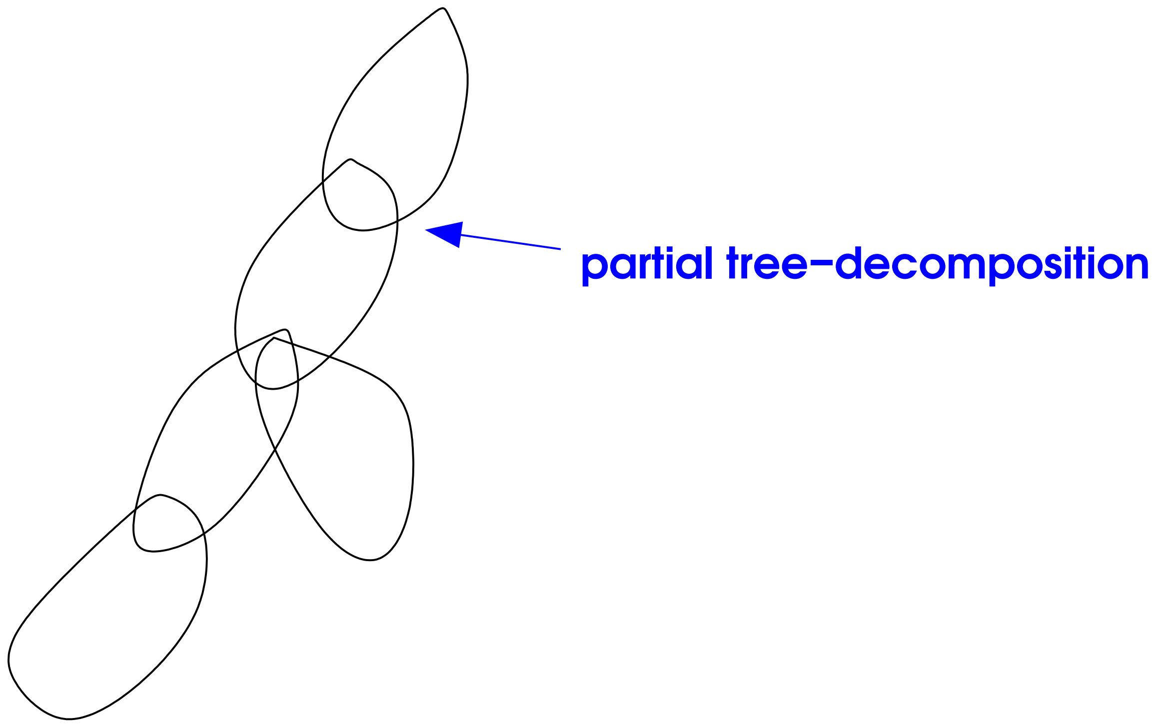
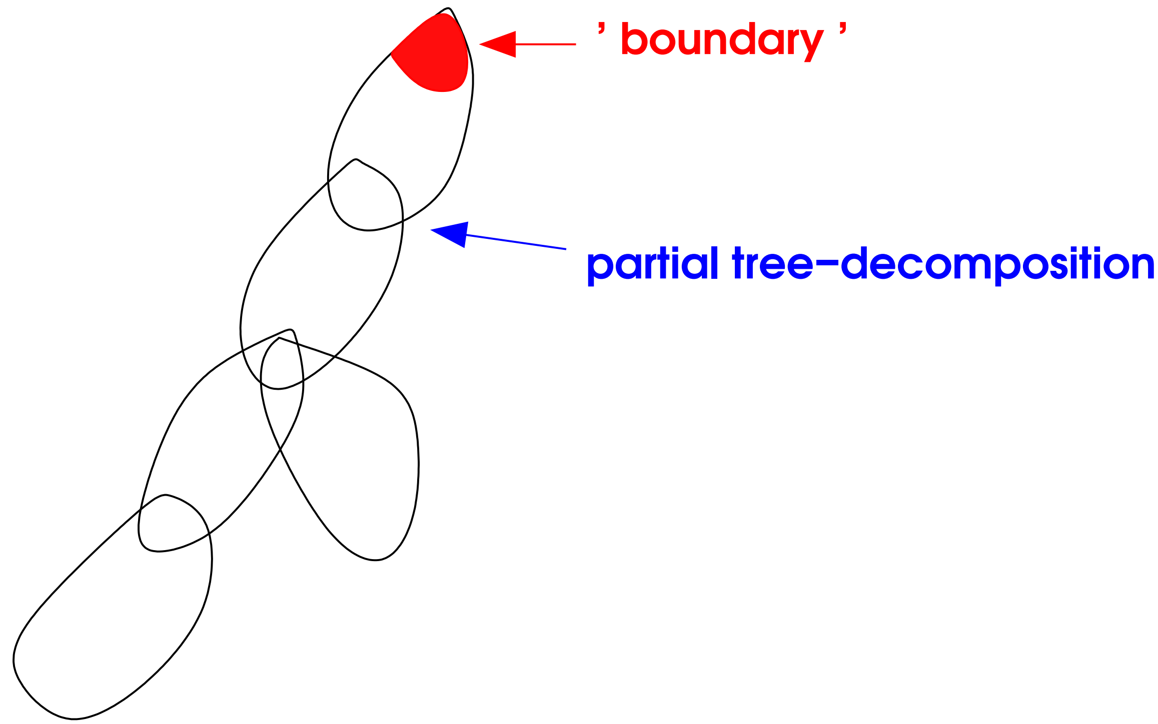
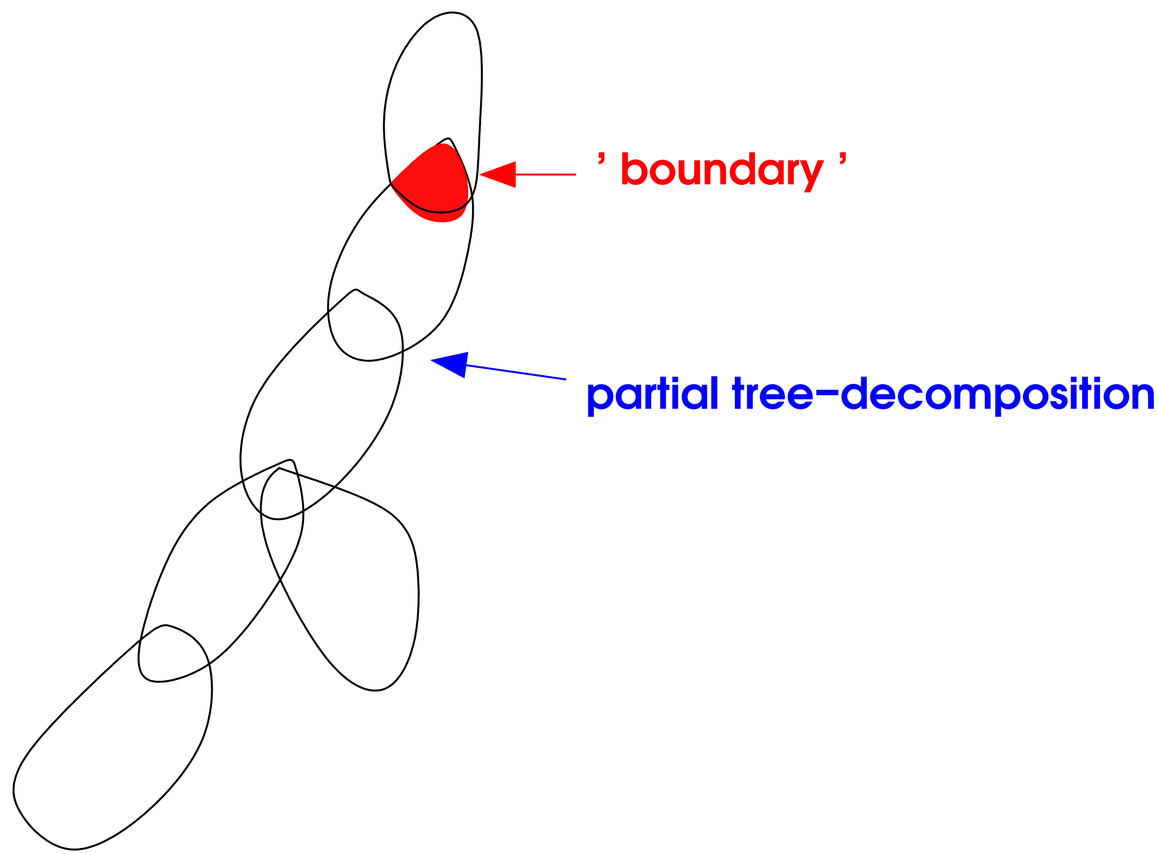
- Common thread: exploit tree-decomposition to obtain good algorithms

- So-called "non-sequential dynamic programming"


partial tree-decomposition



' boundary '
partial tree-decomposition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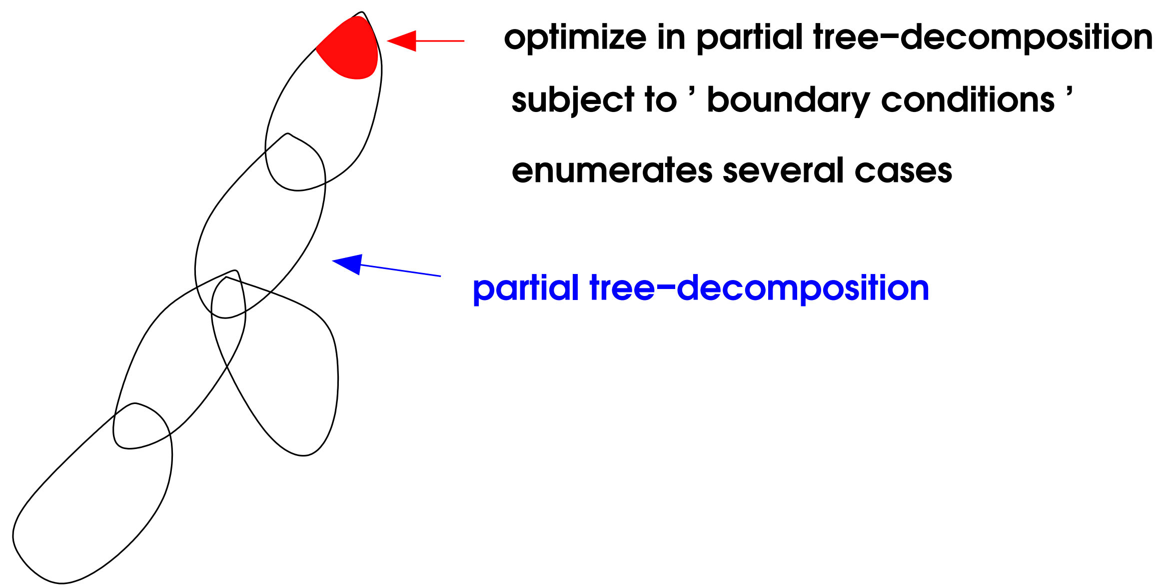
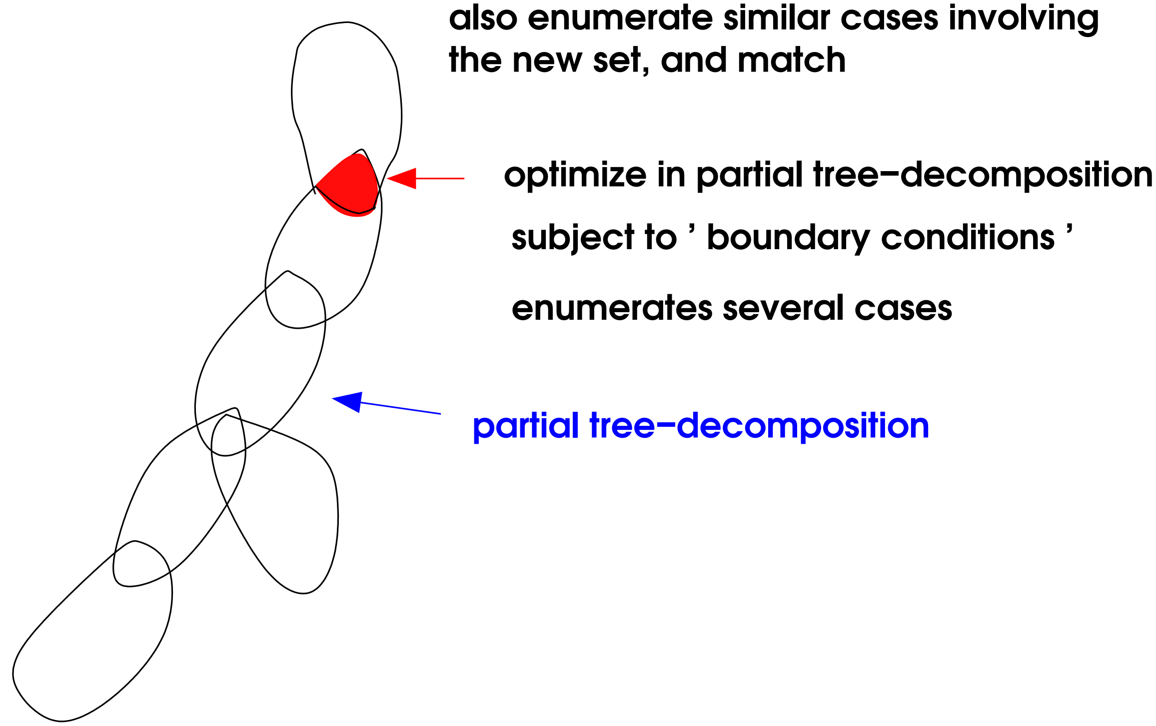

optimize in partial tree-decomposition
subject to ' boundary conditions '
enumerates several cases
partial tree-decomposition


also enumerate similar cases involving
the new set, and match
optimize in partial tree-decomposition
subject to ' boundary conditions '
enumerates several cases
partial tree-decomposition

Graph Minors Project: Robertson and Seymour, 1983 - 2004

$\longrightarrow$ Tree-width as a measure of the complexity of a graph

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- Common thread: exploit tree-decomposition to obtain good algorithms

- So-called "non-sequential dynamic programming"

$\longrightarrow$ Can we do the same for OPF ?

Theorem: Given an instance of **AC-OPF** on a graph with a tree-decomposition of width $\boldsymbol{\omega}$, and $\boldsymbol{n}$ buses, and $\mathbf{0} < \boldsymbol{\epsilon} < \mathbf{1}$,

there is a linear program **LP** such that:

(a) The number of variables and constraints is $\boldsymbol{O(\,2^{2\omega}\,\omega\,n\,\epsilon\log\epsilon^{-1}\,)}$.

(b) An optimal solution to **LP** solves **AC-OPF**, within tolerance $\boldsymbol{\epsilon}$.

More generic statement for AC-OPF

$$\mathbf{min} \quad \sum_{k \in V} C_k$$

$$\text{s.t.} \quad \forall km : \quad P_{km} \;=\; \boldsymbol{g_{km}}(e_k^2 + f_k^2) - \boldsymbol{g_{km}}(e_k e_m + f_k f_m) + \boldsymbol{b_{km}}(e_k f_m - f_k e_m)$$

$$\forall km : \quad Q_{km} \;=\; \boldsymbol{-b_{km}}(e_k^2 + f_k^2) + \boldsymbol{b_{km}}(e_k e_m + f_k f_m) + \boldsymbol{g_{km}}(e_k f_m - f_k e_m)$$

$$\forall km : \quad |P_{km}|^2 + |Q_{km}|^2 \;\leq\; \boldsymbol{U_{km}}$$

$$\forall k : \quad P_k \;=\; \sum_{km \in \delta(k)} P_{km}; \qquad \boldsymbol{P_k^{\mathbf{min}}} \;\leq\; P_k \;\leq\; \boldsymbol{P_k^{\mathbf{max}}}$$

$$\forall k : \quad Q_k \;=\; \sum_{km \in \delta(k)} Q_{km}; \qquad \boldsymbol{Q_k^{\mathbf{min}}} \;\leq\; Q_k \;\leq\; \boldsymbol{Q_k^{\mathbf{max}}}$$

$$\forall k : \quad \left(\boldsymbol{V_k^{\mathbf{min}}}\right)^{\mathbf{2}} \;\leq\; e_k^2 + f_k^2 \;\leq\; \left(\boldsymbol{V_k^{\mathbf{max}}}\right)^{\mathbf{2}}$$

$$\forall k : \quad C_k \;=\; \boldsymbol{F_k}\,(P_k, Q_k, e_k, f_k) \;+\; \sum_{km \in \delta(k)} \boldsymbol{H_{km}}(P_{km}, Q_{km}, e_k, f_k, e_m, f_m)$$

Here, the $\boldsymbol{F_k}$ and $\boldsymbol{H_{km}}$ are quadratics.

A generalization: graphical QCQPs (abridged)

Inputs:

(1) An undirected graph $\boldsymbol{H}$.

(2) For each vertex $\boldsymbol{v}$ of $\boldsymbol{H}$ a set $\boldsymbol{J(v)}$, and for $\boldsymbol{j \in J(v)}$ there is a real variable $\boldsymbol{x_j}$.
Write $\boldsymbol{\mathcal{V}} = \cup_{v \in V(H)} \boldsymbol{J(v)}$.

(3) For each edge $\boldsymbol{\{v, u\}}$ denote by $\boldsymbol{x^{v,u}}$ the vector of all $\boldsymbol{x_j}$ for $\boldsymbol{j \in J(v) \cup J(u)}$.

(4) For each vertex $\boldsymbol{v}$, and each edge $\boldsymbol{\{v, u\}}$ a family of quadratics $\boldsymbol{p^k_{v,u}(x^{v,u})}$ for $\boldsymbol{k = 1, \ldots, N(v)}$.

(5) A vector $\boldsymbol{c} \in \mathbb{R}^{\boldsymbol{\mathcal{V}}}$.

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(5) A vector $\boldsymbol{c} \in \mathbb{R}^{\boldsymbol{\mathcal{V}}}$.

Problem:

$$\text{(GQCQP):} \qquad \min \; c^T x$$

$$\text{subject to:} \qquad \sum_{u \in \delta(v)} p_{v,u,k}(x^{v,u}) \; \geq \; 0, \quad v \in V(H), \quad k = 1, \ldots, N(v)$$

$$0 \; \leq \; x_j \; \leq \; 1, \quad \forall j \in \mathcal{V}.$$

A generalization: mixed-integer graphical QCQPs (abridged)

Inputs:

(1) An undirected graph $\boldsymbol{H}$.

(2) For each vertex $\boldsymbol{v}$ of $\boldsymbol{H}$ a set $\boldsymbol{J(v)}$, and for $\boldsymbol{j} \in \boldsymbol{J(v)}$ there is a real variable $\boldsymbol{x_j}$.
Write $\boldsymbol{\mathcal{V}} = \cup_{v \in V(H)} \boldsymbol{J(v)}$.

(3) For each edge $\boldsymbol{\{v, u\}}$ denote by $\boldsymbol{x^{v,u}}$ the vector of all $\boldsymbol{x_j}$ for $\boldsymbol{j} \in \boldsymbol{J(v)} \cup \boldsymbol{J(u)}$.

(4) For each vertex $\boldsymbol{v}$, and each edge $\boldsymbol{\{v, u\}}$ a family of quadratics $\boldsymbol{p^k_{v,u}(x^{v,u})}$ for $\boldsymbol{k = 1, \ldots, N(v)}$.

(5) A vector $\boldsymbol{c} \in \mathbb{R}^{\boldsymbol{\mathcal{V}}}$.

(6) A partition $\boldsymbol{\mathcal{V}} = \boldsymbol{V_Z} \cup \boldsymbol{V_R}$.

Problem:

$$\text{(MGP):} \qquad \min \ c^T x$$

$$\text{subject to:} \qquad \sum_{u \in \delta(v)} p_{v,u,k}(x^{v,u}) \ \geq \ 0, \quad v \in V(H), \quad k = 1, \ldots, N(v)$$

$$0 \ \leq \ x_j \ \leq \ 1 \quad \forall\, j \in \mathcal{V}_R; \quad x_j \ = \ 0 \textbf{ or } 1 \quad \forall\, j \in \mathcal{V}_Z.$$

(1) An undirected graph $\boldsymbol{H}$.

(2) For each vertex $\boldsymbol{v}$ of $\boldsymbol{H}$ a set $\boldsymbol{J(v)}$, and for $\boldsymbol{j \in J(v)}$ there is a real variable $\boldsymbol{x_j}$. Write $\boldsymbol{\mathcal{V} = \cup_{v \in V(H)} J(v)}$.

(3) For each edge $\boldsymbol{\{v, u\}}$ denote by $\boldsymbol{x^{v,u}}$ the vector of all $\boldsymbol{x_j}$ for $\boldsymbol{j \in J(v) \cup J(u)}$.

(4) For each vertex $\boldsymbol{v}$, and each edge $\boldsymbol{\{v, u\}}$ a family of **polynomials** $\boldsymbol{p^k_{v,u}(x^{v,u})}$ for $\boldsymbol{k = 1, \ldots, N(v)}$.

(5) A vector $\boldsymbol{c} \in \mathbb{R}^{\boldsymbol{\mathcal{V}}}$.

(6) A partition $\boldsymbol{\mathcal{V}} = \boldsymbol{V_Z} \cup \boldsymbol{V_R}$.

(MGP):
$$\min \ c^T x \tag{20a}$$

subject to:
$$\sum_{u \in \delta(v)} p_{v,u,k}(x^{v,u}) \ \geq \ 0, \quad v \in V(H), \quad k = 1, \ldots, N(v) \tag{20b}$$

$$0 \leq x_j \leq 1 \quad \forall\, j \in \mathcal{V}_R; \quad x_j \ = 0 \textbf{ or } 1 \quad \forall\, j \in \mathcal{V}_Z. \tag{20c}$$

Theorem: Given an instance of **MGP** on a graph with a tree-decomposition of width $\boldsymbol{\omega}$, there is an equivalent instance of **MGP** on a graph

- With tree-width $\boldsymbol{\leq 2\omega + 1}$

- Of maximum degree $\mathbf{3}$.

Remark. If we start with an instance of AC-OPF, the equivalent problem is no longer an AC-OPF problem.

Approximation (Glover, 1975)(abridged)

Let $\boldsymbol{x}$ be a variable, with bounds $\mathbf{0} \leq \boldsymbol{x} \leq \mathbf{1}$. Let $\mathbf{0} < \gamma < \mathbf{1}$. Then we can approximate

$$\boldsymbol{x} \;\approx\; \sum_{i=1}^{L} 2^{-i} y_i$$

where each $\boldsymbol{y_i}$ is a **binary variable**. In fact, choosing $\boldsymbol{L} = \lceil \log_2 \gamma^{-1} \rceil$, we have

$$\boldsymbol{x} \;\leq\; \sum_{i=1}^{L} 2^{-i} y_i \;\leq\; \boldsymbol{x} + \gamma.$$

So: given an instance of $\boldsymbol{MGP}$, approximate each continuous variable $\boldsymbol{x_j}$ in this manner.

Theorem: Consider an instance $\mathcal{I}$ of problem **MGP**, with n variables. Then there is another instance, $\mathcal{B}$ of **MGP**, with

(1) $\mathcal{B}$ is defined on the same graph as $\mathcal{I}$.

(2) all variables in $\mathcal{B}$ are binary.

(3) For each continuous variable x_j of $\mathcal{I}$, we now have $\log_2 J^* \log \epsilon^{-1}$ binary variables used to approximate x_j.

(4) Solving $\mathcal{B}$ to exact optimality yields a solution to $\mathcal{I}$ within tolerance ϵ.

J^* = size of largest set $J(v)$. (AC-OPF $\Rightarrow$ $J^* = 2$)

Review

(1) A mixed-integer, graphical polynomial optimization problem on a graph with a tree-decomposition of width ω.

Review

(1) A mixed-integer, graphical polynomial optimization problem on a graph with a tree-decomposition of width $\boldsymbol{\omega}$.

$$\downarrow$$

(2) An equivalent mixed-integer, graphical polynomial optimization problem on a graph with a tree-decomposition of width $\boldsymbol{O(\omega)}$ and **degree** $\boldsymbol{\leq 3}$.

Review

(1) A mixed-integer, graphical polynomial optimization problem on a graph with a tree-decomposition of width $\boldsymbol{\omega}$.

(2) An equivalent mixed-integer, graphical polynomial optimization problem on a graph with a tree-decomposition of width $\boldsymbol{O(\omega)}$ and **degree** $\boldsymbol{\leq 3}$.

(3) An all-binary, graphical polynomial optimization problem on the same graph which is equivalent to the problem in **(2)** within tolerance $\boldsymbol{\epsilon}$. The sets $\boldsymbol{J(v)}$ have grown by a factor of $\boldsymbol{\log_2 J^* \, \log_2 \epsilon^{-1}}$.

Ancient History of this Talk

Fulkerson and Gross (1965), binary packing integer programs

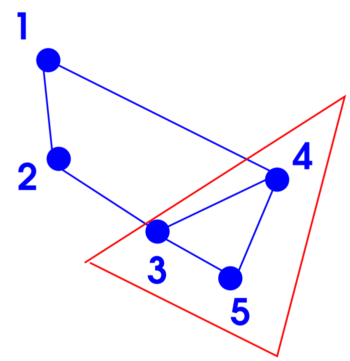
$$\begin{aligned}
\mathrm{IP} = \max \quad & c^T x && (21a)\\
\text{s.t.} \quad & Ax \;\leq\; b, && (21b)\\
& x \in \{0,1\}^n && (21c)
\end{aligned}$$

Here, $\boldsymbol{A}$ is has $\mathbf{0,1}$-valued entries. **Idea:** use the structure of $\boldsymbol{A}$.

The **intersection graph of A**, $\boldsymbol{G_A}$, has:

- A vertex for each column of A.
- An edge between two columns $\boldsymbol{j,k}$ if there is a row $\boldsymbol{i}$ with $\boldsymbol{a_{ij} \neq 0,\; a_{ik} \neq 0}$.

$$\begin{array}{c}
\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array}\\
\begin{bmatrix}
1 & & & 1 & \\
1 & & & & \\
1 & 1 & & & \\
 & 1 & 1 & & \\
 & & 1 & 1 & 1
\end{bmatrix}
\end{array}$$



Each row of A induces a clique of $\boldsymbol{G_A}$.

Review

(1) A mixed-integer, graphical polynomial optimization problem on a graph with a tree-decomposition of width $\boldsymbol{\omega}$.

(2) An equivalent mixed-integer, graphical polynomial optimization problem on a graph with a tree-decomposition of width $\boldsymbol{O(\omega)}$ and **degree** $\leq \mathbf{3}$.

(3) An all-binary, graphical polynomial optimization problem on the same graph which is equivalent to the problem in **(2)** within tolerance $\boldsymbol{\epsilon}$. The sets $\boldsymbol{J(v)}$ have grown by a factor of $\boldsymbol{\log_2 J^* \ \log_2 \epsilon^{-1}}$.

(4) Corollary. The intersection graph of the problem in **(3)** has a tree-decomposition of width at most

$$\boldsymbol{O(\omega\, J^* \log_2 J^* \ \log_2 \epsilon^{-1})}$$

Note: There are **two** graphs. The initial graph used to define the problem, and the intersection graph for the constraints in **(3)**.

Pièce de Résistance

Theorem. Given an all-binary problem on n variables and whose intersection graph has a tree-decomposition of width k, then there is an exact linear programming representation using

$$O(2^k n)$$

variables and constraints.

Construction similar to Lovász-Schrijver, Sherali-Adams, Lasserre, Bienstock-Zuckerberg

Pièce de Résistance

Theorem. Given an all-binary problem on $\boldsymbol{n}$ variables and whose intersection graph has a tree-decomposition of width $\boldsymbol{k}$, then there is an exact linear programming representation using

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Construction similar to Lovász-Schrijver, Sherali-Adams, Lasserre, Bienstock-Zuckerberg

(A) A mixed-integer, graphical polynomial optimization problem, with $\boldsymbol{N}$ variables, on a graph with a tree-decomposition of width $\boldsymbol{\omega}$.
$\boldsymbol{J^*}$ = size of largest set $\boldsymbol{J(v)}$. (AC-OPF $\boldsymbol{J^* = 2}$)

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$\boldsymbol{J^*}$ = size of largest set $\boldsymbol{J(v)}$. (AC-OPF $\boldsymbol{J^* = 2}$)

$$\downarrow$$

(B) A linear program that solves the problem in **(A)** within tolerance $\boldsymbol{\epsilon}$, of size

$$\boldsymbol{O(\, 2^{O(\omega J^*)}\, \omega\, J^*\, \epsilon^{-1}\, N)}$$

Should we able to do better?

Probably.

But.

- There are trivial AC-OPF problems where there is a unique feasible solution and it is irrational.
  Under the bit model of computing we cannot produce an "exact" answer.

- AC-OPF is weakly NP-hard on *trees*. Lavaei and Low (2011), a more recent proof by Coffrin and van Hentenryck.

- AC-OPF is strongly NP-hard on general graphs. A. Verma (2009). So no strong approximation algorithms exist unless P = NP.

Optimal Resilient Distribution Grid Design

Russell Bent

Joint work with Scott Backhaus, Brent Daniel, Harsha Nagarajan, and Emre Yamangil (see poster)

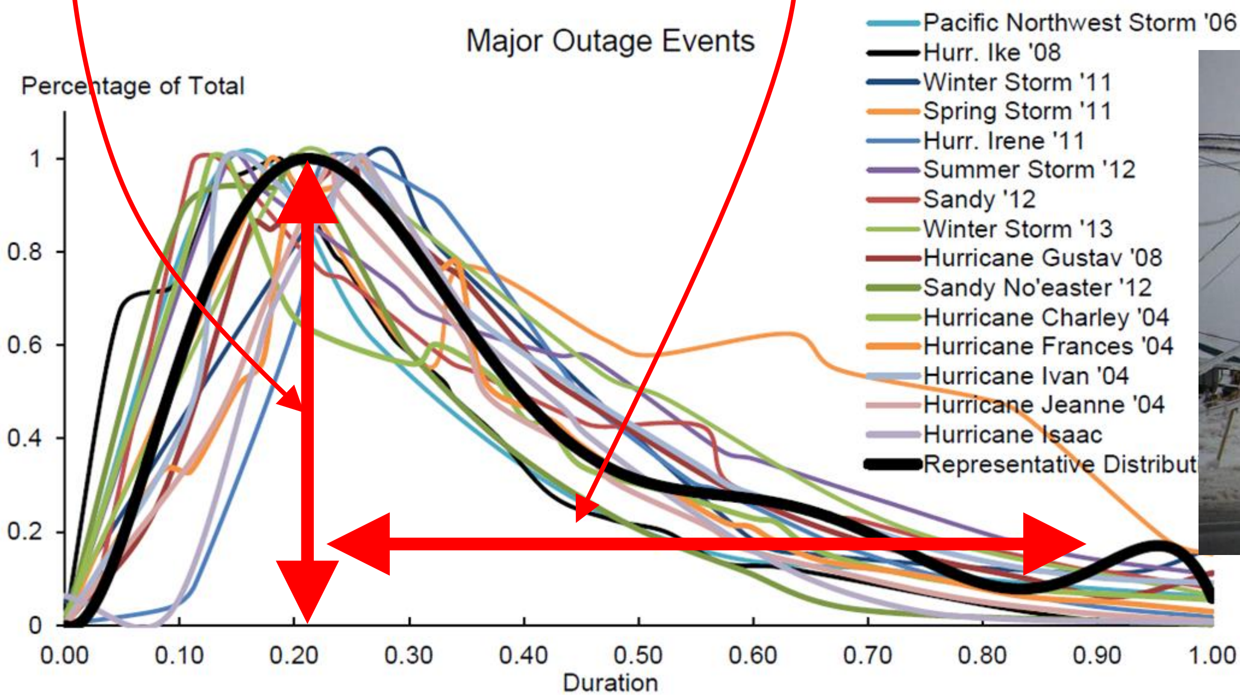
LA-UR-15-20362

This work was supported by the Microgrid Program of the Office of Electricity within the U.S. Department of Energy

Definition of Resilience

<u>Presidential Policy Directive - Critical Infrastructure Security and Resilience</u>

> ***"The ability to prepare for and adapt to changing conditions and withstand and recover rapidly from disruptions. Resilience includes the ability to <u>withstand</u> and <u>recover</u> from deliberate attacks, accidents, or naturally occurring threats or incidents."***



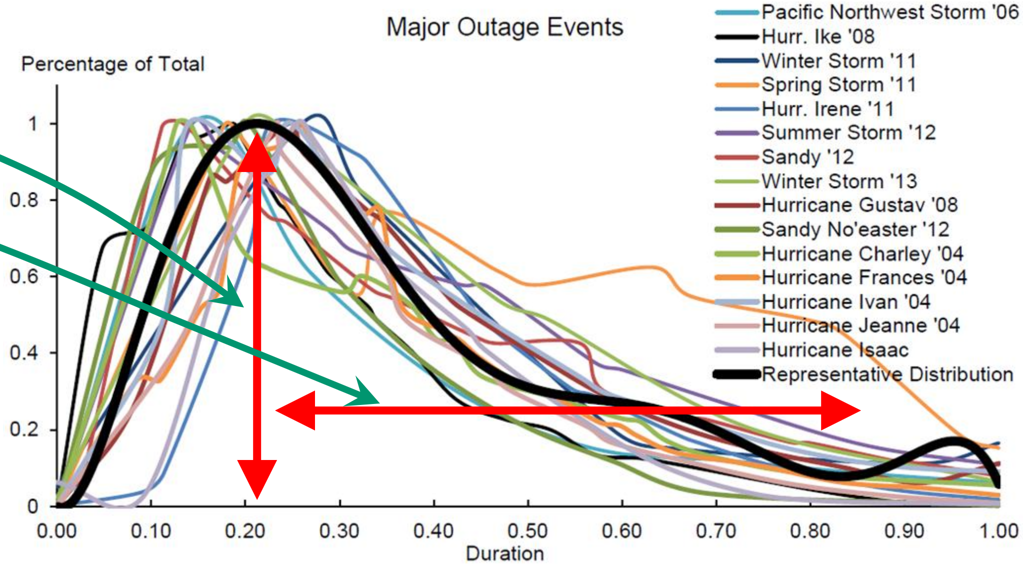
Source: Department of Energy, Office of Electrcity Delivery and Energy Reliability

Problem Overview: Our Goals

Develop new tools, methodologies, and algorithms to enable the design of resilient power distribution systems—utility scale

Hardening/Resilience options

- *Asset hardening*
- *System design*
- *System operations*
- *Repair scheduling*
- *Emergency operations*



Source: Department of Energy, Office of Electrcity Delivery and Energy Reliability

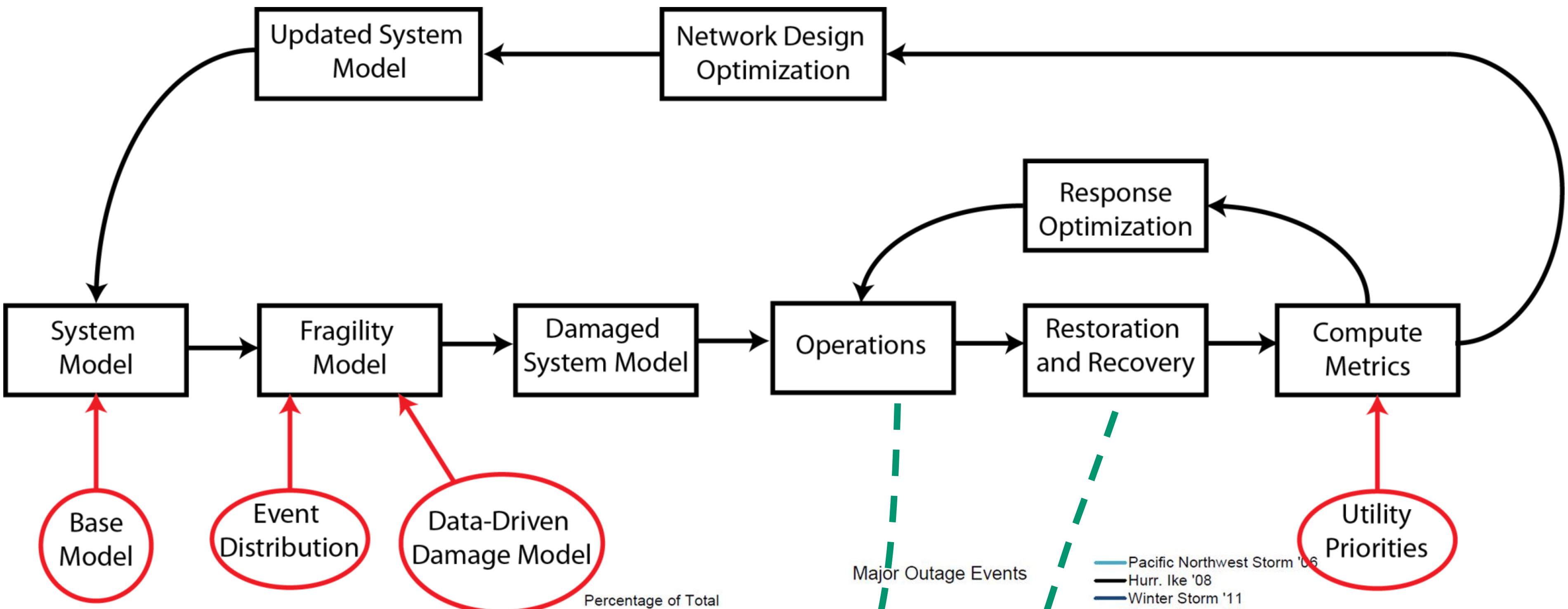
Flexibility for the user

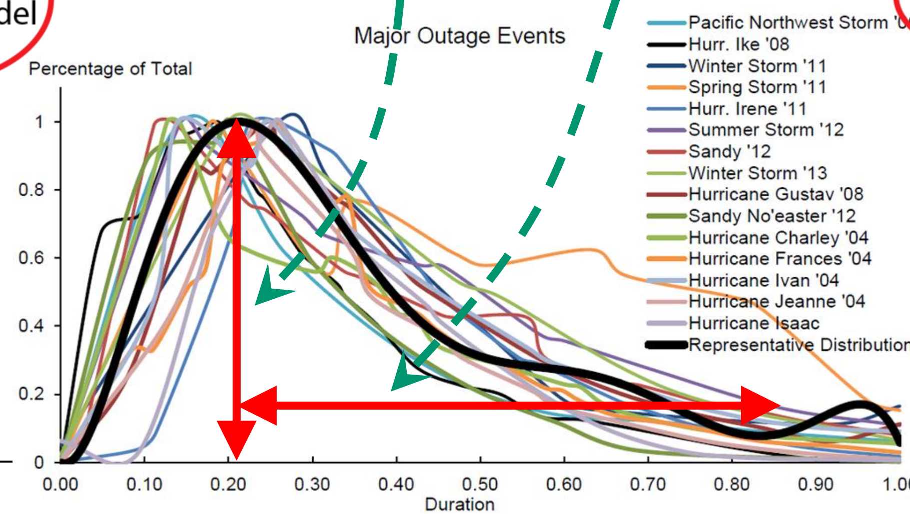
- *User's base network model*
- *User-defined resilience metrics*
- *User suggests upgrades*
- *User-defined costs*
- *User-defined threat and scenarios*

Capabilities

- *Assess current resilience posture*
- *Optimize over user-suggested upgrades to improve resilience considering budget*

Problem Overview





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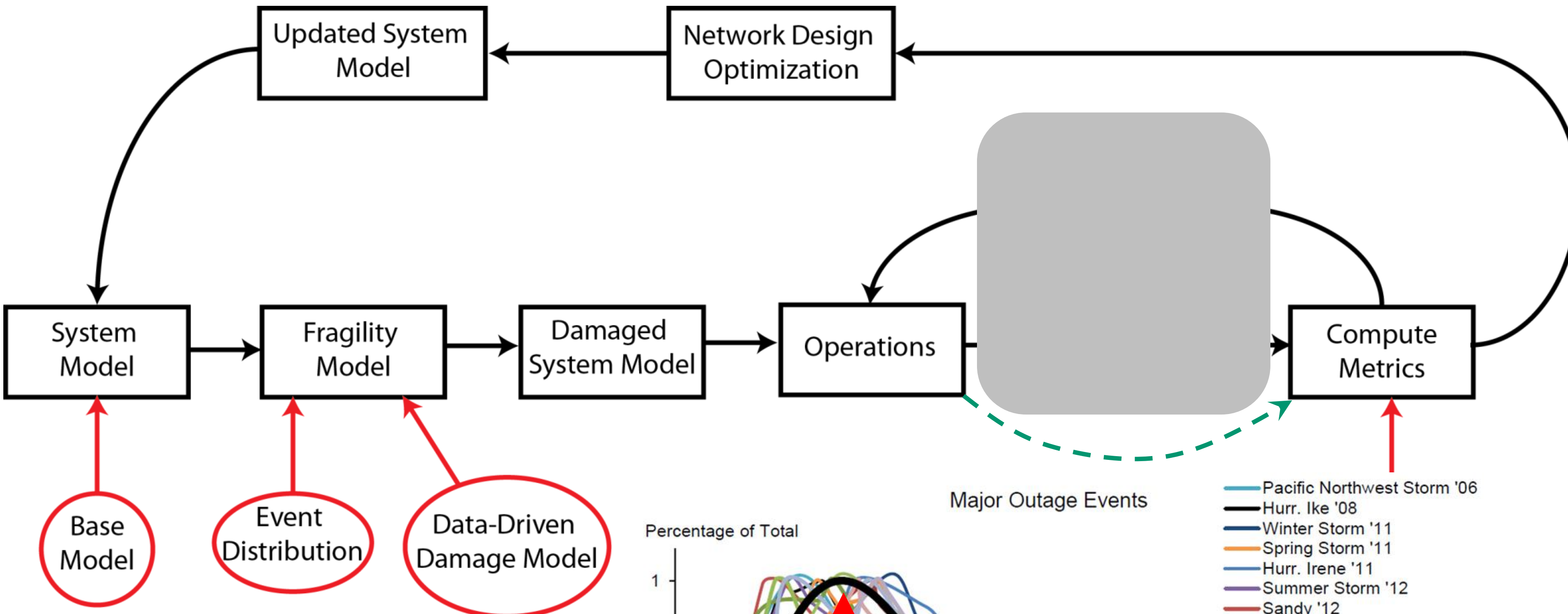
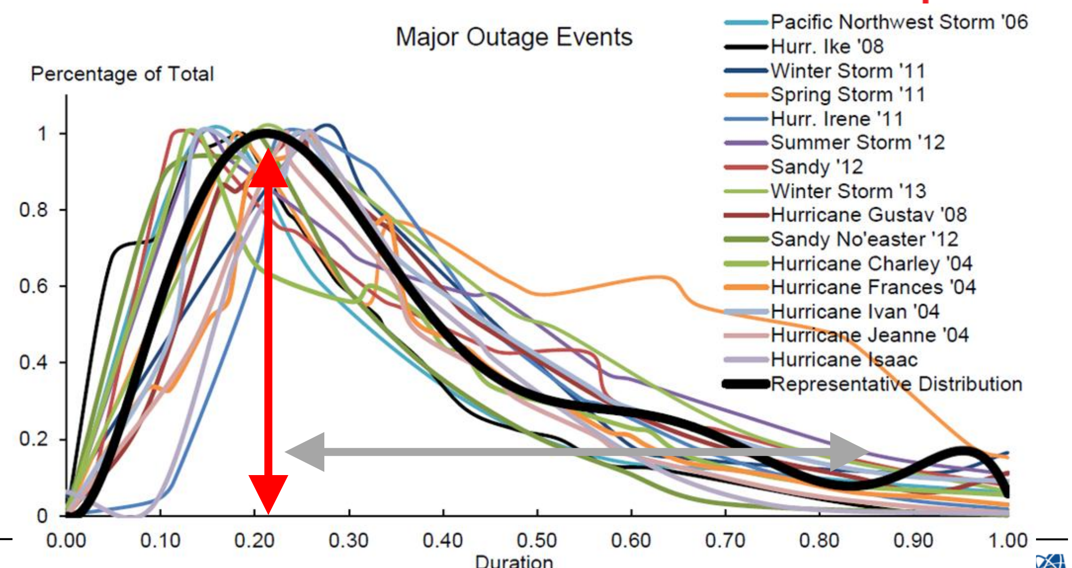
Today's Talk





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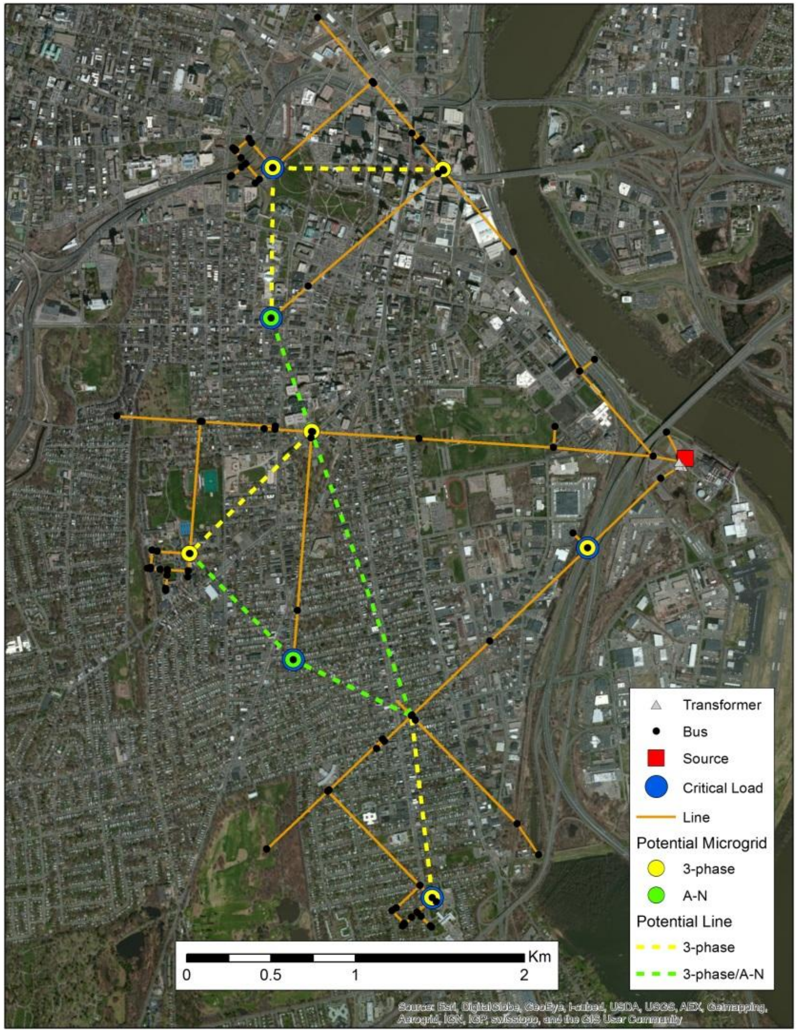
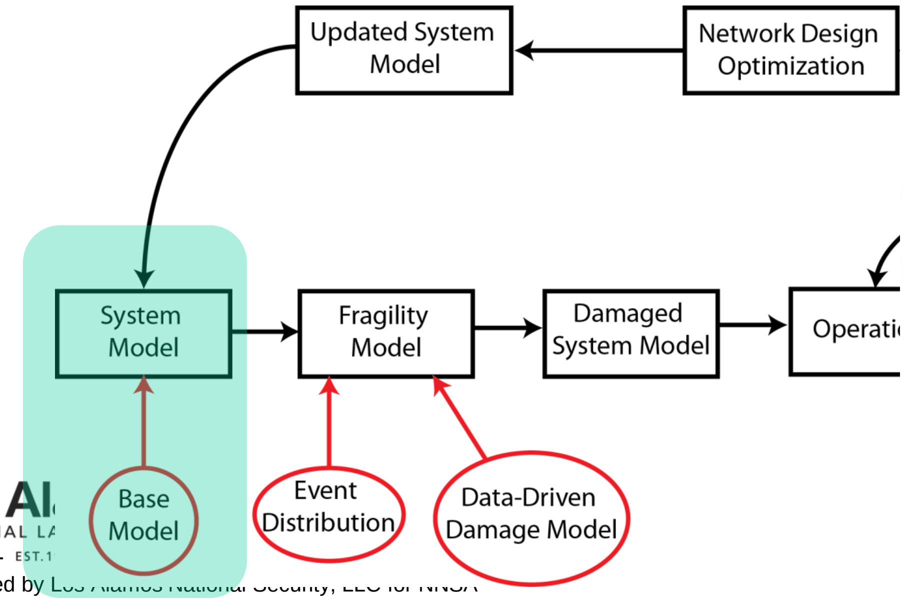
Resilience Design Process Flow—System Model

Flexibility for the user

- *User's base network model*
- *User-defined resilience metrics, e.g. critical load service*
- *User suggests upgrades*
- *User-defined costs*
- *User-defined threat and scenarios*

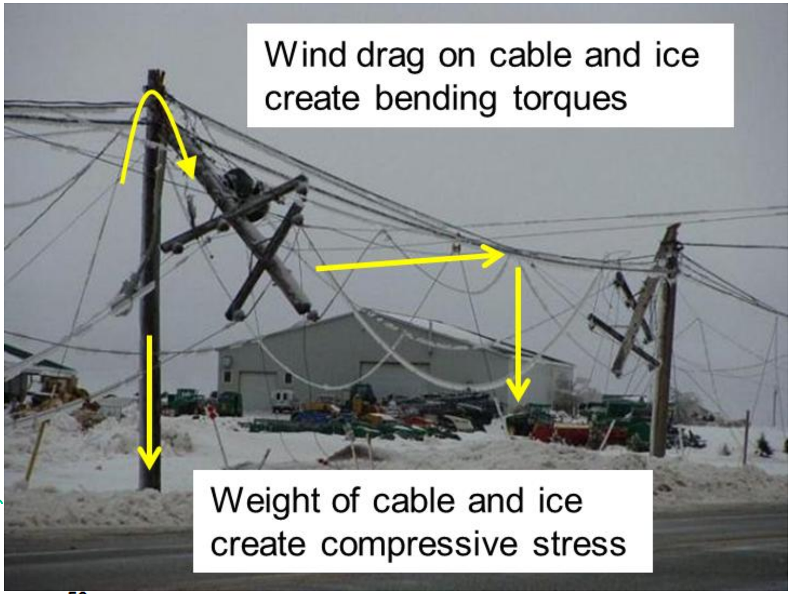


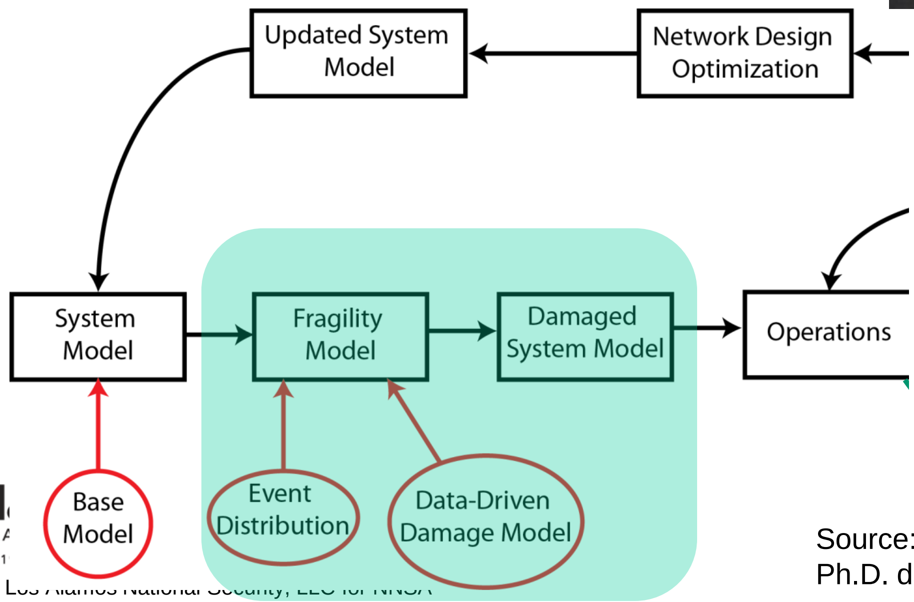


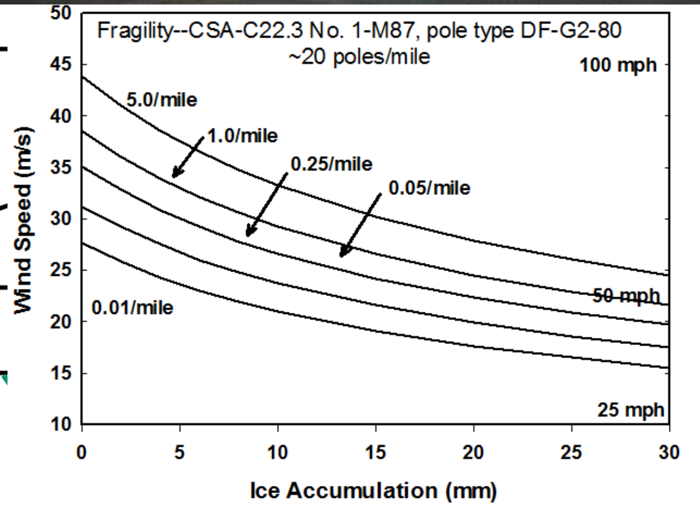
Resilience Design Process Flow—Direct Impacts

Flexibility for the user

User's base network model
User-defined resilience metrics, e.g.
critical load service
User suggests upgrades
User-defined costs
User-defined threat and scenarios

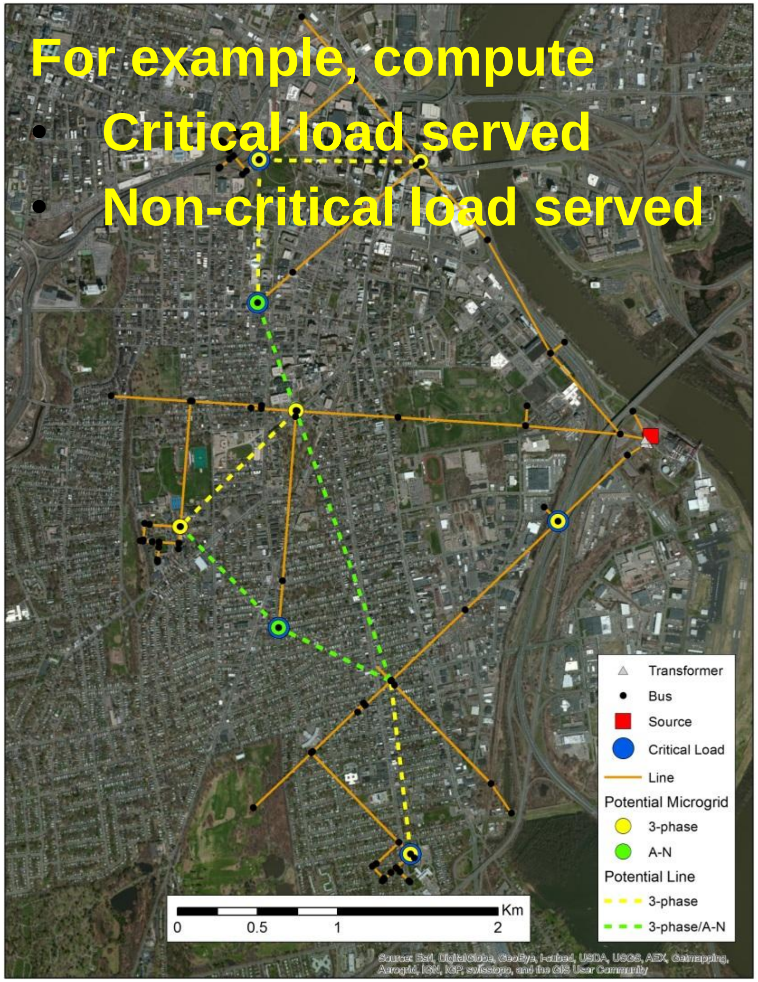


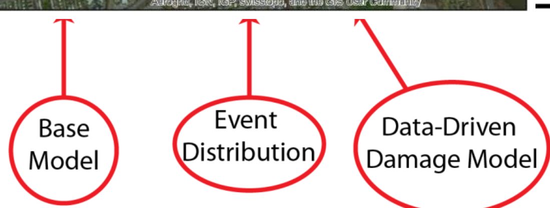




Source: Y. Sa, *Reliability analysis of electric distribution lines* Ph.D. dissertation, McGill University, Montreal, Canada, 2002

Resilience Design Process Flow—Secondary





Flexibility for the user

User's base network model

User-defined resilience metrics, e.g. critical load service

User suggests upgrades

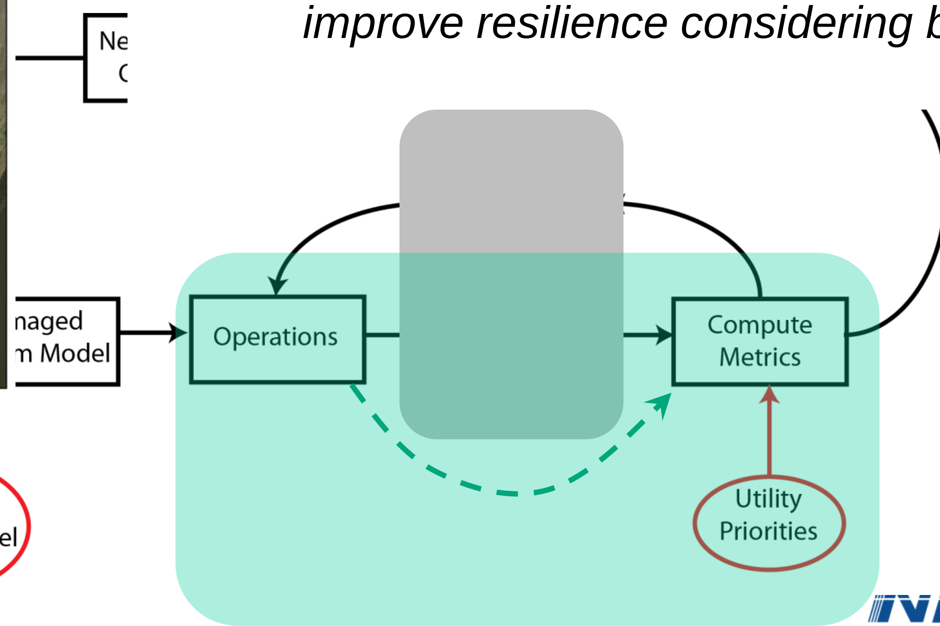
User-defined costs

User-defined threat and scenarios

Capabilities

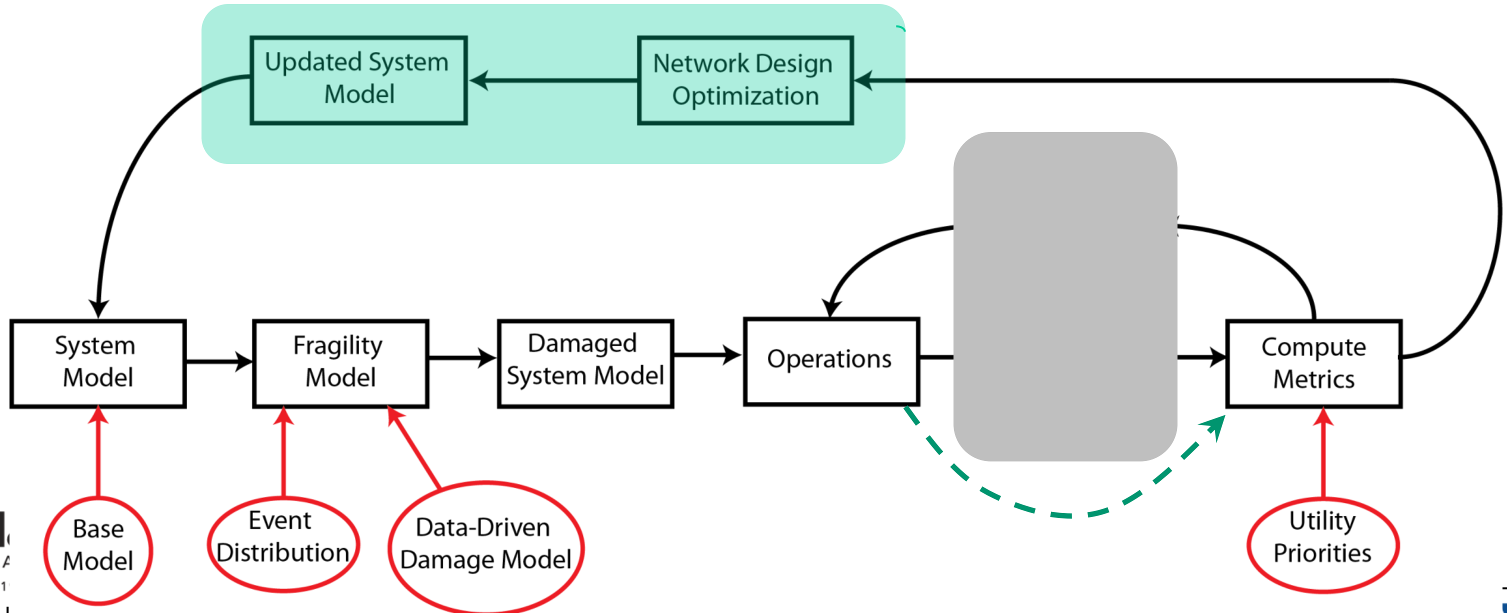
Assess current resilience posture

Optimize over user-suggested upgrade to improve resilience considering budget



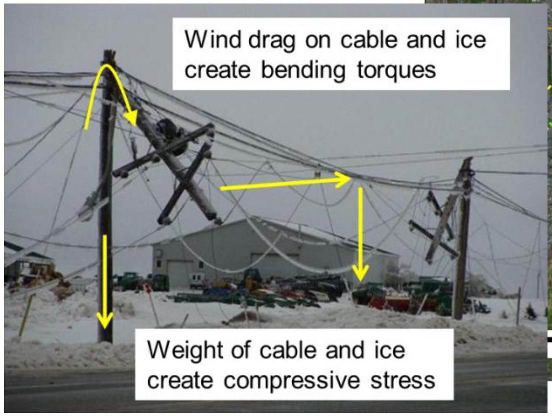
Resilience Design Process Flow—Design Network

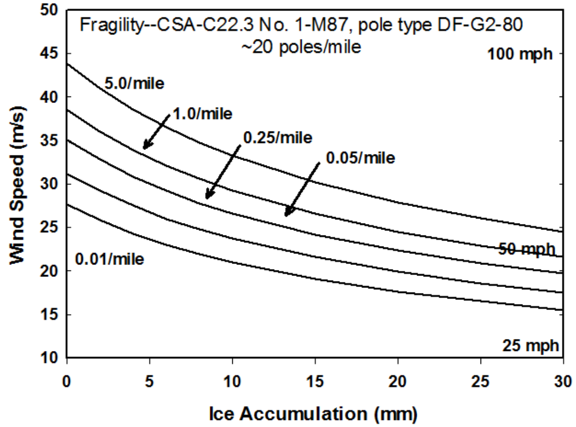
- Hardening/Resilience options
  - *Asset hardening*
  - *System design*
  - *System operations*
  - *Repair scheduling*
  - *Emergency operations*
- Capabilities
  - *Assess current resilience posture*
  - *Optimize over user-suggested upgrade to improve resilience considering budget*



Resiliency Model Details

- **Distribution power system**
  - Power lines, loads, generation
- **Hardening and Resilience Options**
  - Distributed generation
  - 3-phase or 1-phase interties
    - Above ground or underground
  - Add switches to:
    - Reconfigure circuits
    - Shed circuits and/or loads
  - Harden existing components
    - Reduce damage probabilities

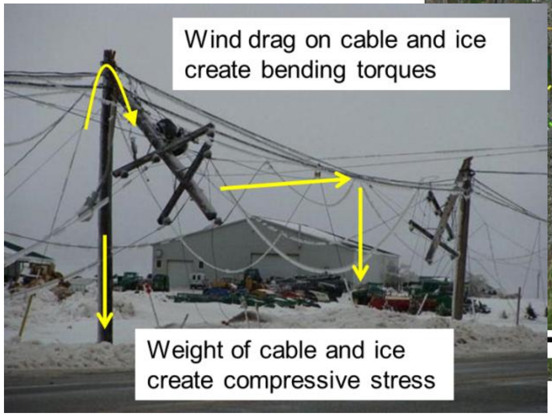


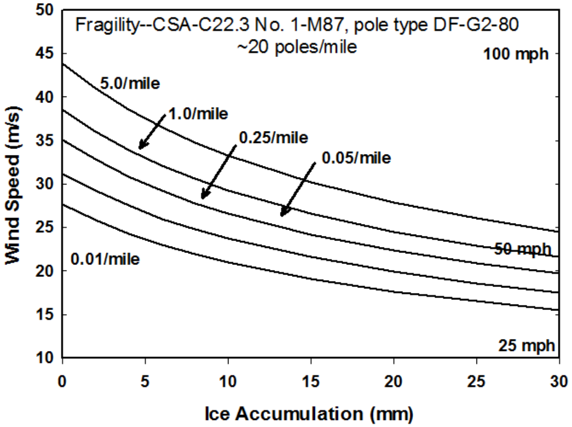


Source: Y. Sa, *Reliability analysis of electric distribution lines* Ph.D. dissertation, McGill University, Montreal, Canada, 2002

Resiliency Model Details

- **Damage Scenarios**
  - Historical data
  - Probability distribution
- **Operating and Resilience Constraints**
  - Radial operations
  - Load satisfaction
    - Critical and non-critical load
- **Objective**
  - Minimize cost





Source: Y. Sa, *Reliability analysis of electric distribution lines* Ph.D. dissertation, McGill University, Montreal, Canada, 2002

Optimization Model

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

s.t. $x_{ij}^s \le x_{ij}, \tau_{ij}^s \le \tau_{ij}, t_{ij}^s \le t_{ij}, z_i^{ks} \le z_i^k, u_i^s \le u_i$

$$-x_{ij0}^s Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{sk} \le x_{ij1}^s Q_{ij}^k$$

$$x_{ij0}^s + x_{ij1}^s \le x_{ij}^s$$

$$-(1-\tau_{ij}^s)\, Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{ks} \le (1-\tau_{ij}^s)\, Q_{ij}^k$$

$$-\beta_{ij}\frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le f_{ij}^{k's} - \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le \beta_{ij}\frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|}$$

$$z_i^k \le M_i^k u_i$$

$x_{ij}^s = t_{ij}^s$ when x is damaged

$$\mathrm{l}_\mathrm{i}^{\mathrm{ks}} = y_i^s d_i^k$$

$$0 \le g_i^{sk} \le z_i^{ks} + g_i^{k+}$$

$$\mathrm{g}_\mathrm{i}^{\mathrm{ks}} - l_i^{ks} - \sum_{j\in N} f_{ij}^{ks} = 0$$

$$0 \le z_i^{ks} \le u_i^s Z_i^k$$

$$\sum_{ij\in s}\left(\overline{x}_{ij}^s + (1-\overline{\tau}_{ij})\right) \le |s|-1$$

$$\tau_{ij}^s \ge x_{ij}^s + \overline{\tau}_{ij}^s - 1,\ x_{ij}^s \le \overline{x}_{ij}^s$$

$$\sum_{i\in CL,k\in p_i} l_i^{ks} \ge \lambda \sum_{i\in CL,k\in p_i} d_i^k$$

$$\sum_{i\in N\setminus L,k\in p_i} l_i^{ks} \ge \gamma \sum_{i\in N\setminus L,k\in p_i} d_i^k$$

$$x, y, \tau, u, t \in \{0,1\}$$

<u>Key Features</u>

- Least cost design for a set of scenarios
- Three-phase unbalanced real power flows
- Enforces radial operations
- Enforces phase balance tolerance
- Discrete variables for load shedding (per scenario), line switching (per scenario), capital construction (first stage)

- Relaxes unbalanced 3 phase power flows to a multi-commodity flow
- Assumption/Justification: Radial operations + Initial network is voltage feasible, upgrades tend to move loads closer to generation, which improves voltage and lowers line loading.

Optimization Model

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

Minimize expansion cost

s.t. $x_{ij}^s \le x_{ij}, \tau_{ij}^s \le \tau_{ij}, t_{ij}^s \le t_{ij}, z_i^{ks} \le z_i^k, u_i^s \le u_i$

$$-x_{ij0}^s Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{sk} \le x_{ij1}^s Q_{ij}^k$$

$$x_{ij0}^s + x_{ij1}^s \le x_{ij}^s$$

$$-(1-\tau_{ij}^s)\, Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{ks} \le (1-\tau_{ij}^s)\, Q_{ij}^k$$

$$-\beta_{ij}\frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le f_{ij}^{k's} - \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le \beta_{ij}\frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|}$$

$x_{ij}^s = t_{ij}^s$ when x is damaged

$$\mathrm{l}_{\mathrm{i}}^{\mathrm{ks}} = y_i^s d_i^k$$

$$0 \le g_i^{sk} \le z_i^{ks} + g_i^{k+}$$

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$$x, y, \tau, u, t \in \{0,1\}$$

Optimization Model

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

s.t. $\boxed{x_{ij}^s \le x_{ij}\ , \tau_{ij}^s \le \tau_{ij}, t_{ij}^s \le t_{ij}, z_i^{ks} \le z_i^k, u_i^s \le u_i}$

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$$x, y, \tau, u, t \in \{0,1\}$$

Auxiliary variables for linking first and second stage. Useful for decomposition

Optimization Model

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

s.t. $x_{ij}^s \le x_{ij}\ , \tau_{ij}^s \le \tau_{ij}, t_{ij}^s \le t_{ij}, z_i^{ks} \le z_i^k, u_i^s \le u_i$

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$$x_{ij0}^s + x_{ij1}^s \le x_{ij}^s$$

$$-(1-\tau_{ij}^s)\,Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{ks} \le (1-\tau_{ij}^s)\,Q_{ij}^k$$

Line capacity constraints. Capacity is 0 when line is unavailable or open.

$$-\beta_{ij}\frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le f_{ij}^{k's} - \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le \beta_{ij}\frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|}$$

$x_{ij}^s = t_{ij}^s$ when x is damaged

$$\mathrm{l}_\mathrm{i}^\mathrm{ks} = y_i^s d_i^k$$

$$0 \le g_i^{sk} \le z_i^{ks} + g_i^{k+}$$

$$\mathrm{g}_\mathrm{i}^\mathrm{ks} - l_i^{ks} - \sum_{j\in N} f_{ij}^{ks} = 0$$

$$0 \le z_i^{ks} \le u_i^s Z_i^k$$

$$\sum_{ij\in s}\left(\overline{x}_{ij}^s + (1-\overline{\tau}_{ij})\right) \le |s| - 1$$

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Optimization Model

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

s.t. $x_{ij}^s \le x_{ij}$, $\tau_{ij}^s \le \tau_{ij}, t_{ij}^s \le t_{ij}, z_i^{ks} \le z_i^k, u_i^s \le u_i$

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$$-(1-\tau_{ij}^s)\,Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{ks} \le (1-\tau_{ij}^s)\,Q_{ij}^k$$

$$-\beta_{ij}\frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le f_{ij}^{k's} - \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le \beta_{ij}\frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|}$$

Phase imbalance tolerance

$x_{ij}^s = t_{ij}^s$ when x is damaged

$$\mathrm{l}_\mathrm{i}^{\mathrm{ks}} = y_i^s d_i^k$$

$$0 \le g_i^{sk} \le z_i^{ks} + g_i^{k+}$$

$$\mathrm{g}_\mathrm{i}^{\mathrm{ks}} - l_i^{ks} - \sum_{j\in N} f_{ij}^{ks} = 0$$

$$0 \le z_i^{ks} \le u_i^s Z_i^k$$

$$\sum_{ij\in s}\left(\overline{x}_{ij}^s + (1-\overline{\tau}_{ij})\right) \le |s|-1$$

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$$x, y, \tau, u, t \in \{0,1\}$$

Optimization Model

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

s.t. $x_{ij}^s \leq x_{ij}$, $\tau_{ij}^s \leq \tau_{ij}, t_{ij}^s \leq t_{ij}, z_i^{ks} \leq z_i^k, u_i^s \leq u_i$

$$-x_{ij0}^s Q_{ij}^k \leq \sum_{k\in p_{ij}} f_{ij}^{sk} \leq \; x_{ij1}^s Q_{ij}^k$$

$$x_{ij0}^s + x_{ij1}^s \leq x_{ij}^s$$

$$-(1-\tau_{ij}^s)\, Q_{ij}^k \leq \sum_{k\in p_{ij}} f_{ij}^{ks} \leq \; (1-\tau_{ij}^s)\, Q_{ij}^k$$

$$-\beta_{ij}\frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \leq f_{ij}^{k's} - \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \leq \beta_{ij}\frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|}$$

$x_{ij}^s = t_{ij}^s$ when x is damaged

Links damaged lines with hardening variables

$$\mathrm{l}_\mathrm{i}^{\mathrm{ks}} = y_i^s d_i^k$$

$$0 \leq g_i^{sk} \leq z_i^{ks} + g_i^{k+}$$

$$\mathrm{g}_\mathrm{i}^{\mathrm{ks}} - l_i^{ks} - \sum_{j\in N} f_{ij}^{ks} = 0$$

$$0 \leq z_i^{ks} \leq u_i^s Z_i^k$$

$$\sum_{ij\in s}\left(\overline{x}_{ij}^s + (1-\overline{\tau}_{ij})\right) \leq \; |s|-1$$

$$\tau_{ij}^s \geq x_{ij}^s + \overline{\tau}_{ij}^s - 1,\; x_{ij}^s \leq \overline{x}_{ij}^s$$

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$$x, y, \tau, u, t \in \{0,1\}$$

Optimization Model

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

s.t. $x_{ij}^s \le x_{ij}, \tau_{ij}^s \le \tau_{ij}, t_{ij}^s \le t_{ij}, z_i^{ks} \le z_i^k, u_i^s \le u_i$

$$-x_{ij0}^s Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{sk} \le x_{ij1}^s Q_{ij}^k$$

$$x_{ij0}^s + x_{ij1}^s \le x_{ij}^s$$

$$-(1-\tau_{ij}^s)\, Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{ks} \le (1-\tau_{ij}^s)\, Q_{ij}^k$$

$$-\beta_{ij}\frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le f_{ij}^{k's} - \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le \beta_{ij}\frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|}$$

$x_{ij}^s = t_{ij}^s$ when x is damaged

$$\boxed{\mathrm{l}_\mathrm{i}^{\mathrm{ks}} = y_i^s d_i^k}$$

Load switching

$$0 \le g_i^{sk} \le z_i^{ks} + g_i^{k+}$$

$$\mathrm{g}_\mathrm{i}^{\mathrm{ks}} - l_i^{ks} - \sum_{j\in N} f_{ij}^{ks} = 0$$

$$0 \le z_i^{ks} \le u_i^s Z_i^k$$

$$\sum_{ij\in s}\left(\overline{x}_{ij}^s + (1-\overline{\tau}_{ij})\right) \le |s|-1$$

$$\tau_{ij}^s \ge x_{ij}^s + \overline{\tau}_{ij}^s - 1,\ x_{ij}^s \le \overline{x}_{ij}^s$$

$$\sum_{i\in CL,k\in p_i} l_i^{ks} \ge \lambda \sum_{i\in CL,k\in p_i} d_i^k$$

$$\sum_{i\in N\backslash L,k\in p_i} l_i^{ks} \ge \gamma \sum_{i\in N\backslash L,k\in p_i} d_i^k$$

$$x, y, \tau, u, t \in \{0,1\}$$

Optimization Model

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

s.t. $x_{ij}^s \leq x_{ij}$, $\tau_{ij}^s \leq \tau_{ij}, t_{ij}^s \leq t_{ij}, z_i^{ks} \leq z_i^k, u_i^s \leq u_i$

$$-x_{ij0}^s Q_{ij}^k \leq \sum_{k\in p_{ij}} f_{ij}^{sk} \leq x_{ij1}^s Q_{ij}^k$$

$$x_{ij0}^s + x_{ij1}^s \leq x_{ij}^s$$

$$-(1 - \tau_{ij}^s)\, Q_{ij}^k \leq \sum_{k\in p_{ij}} f_{ij}^{ks} \leq (1 - \tau_{ij}^s)\, Q_{ij}^k$$

$$-\beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \leq f_{ij}^{k's} - \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \leq \beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|}$$

$x_{ij}^s = t_{ij}^s$ when x is damaged

$$\mathrm{l}_\mathrm{i}^\mathrm{ks} = y_i^s d_i^k$$

$$\boxed{0 \leq g_i^{sk} \leq z_i^{ks} + g_i^{k^+}}$$

Power produced

$$\mathrm{g}_\mathrm{i}^\mathrm{ks} - l_i^{ks} - \sum_{j\in N} f_{ij}^{ks} = 0$$

$$0 \leq z_i^{ks} \leq u_i^s Z_i^k$$

$$\sum_{ij\in s} \left(\overline{x}_{ij}^s + (1 - \overline{\tau}_{ij})\right) \leq |s| - 1$$

$$\tau_{ij}^s \geq x_{ij}^s + \overline{\tau}_{ij}^s - 1,\ x_{ij}^s \leq \overline{x}_{ij}^s$$

$$\sum_{i\in CL,k\in p_i} l_i^{ks} \geq \lambda \sum_{i\in CL,k\in p_i} d_i^k$$

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$$x, y, \tau, u, t \in \{0,1\}$$

Optimization Model

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

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$$\boxed{\mathrm{g}_\mathrm{i}^\mathrm{ks} - l_i^{ks} - \sum_{j\in N} f_{ij}^{ks} = 0}$$

Nodal flow balance

$$0 \leq z_i^{ks} \leq u_i^s Z_i^k$$

$$\sum_{ij\in s} \left(\overline{x}_{ij}^s + (1-\overline{\tau}_{ij})\right) \leq |s| - 1$$

$$\tau_{ij}^s \geq x_{ij}^s + \overline{\tau}_{ij}^s - 1,\ x_{ij}^s \leq \overline{x}_{ij}^s$$

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$$x, y, \tau, u, t \in \{0,1\}$$

Optimization Model

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

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$$\boxed{0 \le z_i^{ks} \le u_i^s Z_i^k}$$

Links generation construction and capacity

$$\sum_{ij\in s}\left(\bar{x}_{ij}^s + (1-\bar{\tau}_{ij})\right) \le |s| - 1$$

$$\tau_{ij}^s \ge x_{ij}^s + \bar{\tau}_{ij}^s - 1,\ x_{ij}^s \le \bar{x}_{ij}^s$$

$$\sum_{i\in CL,k\in p_i} l_i^{ks} \ge \lambda \sum_{i\in CL,k\in p_i} d_i^k$$

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$$x, y, \tau, u, t \in \{0,1\}$$

Optimization Model

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij}t_{ij}$

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$$-x_{ij0}^s Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{sk} \le x_{ij1}^s Q_{ij}^k$$

$$x_{ij0}^s + x_{ij1}^s \le x_{ij}^s$$

$$-(1-\tau_{ij}^s)\, Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{ks} \le (1-\tau_{ij}^s)\, Q_{ij}^k$$

$$-\beta_{ij}\frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le f_{ij}^{k's} - \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le \beta_{ij}\frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|}$$

$x_{ij}^s = t_{ij}^s$ when x is damaged

$$\mathrm{l}_\mathrm{i}^\mathrm{ks} = y_i^s d_i^k$$

$$0 \le g_i^{sk} \le z_i^{ks} + g_i^{k+}$$

$$\mathrm{g}_\mathrm{i}^\mathrm{ks} - l_i^{ks} - \sum_{j\in N} f_{ij}^{ks} = 0$$

$$0 \le z_i^{ks} \le u_i^s Z_i^k$$

$$\sum_{ij\in s}\left(\overline{x}_{ij}^s + (1-\overline{\tau}_{ij})\right) \le |s| - 1$$

$$\tau_{ij}^s \ge x_{ij}^s + \overline{\tau}_{ij}^s - 1,\ x_{ij}^s \le \overline{x}_{ij}^s$$

Enforces radial operation

$$\sum_{i\in CL,k\in p_i} l_i^{ks} \ge \lambda \sum_{i\in CL,k\in p_i} d_i^k$$

$$\sum_{i\in N\setminus L,k\in p_i} l_i^{ks} \ge \gamma \sum_{i\in N\setminus L,k\in p_i} d_i^k$$

$$x, y, \tau, u, t \in \{0,1\}$$

Optimization Model

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

s.t. $x_{ij}^s \le x_{ij}$, $\tau_{ij}^s \le \tau_{ij}, t_{ij}^s \le t_{ij}, z_i^{ks} \le z_i^k, u_i^s \le u_i$

$$-x_{ij0}^s Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{sk} \le x_{ij1}^s Q_{ij}^k$$

$$x_{ij0}^s + x_{ij1}^s \le x_{ij}^s$$

$$-(1-\tau_{ij}^s)\, Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{ks} \le (1-\tau_{ij}^s)\, Q_{ij}^k$$

$$-\beta_{ij}\frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le f_{ij}^{k's} - \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le \beta_{ij}\frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|}$$

$x_{ij}^s = t_{ij}^s$ when x is damaged

$$l_i^{ks} = y_i^s d_i^k$$

$$0 \le g_i^{sk} \le z_i^{ks} + g_i^{k+}$$

$$g_i^{ks} - l_i^{ks} - \sum_{j\in N} f_{ij}^{ks} = 0$$

$$0 \le z_i^{ks} \le u_i^s Z_i^k$$

$$\sum_{ij\in s}\left(\overline{x}_{ij}^s + (1-\overline{\tau}_{ij})\right) \le |s| - 1$$

$$\tau_{ij}^s \ge x_{ij}^s + \overline{\tau}_{ij}^s - 1,\ x_{ij}^s \le \overline{x}_{ij}^s$$

$$\sum_{i\in CL,k\in p_i} l_i^{ks} \ge \lambda \sum_{i\in CL,k\in p_i} d_i^k$$

$$\sum_{i\in N\setminus L,k\in p_i} l_i^{ks} \ge \gamma \sum_{i\in N\setminus L,k\in p_i} d_i^k$$

$$x, y, \tau, u, t \in \{0,1\}$$

Resilience criteria—minimum amount of load served

Is generalized to a chance constraint

Algorithm Overview

Exact Algorithms

- **CPLEX 12.6—no parameter tuning**
  - Difficult problem – 50-60k binary variables
- **Decomposition**
  - Benders, Dantzig-Wolfe, Scenario

Heuristics

- **Greedy**
  - Union of single scenario solutions
  - Based on industry algorithms
- **Variable Neighborhood Search**
  - Ruin and Recreate—hybrid of exact methods and local search
  - Iteratively relax variable assignments (ruin)
  - Use exact method to find optimal variable assignments for relaxed variables, given the fixed partial solution (recreate)

Scenario Based Decomposition

Solve over all damage scenarios

$$ResilientDesign(S)$$

Select 1 scenario

$$s \leftarrow chooseScenario(S)$$

Design network for damage scenario 1

$$\sigma \rightarrow solveMIP(s)$$

Is solution feasible for remaining scenarios

$$while\ (\sim Feasible(\sigma, S \backslash s))$$

If NOT, add an infeasible scenario to the set under consideration

$$s \rightarrow s \cup chooseScenario(S \backslash s)$$

Find a new solution

$$\sigma \rightarrow solveMIP(s)$$

Iterate until solution is feasible for all scenarios

Outperformed other decomposition strategies—second stage influences feasibility, not optimality. Continuous investment variables also adds difficulty

Variable Neighborhood Search

$ResilientDesign(S, maxTime, maxRestarts, maxIterations)$

$\boxed{\sigma^{LP} \leftarrow Solve(P^{LP}), \sigma^* \leftarrow \sigma', restart \leftarrow false}$

while $(t < maxTime$ and $i < maxRestarts)$

$j \leftarrow 0$

$n \leftarrow |x \in X : |\sigma^*(x) - \sigma^{LP}(x)| \neq 0|$

$J \leftarrow \langle \pi_1, \pi_2, \dots \pi_{|J|} \rangle \in X : |\sigma^*(\pi_i) - \sigma^{LP}(\pi_i)| \leq |\sigma^*(\pi_{i+1}) - \sigma^{LP}(\pi_{i+1})|$

if $(restart = true)$

$i \leftarrow i + 1$

$step \leftarrow \frac{4n}{d}, k \leftarrow |X| - step$

$shuffle(J)$

else

$step \leftarrow \frac{n}{d}, k \leftarrow |X| - step$

while $(t < maxTime\ and\ j \leq maxIterations)$

$\sigma' \leftarrow Solve(P(\sigma^*, J(1, \dots k))$

if $\left(f(\sigma') < f(\sigma^*)\right)$

$\sigma^* \leftarrow \sigma', i \leftarrow 0, restart \leftarrow false, j \leftarrow maxIterations$

else

$i \leftarrow i + 1, k \leftarrow k - \frac{step}{\ }$

Solve the LP relaxation

Intuition: LP relaxation guides the search procedure

Variable Neighborhood Search

$ResilientDesign(S, maxTime, maxRestarts, maxIterations)$

$\sigma^{LP} \leftarrow Solve(P^{LP}), \sigma^* \leftarrow \sigma', restart \leftarrow false$

while $(t < maxTime$ and $i < maxRestarts)$

$j \leftarrow 0$

$\boxed{n \leftarrow |x \in X : |\sigma^*(x) - \sigma^{LP}(x)| \neq 0|}$

$J \leftarrow \langle \pi_1, \pi_2, \ldots \pi_{|J|} \rangle \in X : |\sigma^*(\pi_i) - \sigma^{LP}(\pi_i)| \leq |\sigma^*(\pi_{i+1}) - \sigma^{LP}(\pi_{i+1})|$

if $(restart = true)$

$i \leftarrow i + 1$

$step \leftarrow \frac{4n}{d}, k \leftarrow |X| - step$

$shuffle(J)$

else

$step \leftarrow \frac{n}{d}, k \leftarrow |X| - step$

while $(t < maxTime$ and $j \leq maxIterations)$

$\sigma' \leftarrow Solve(P(\sigma^*, J(1, \ldots k))$

if $\left(f(\sigma') < f(\sigma^*)\right)$

$\sigma^* \leftarrow \sigma', i \leftarrow 0, restart \leftarrow false, j \leftarrow maxIterations$

else

$i \leftarrow i + 1, k \leftarrow k - \frac{step}{\ldots}$

Count differences between current best solution and relaxation

Intuition: *n* is a parameter used to control the size of the neighborhood. Larger differences between the LP relaxation and the incumbent solution indicate that a larger neighborhood should be considered.

Variable Neighborhood Search

$ResilientDesign(S, maxTime, maxRestarts, maxIterations)$

$\sigma^{LP} \leftarrow Solve(P^{LP}), \sigma^{*} \leftarrow \sigma', restart \leftarrow false$

while $(t < maxTime$ and $i < maxRestarts)$

$j \leftarrow 0$

$n \leftarrow |x \in X : |\sigma^{*}(x) - \sigma^{LP}(x)| \neq 0|$

$\boxed{J \leftarrow \langle \pi_1, \pi_2, \ldots \pi_{|J|} \rangle \in X : |\sigma^{*}(\pi_i) - \sigma^{LP}(\pi_i)| \leq |\sigma^{*}(\pi_{i+1}) - \sigma^{LP}(\pi_{i+1})|}$

if $(restart = true)$

$i \leftarrow i + 1$

$step \leftarrow \frac{4n}{d}, k \leftarrow |X| - step$

$shuffle(J)$

else

$step \leftarrow \frac{n}{d}, k \leftarrow |X| - step$

while $(t < maxTime \text{ and } j \leq maxIterations)$

$\sigma' \leftarrow Solve(P(\sigma^{*}, J(1, \ldots k))$

if $\left(f(\sigma') < f(\sigma^{*})\right)$

$\sigma^{*} \leftarrow \sigma', i \leftarrow 0, restart \leftarrow false, j \leftarrow maxIterations$

else

$i \leftarrow i + 1, k \leftarrow k - \frac{step}{}$

Order variables by difference from LP relaxation

Intuition: Variables whose assignments differ from the LP relaxation have more potential to improve the incumbent solution.

Variable Neighborhood Search

$ResilientDesign(S, maxTime, maxRestarts, maxIterations)$

$\sigma^{LP} \leftarrow Solve(P^{LP}), \sigma^* \leftarrow \sigma', restart \leftarrow false$

while ($t < maxTime$ and $i < maxRestarts$)

$j \leftarrow 0$

$n \leftarrow |x \in X : |\sigma^*(x) - \sigma^{LP}(x)| \neq 0|$

$J \leftarrow \langle \pi_1, \pi_2, \ldots \pi_{|J|} \rangle \in X : |\sigma^*(\pi_i) - \sigma^{LP}(\pi_i)| \leq |\sigma^*(\pi_{i+1}) - \sigma^{LP}(\pi_{i+1})|$

if ($restart = true$)

$i \leftarrow i + 1$

$step \leftarrow \frac{4n}{d}, k \leftarrow |X| - step$

$shuffle(J)$

else

$step \leftarrow \frac{n}{d}, k \leftarrow |X| - step$

while ($t < maxTime$ and $j \leq maxIterations$)

$\boxed{\sigma' \leftarrow Solve(P(\sigma^*, J(1, \ldots k))}$

Compute best solution in neighborhood J(1…k)

if $\left(f(\sigma') < f(\sigma^*)\right)$

$\sigma^* \leftarrow \sigma', i \leftarrow 0, restart \leftarrow false, j \leftarrow maxIterations$

else

$i \leftarrow i + 1, k \leftarrow k - \frac{step}{\ldots}$

Intuition: Ruin and recreate

Variable Neighborhood Search

$ResilientDesign(S, maxTime, maxRestarts, maxIterations)$

$\sigma^{LP} \leftarrow Solve(P^{LP}), \sigma^* \leftarrow \sigma', restart \leftarrow false$

while $(t < maxTime$ and $i < maxRestarts)$

$j \leftarrow 0$

$n \leftarrow |x \in X : |\sigma^*(x) - \sigma^{LP}(x)| \neq 0|$

$J \leftarrow \langle \pi_1, \pi_2, \ldots \pi_{|J|} \rangle \in X : |\sigma^*(\pi_i) - \sigma^{LP}(\pi_i)| \leq |\sigma^*(\pi_{i+1}) - \sigma^{LP}(\pi_{i+1})|$

if $(restart = true)$

$i \leftarrow i + 1$

$step \leftarrow \frac{4n}{d}, k \leftarrow |X| - step$

$shuffle(J)$

else

$step \leftarrow \frac{n}{d}, k \leftarrow |X| - step$

while $(t < maxTime \text{ and } j \leq maxIterations)$

$\sigma' \leftarrow Solve(P(\sigma^*, J(1, \ldots k))$

if $(f(\sigma') < f(\sigma^*))$

$\sigma^* \leftarrow \sigma', i \leftarrow 0, restart \leftarrow false, j \leftarrow maxIterations$

else

$i \leftarrow i + 1, k \leftarrow k - \frac{step}{\ldots}$

Update best solution

UNCLASSIFIED

Variable Neighborhood Search

$ResilientDesign(S, maxTime, maxRestarts, maxIterations)$

$\sigma^{LP} \leftarrow Solve(P^{LP}), \sigma^* \leftarrow \sigma', restart \leftarrow false$

while $(t < maxTime$ and $i < maxRestarts)$

$j \leftarrow 0$

$n \leftarrow |x \in X : |\sigma^*(x) - \sigma^{LP}(x)| \neq 0|$

$J \leftarrow \langle \pi_1, \pi_2, \ldots \pi_{|J|} \rangle \in X : |\sigma^*(\pi_i) - \sigma^{LP}(\pi_i)| \leq |\sigma^*(\pi_{i+1}) - \sigma^{LP}(\pi_{i+1})|$

if $(restart = true)$

$i \leftarrow i + 1$

$step \leftarrow \frac{4n}{d}, k \leftarrow |X| - step$

$shuffle(J)$

else

$step \leftarrow \frac{n}{d}, k \leftarrow |X| - step$

while $(t < maxTime \text{ and } j \leq maxIterations)$

$\sigma' \leftarrow Solve(P(\sigma^*, J(1, \ldots k))$

if $\left(f(\sigma') < f(\sigma^*)\right)$

$\sigma^* \leftarrow \sigma', i \leftarrow 0, restart \leftarrow false, j \leftarrow maxIterations$

else

$i \leftarrow i + 1, k \leftarrow k - step$

Intuition: When a better solution is not found, increase the size of the neighborhood

Increase neighborhood size

Variable Neighborhood Search

$ResilientDesign(S, maxTime, maxRestarts, maxIterations)$

$\sigma^{LP} \leftarrow Solve(P^{LP}), \sigma^* \leftarrow \sigma', restart \leftarrow false$

while ($t < maxTime$ and $i < maxRestarts$)

$j \leftarrow 0$

$n \leftarrow |x \in X : |\sigma^*(x) - \sigma^{LP}(x)| \neq 0|$

$J \leftarrow \langle \pi_1, \pi_2, \ldots \pi_{|J|} \rangle \in X : |\sigma^*(\pi_i) - \sigma^{LP}(\pi_i)| \leq |\sigma^*(\pi_{i+1}) - \sigma^{LP}(\pi_{i+1})|$

if ($restart = true$)

$i \leftarrow i + 1$

$step \leftarrow \frac{4n}{d}, k \leftarrow |X| - step$

$shuffle(J)$

Shuffle ordering after restart

else

$step \leftarrow \frac{n}{d}, k \leftarrow |X| - step$

while ($t < maxTime$ and $j \leq maxIterations$)

$\sigma' \leftarrow Solve(P(\sigma^*, J(1, \ldots k))$

if $\left(f(\sigma') < f(\sigma^*)\right)$

$\sigma^* \leftarrow \sigma', i \leftarrow 0, restart \leftarrow false, j \leftarrow maxIterations$

else

$i \leftarrow i + 1, k \leftarrow k - \frac{step}{}$

Intuition: Consider different sets of variables to relax

Variable Neighborhood Search

$ResilientDesign(S, maxTime, maxRestarts, maxIterations)$

$\sigma^{LP} \leftarrow Solve(P^{LP}), \sigma^* \leftarrow \sigma', restart \leftarrow false$

while ($t < maxTime$ and $i < maxRestarts$)

$j \leftarrow 0$

$n \leftarrow |x \in X : |\sigma^*(x) - \sigma^{LP}(x)| \neq 0|$

$J \leftarrow \langle \pi_1, \pi_2, \ldots \pi_{|J|} \rangle \in X : |\sigma^*(\pi_i) - \sigma^{LP}(\pi_i)| \leq |\sigma^*(\pi_{i+1}) - \sigma^{LP}(\pi_{i+1})|$

if ($restart = true$)

$i \leftarrow i + 1$

$\boxed{step \leftarrow \frac{4n}{d}, k \leftarrow |X| - step}$

$shuffle(J)$

else

$\boxed{step \leftarrow \frac{n}{d}, k \leftarrow |X| - step}$

while ($t < maxTime \ and \ j \leq maxIterations$)

$\sigma' \leftarrow Solve(P(\sigma^*, J(1, \ldots k))$

if $(f(\sigma') < f(\sigma^*))$

$\sigma^* \leftarrow \sigma', i \leftarrow 0, restart \leftarrow false, j \leftarrow maxIterations$

else

$i \leftarrow i + 1, k \leftarrow k - \frac{step}{\ldots}$

Adjust neighborhood parameters

Intuition: Neighborhood size is based on differences between LP relaxation and incumbent solution.

Variable Neighborhood Search with Decomposition

$$ResilientDesign(S)$$

Solve over all damage scenarios

$$s \leftarrow chooseScenario(S)$$

Select 1 scenario

$$\sigma \rightarrow solveVNS(s)$$

Design network for damage scenario 1

$$while\ (\sim Feasible(\sigma, S \backslash s))$$

Is solution feasible for remaining scenarios

$$s \rightarrow s \cup chooseScenario(S \backslash s)$$

If NOT, add an infeasible scenario to the set under consideration

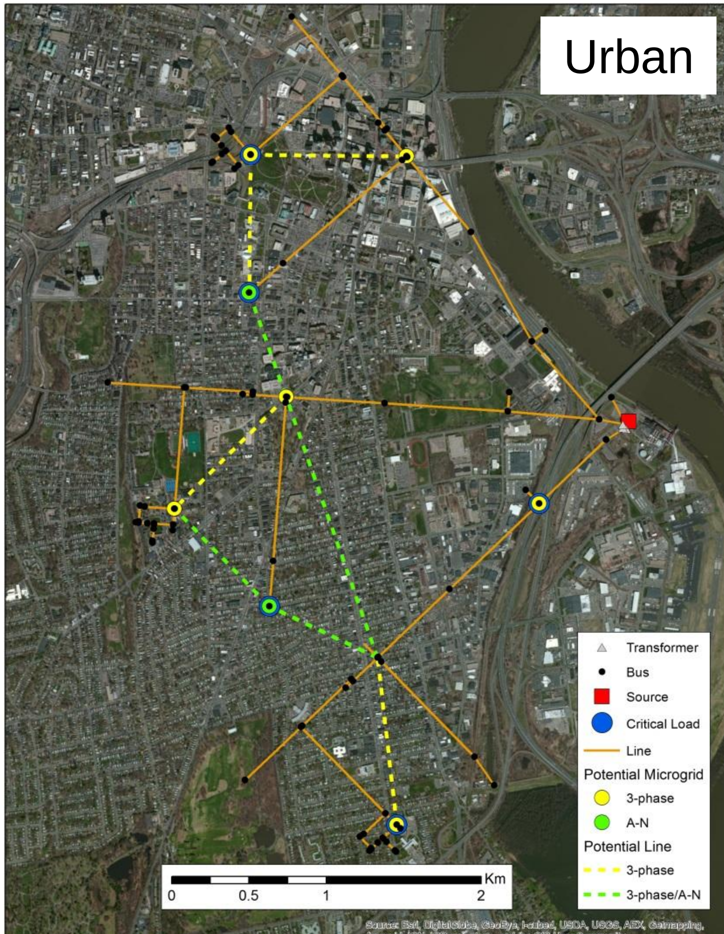
$$\sigma \rightarrow solveVNS(s)$$

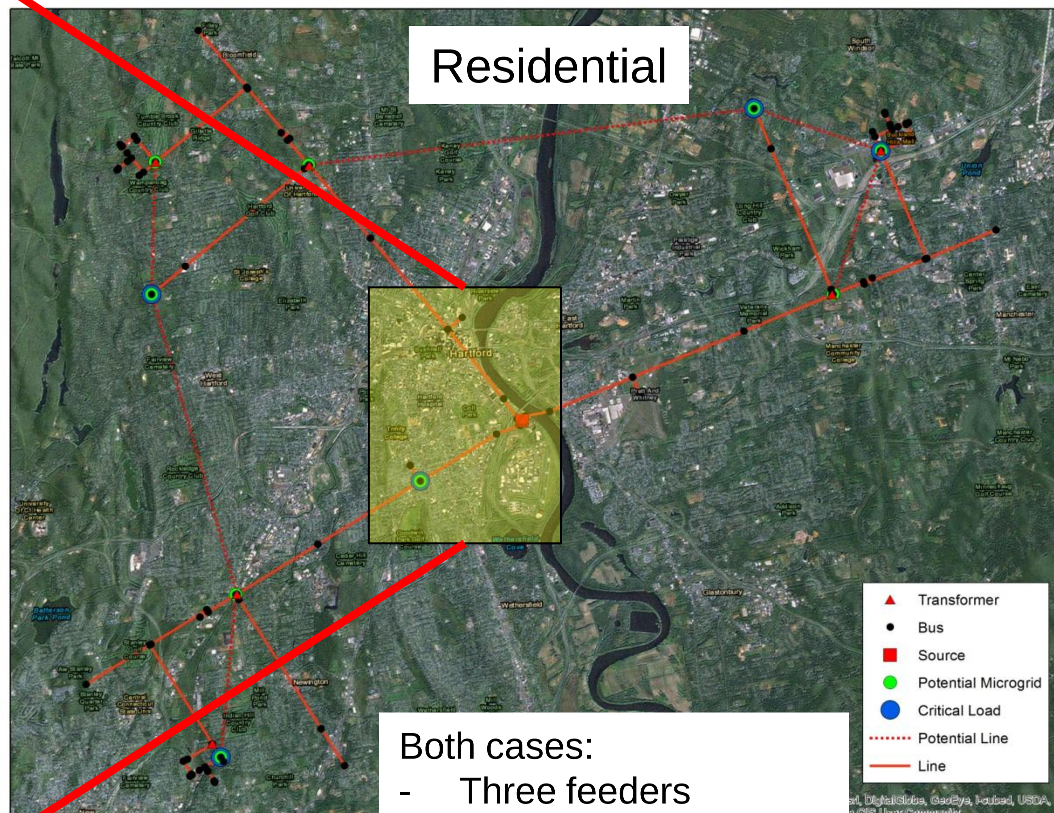
Find a new solution

Iterate until solution is feasible for all scenarios

Test Cases

Two base-model configurations—"Dense Urban" and "Sparse Residential"
Range of damage intensity—Light damage to Heavy damage
Different trade off between 1) distributed generation 2) new interties 3) hardening
Based on IEEE 34 (100 Scenarios, 109 nodes, 148 edges)





Both cases:

- Three feeders
- 5.1 MW of total load
- 2.1 MW of critical load

Assumptions

Distributed generators provide firm generation, e.g. natural gas CHP

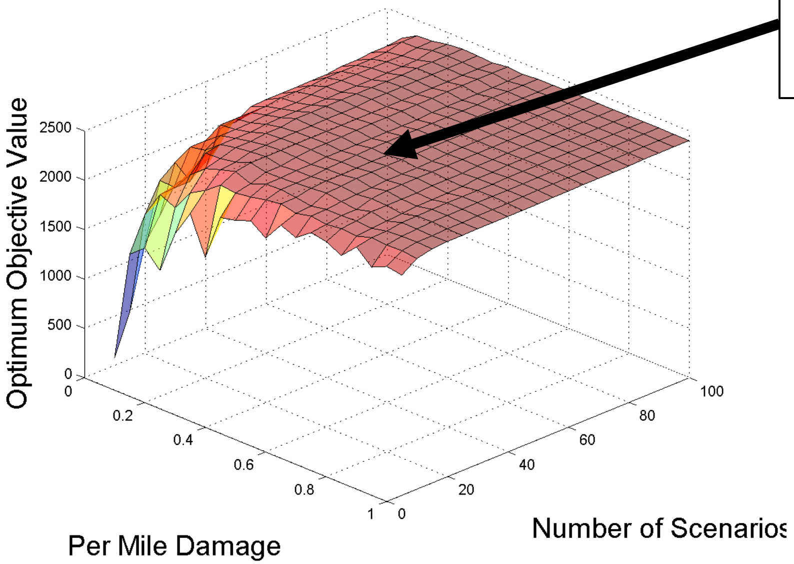
Circuits or sections of circuits configured as trees

Loads and/or generators stay on the phases where they were installed

Costs…… (can be modified based on user specifications)

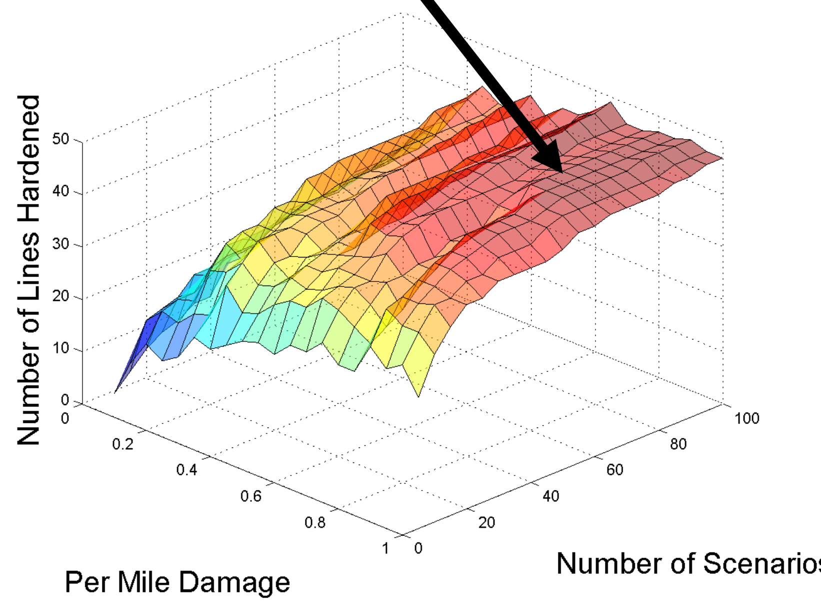
Device	Type	Cost Range	Suggested cost	Source
$c_{i,j}$	overhead 3-phase	\$60k-\$150k/mile	\$95k/mile	State of
$c_{i,j}$	overhead 1-phase	\$40k-\$75k/mile	\$55k/mile	Virginia Study on
$c_{i,j}$	underground 3-phase	\$40k-\$1,500k/mile	\$500k/mile	Underground Circuits
$c_{i,j}$	underground 1-phase	\$40k-\$1,500k/mile	\$100k/mile	-
$\kappa_{i,j}$	automatic, 3-phase, overhead	–	\$15k	Tom Bialek (SDG&E)
$\kappa_{i,j}$	automatic, 3-phase, underground	–	\$30k	-
$\kappa_{i,j}$	manual, 3-phase, overhead	–	\$7.5k	-
$\kappa_{i,j}$	manual, 3-phase, underground	–	\$20k	-
$\kappa_{i,j}$	automatic, 1-phase, overhead	–	\$10k	-
$\kappa_{i,j}$	automatic, 1-phase, underground	–	\$25k	-
$\kappa_{i,j}$	manual, 1-phase, overhead	–	\$5	-
$\kappa_{i,j}$	manual, 1-phase, underground	–	\$15k	-
$\zeta_{i,j}$	Natural gas CHP variable	–	\$1,500k/MW	EIA 2025 Study
$\zeta_{i,j}$	Natural gas CHP fixed	–	\$500k	

100 Scenarios are sufficient (empirically)



Solution quality doesn't change much

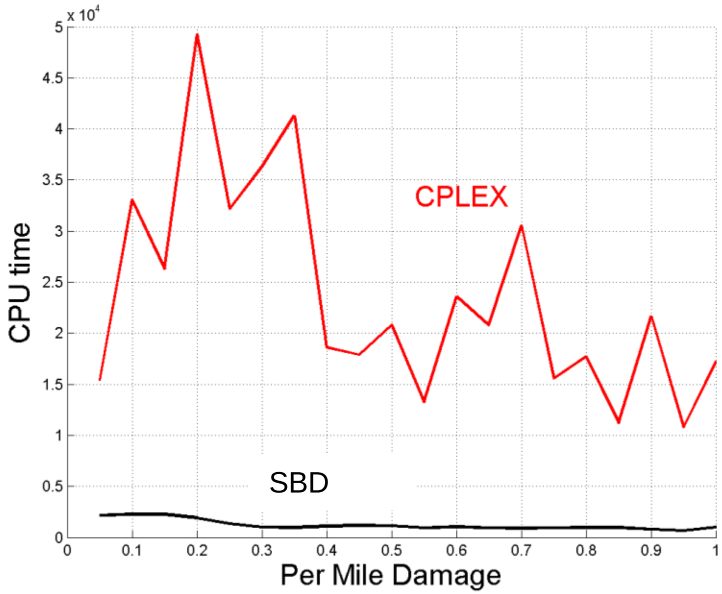
Solution changes slightly

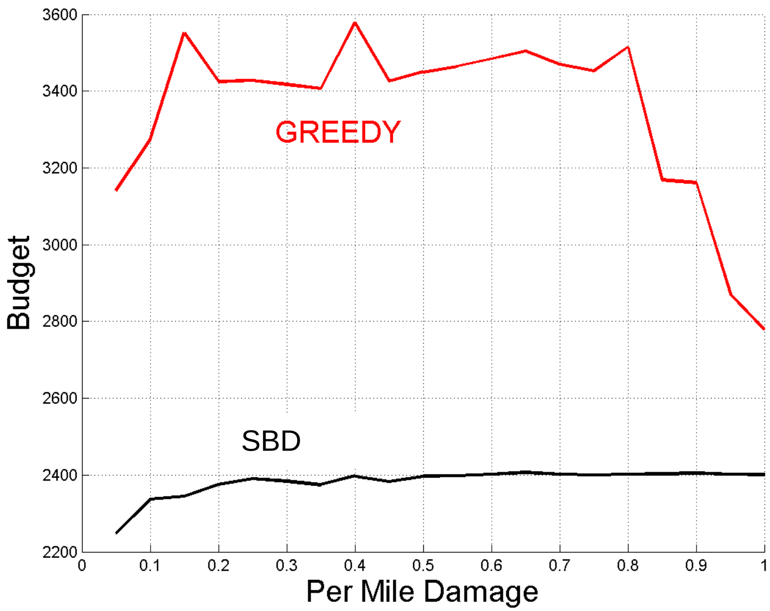


Residential Problem

Algorithm Comparisons

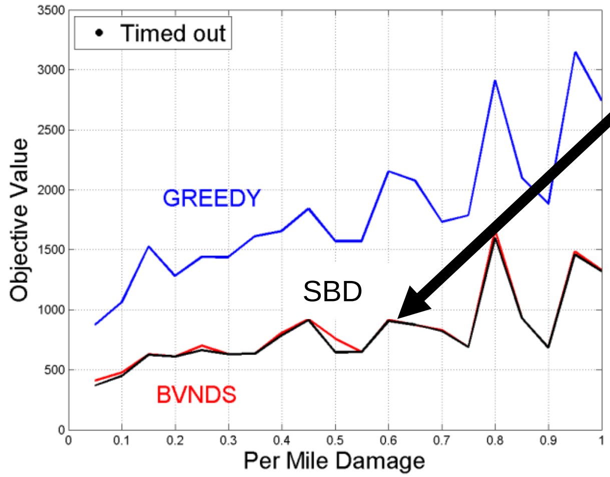
Residential Problem



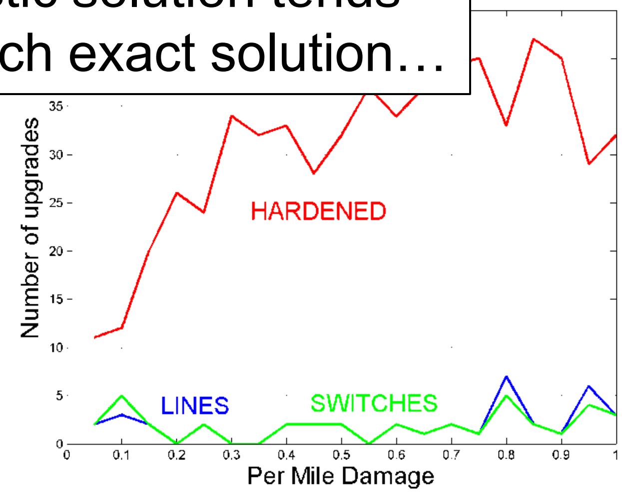


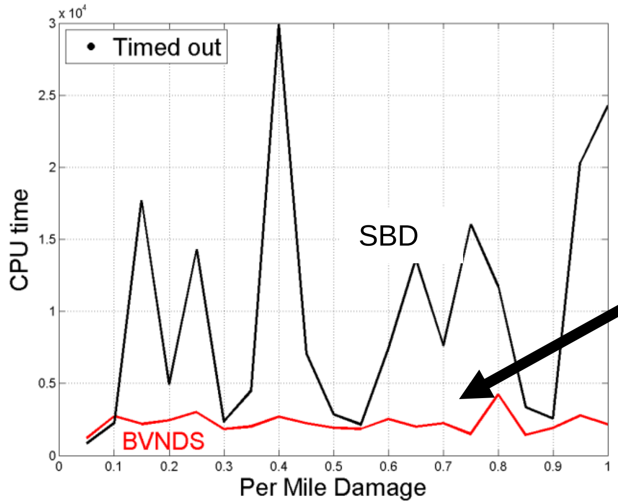
Gaps widen as problems become larger

Exact vs. Heuristics

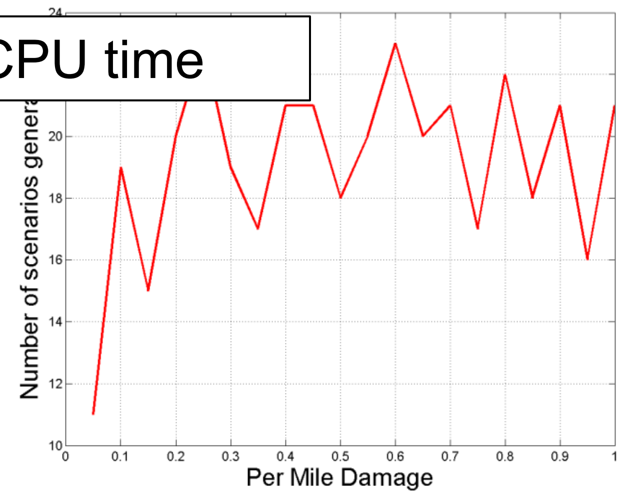


Heuristic solution tends to match exact solution…



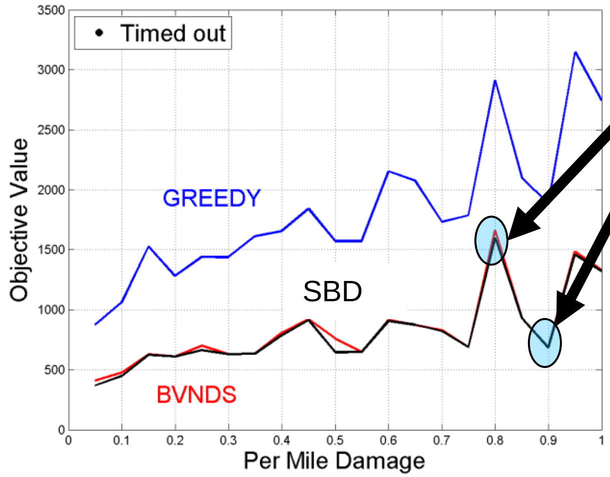


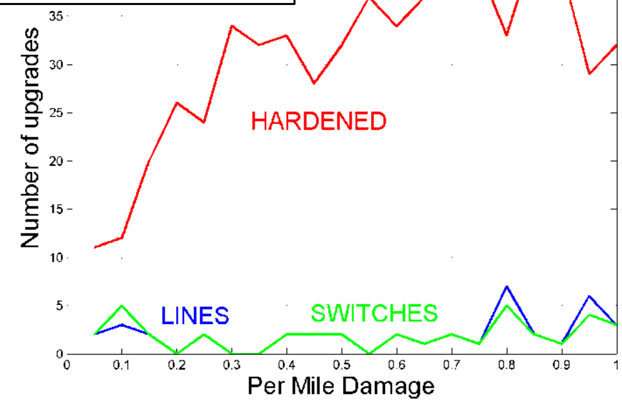
… with less CPU time

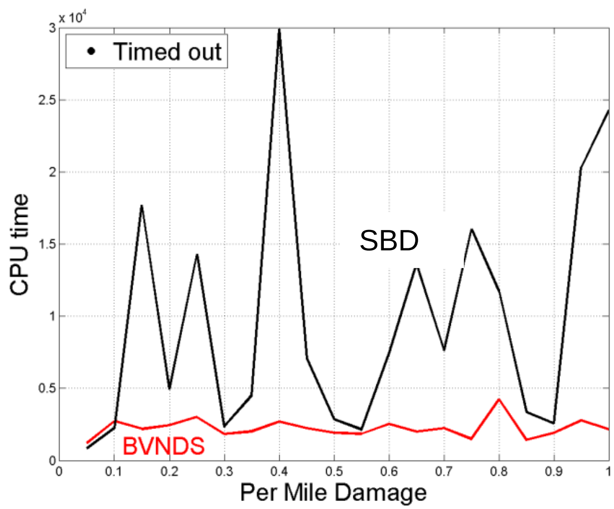


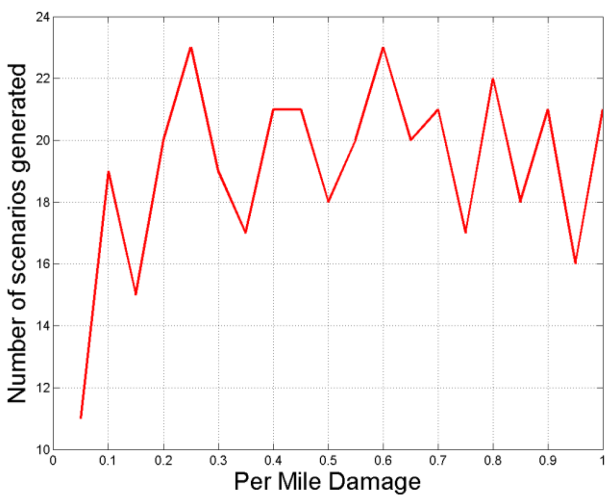
Exact vs. Heuristics

Why the objective fluctuations?



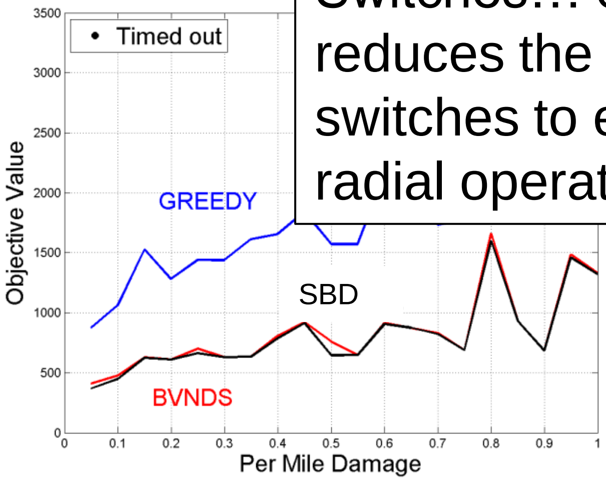


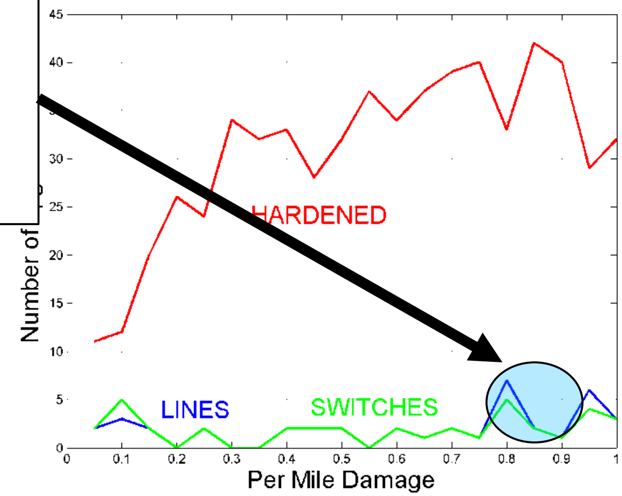


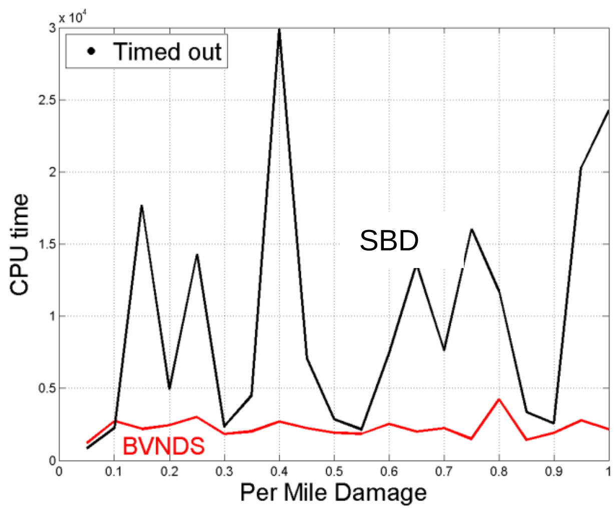


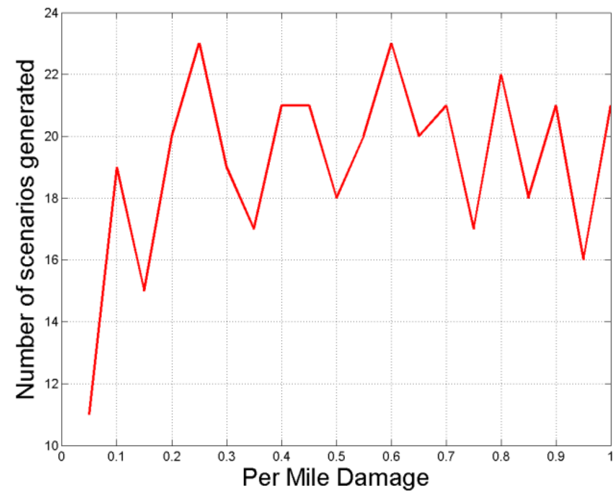
Exact vs. Heuristics

Switches… damage reduces the need for switches to enforce radial operations



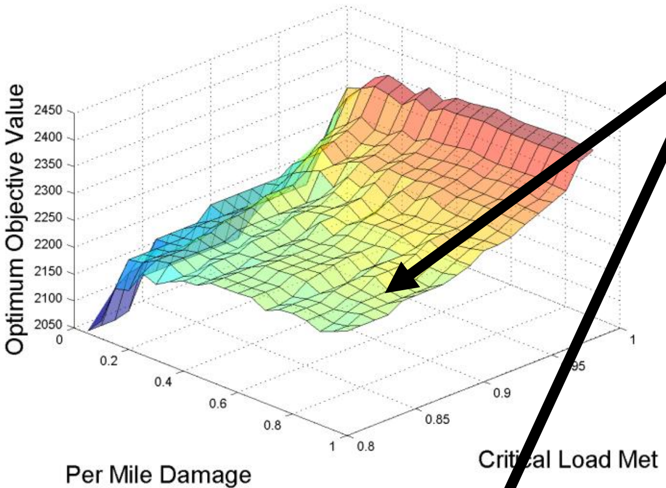


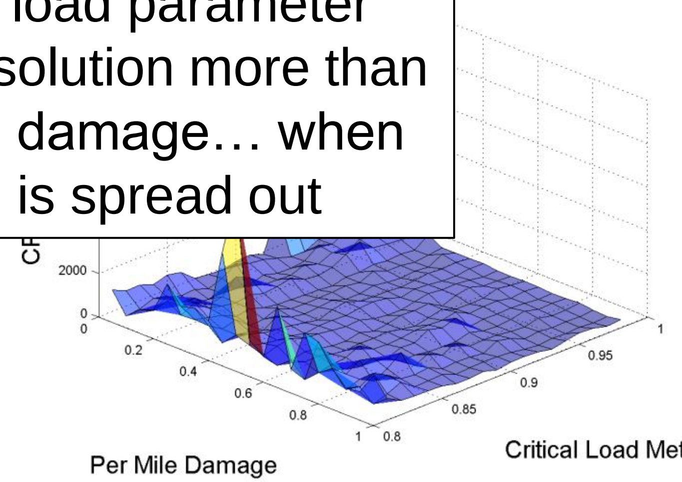


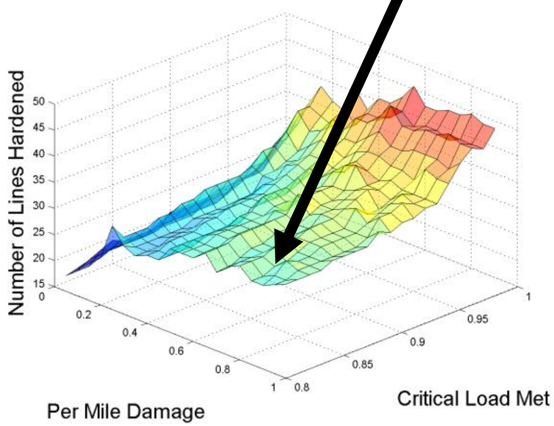


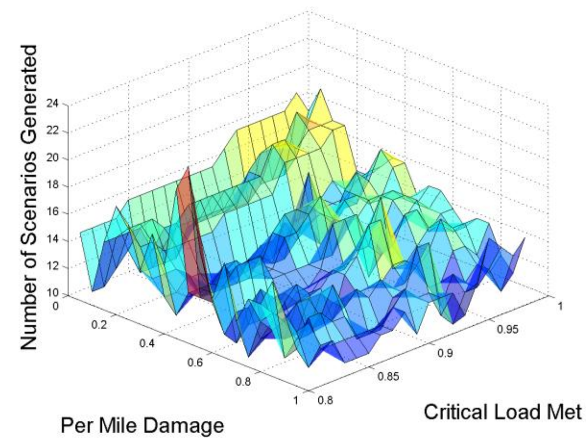
Resilience Criteria: Residential

Critical load parameter drives solution more than system damage… when system is spread out

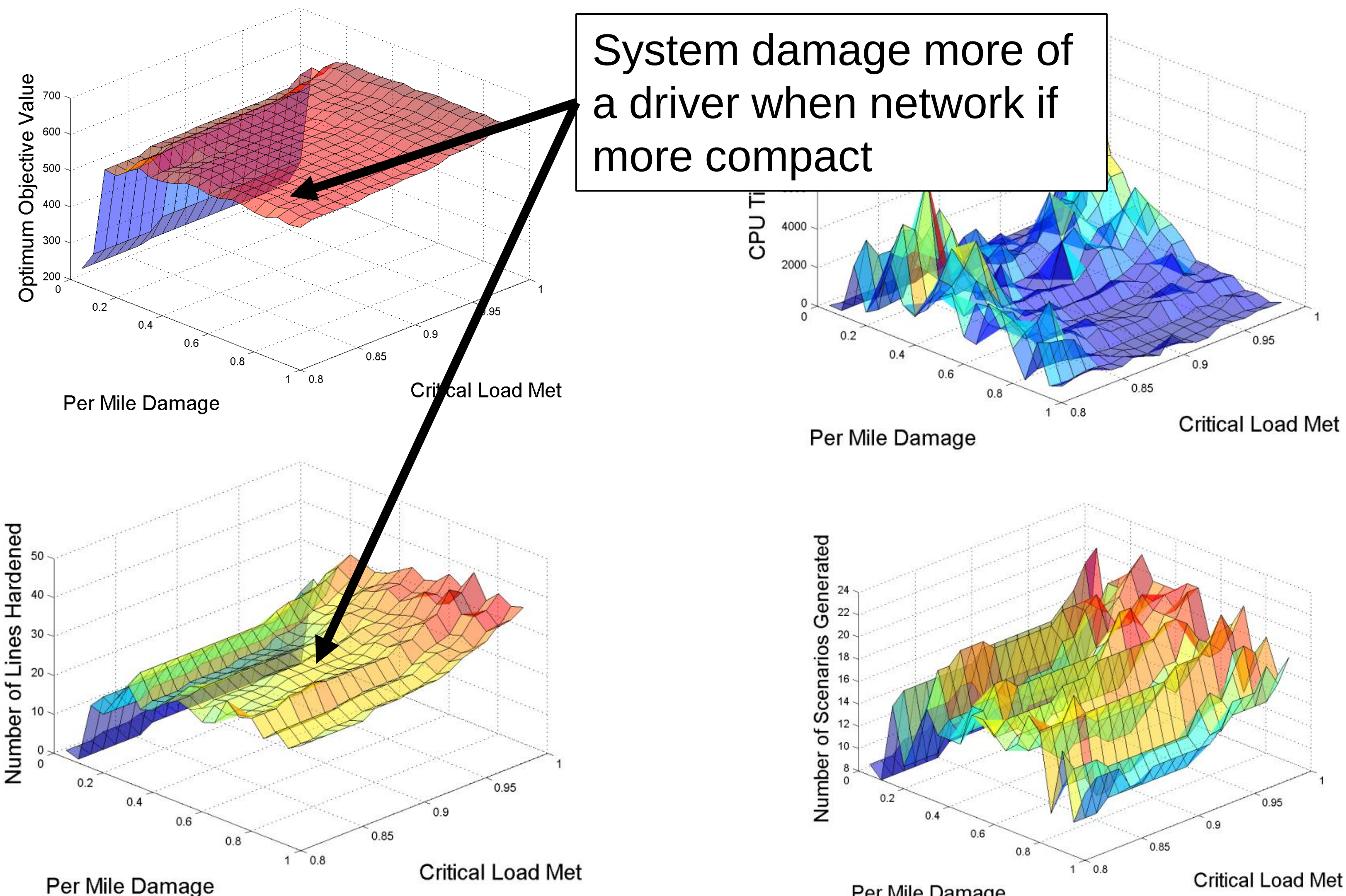






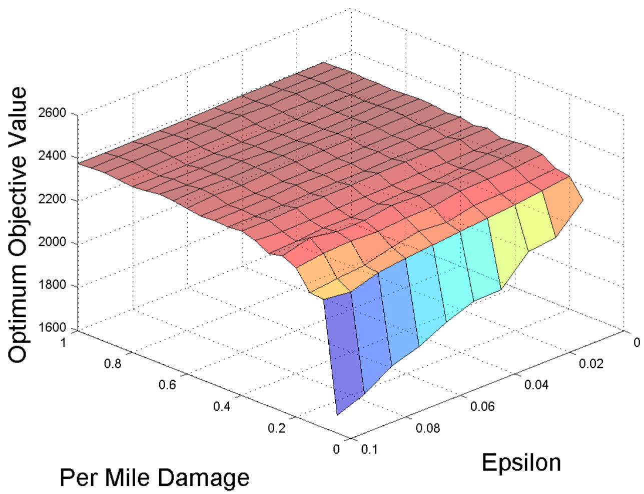


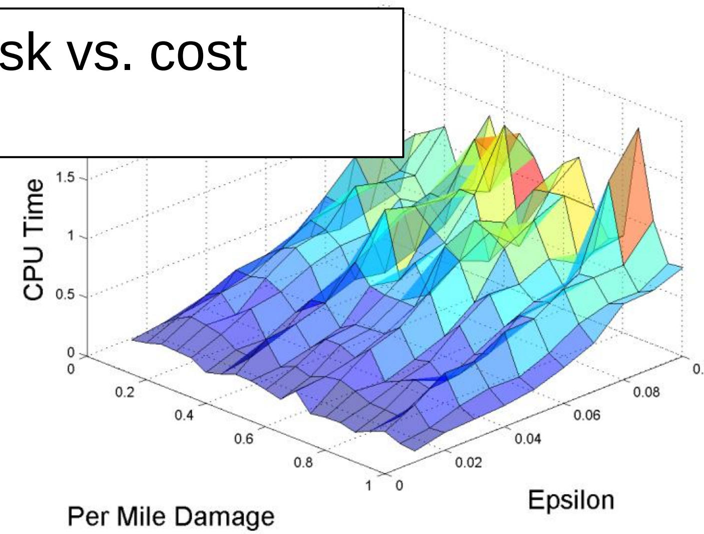
Resilience Criteria: Urban

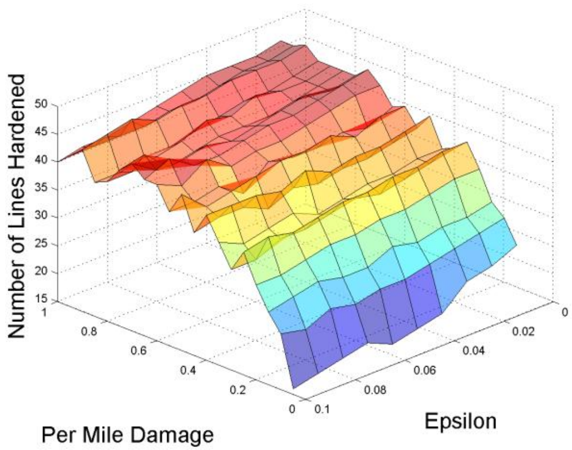


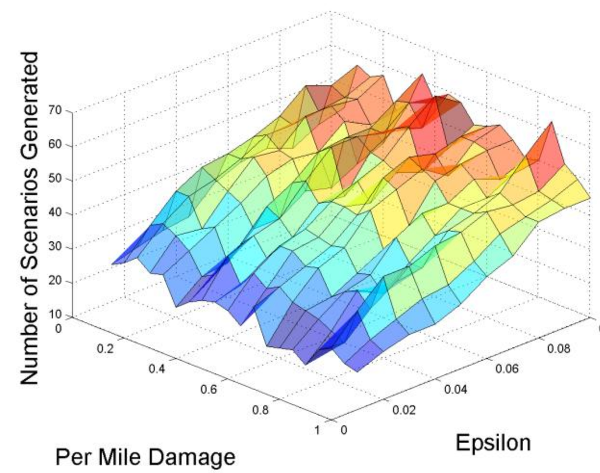
Chance Constraints

Assess risk vs. cost tradeoff

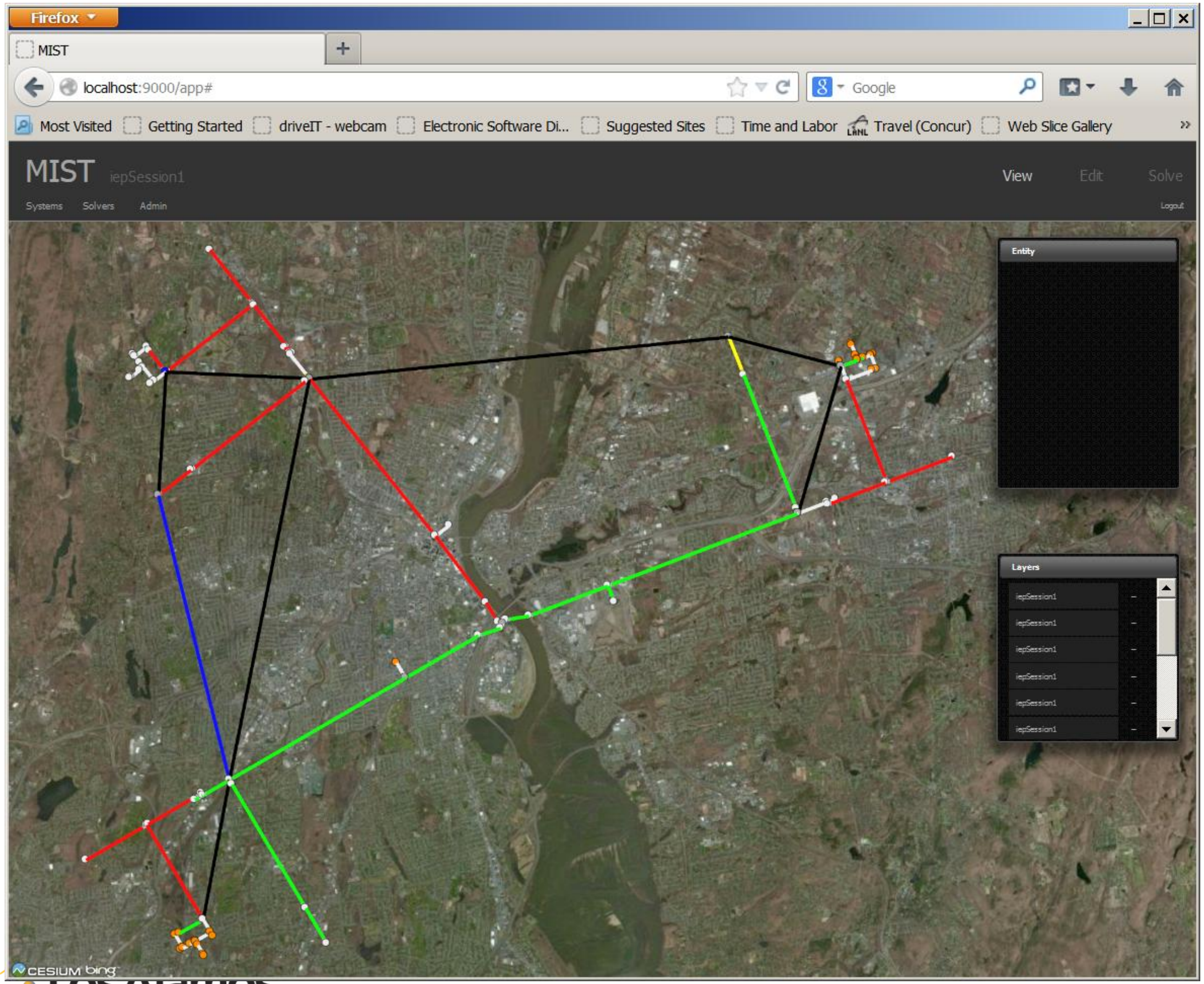








System interaction via MIST



- Critical Load
- Generation
- Damaged Lines
- Hardened Lines
- New Lines
- Unbuilt New Lines
- Switches

Future Work

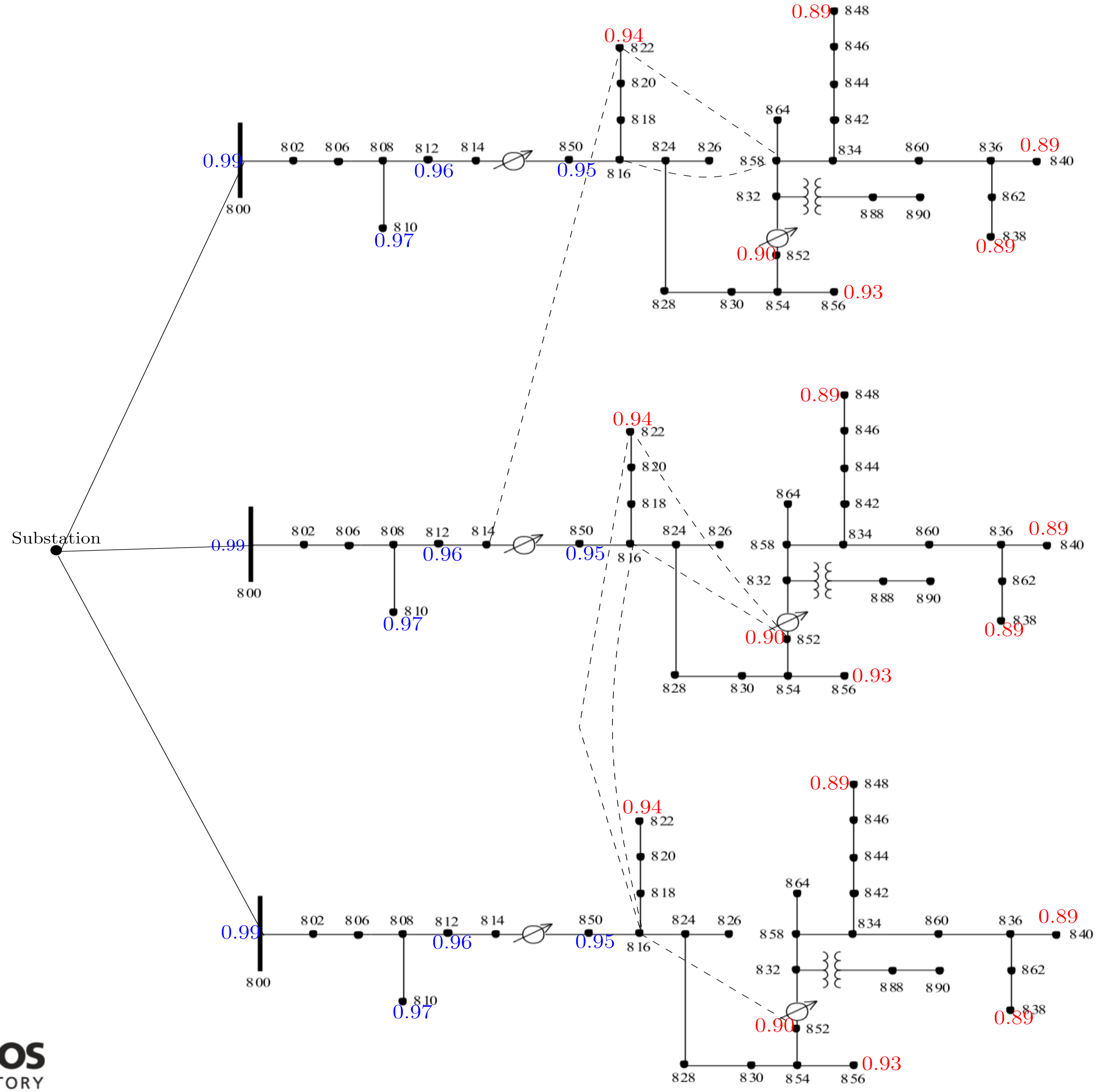
Reference

- E. Yamangil, R. Bent, S. Backhaus. *Optimal Resilient Distribution Grid Design Under Stochastic Events*, AAAI 2015

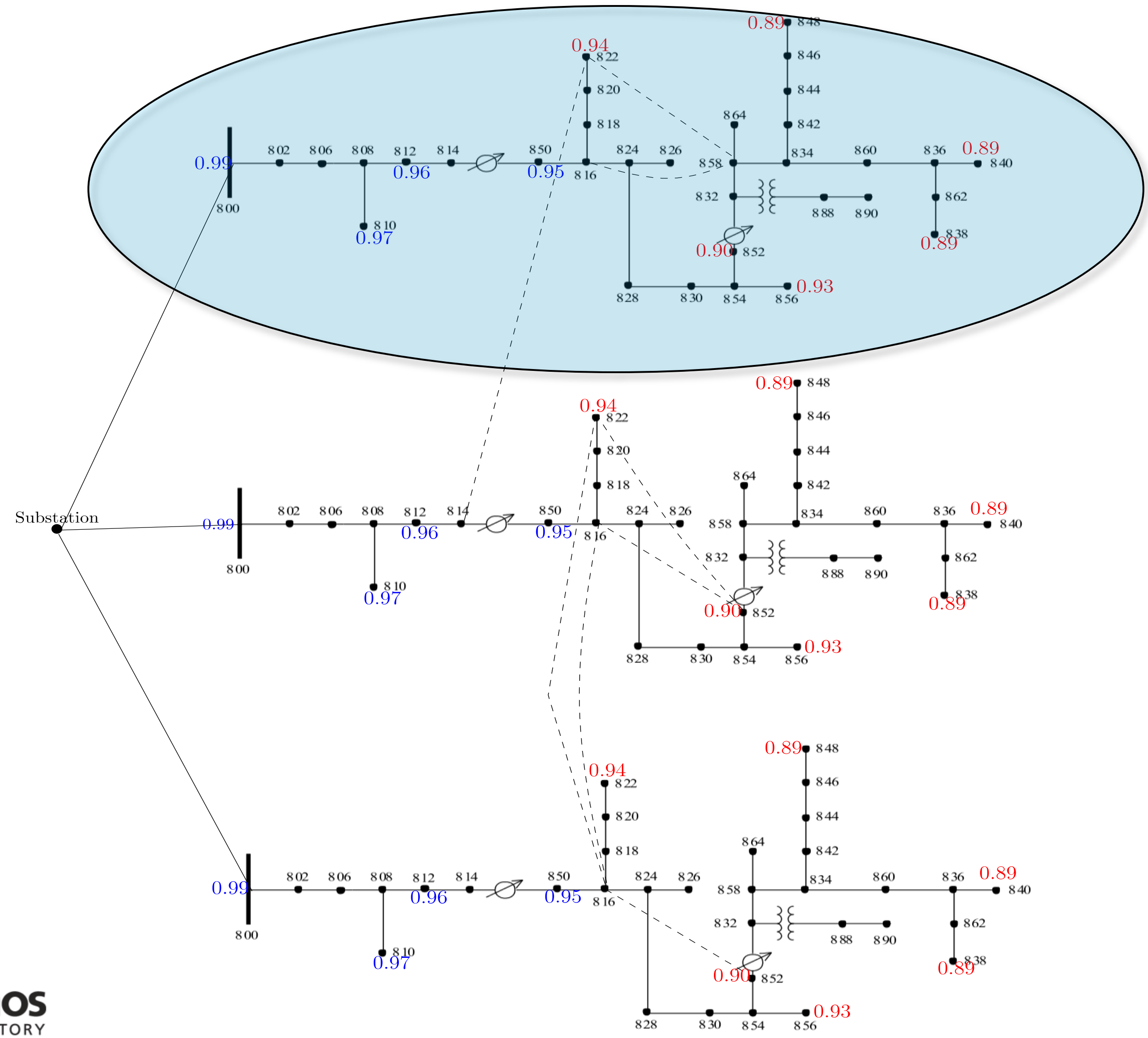
Generalizations

- Multi-Commodity Flow Relaxation
  - Voltage and Reactive power are ignored.
  - Initial network is voltage feasible, upgrades tend to move loads closer to generation, which improves voltage and lowers line flows
  - May not always hold
    - No good/L-shaped cuts
    - DistFlow formulation – derive a linearization of 3 phase formulation
      - Gan and Low 2014
- Larger networks
  - Graph-based decompositions

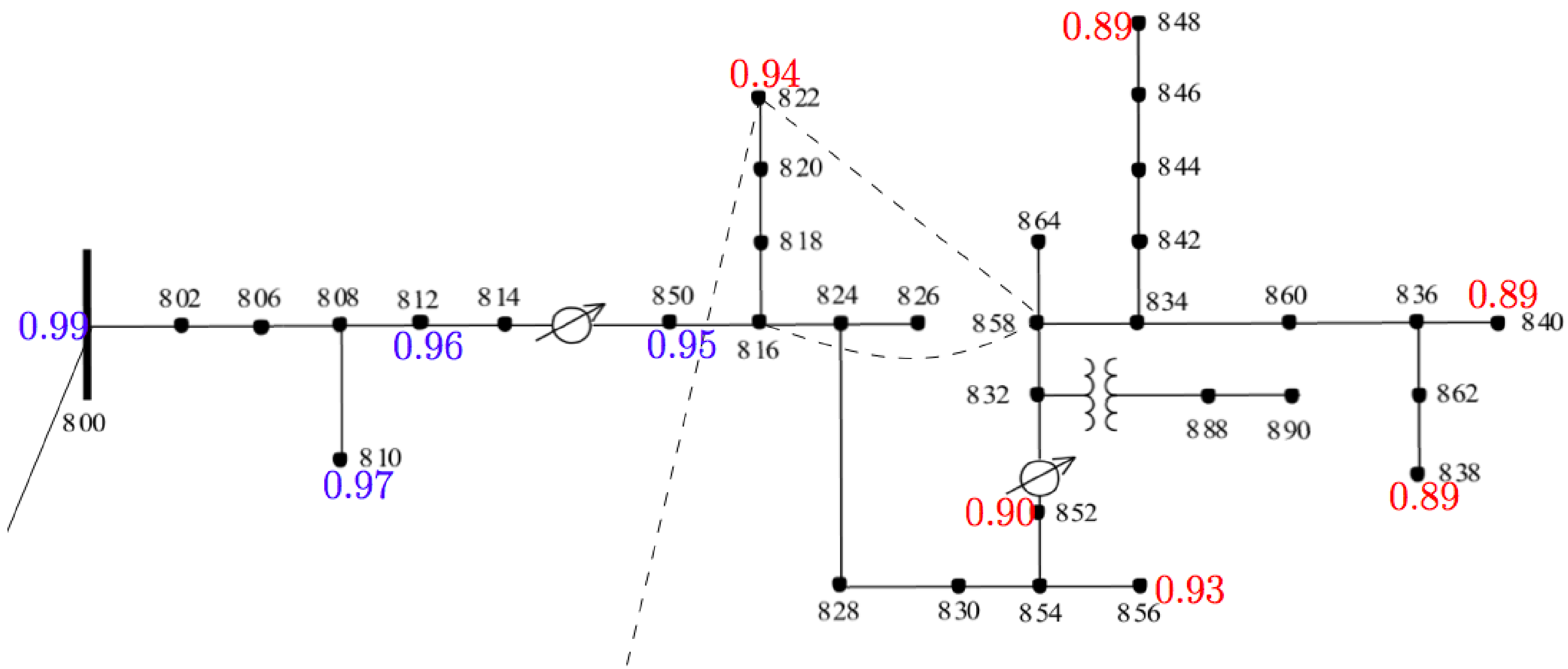
Algorithm Enhancements (example)



Algorithm Enhancements (example)



Algorithm Enhancements (example)



Algorithm Enhancements

A. Cutting plane algorithm

1. Solve the MILP based on the decomposition approaches. Let T_1 be a radial network satisfying a scenario.
2. Solve the scenario base topology (T_1) using a power flow solver (i.e. GridLab-D) to obtain the voltage and reactive power profile.
3. If the voltage profile and line limits are within the prescribed bounds, the MILP solutions satisfies the 3-phase AC power flow equations. Else, augment the MILP with the following "no good" cut:

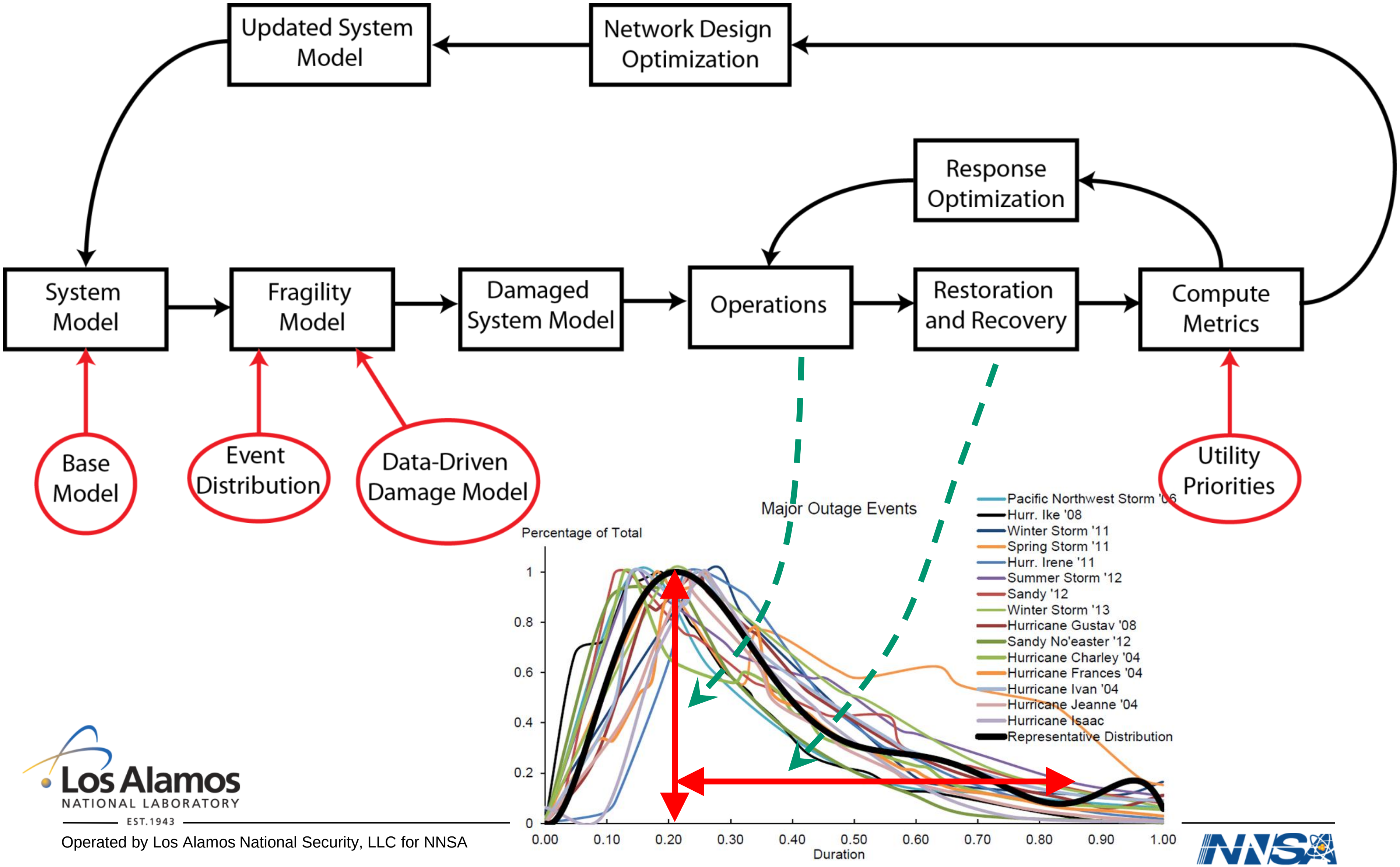
$$\sum_{e \in T_1} x_e \leq (|T_1| - 1)$$

where $|T_1|$ represents the number of edges in T_1

Disclaimer: Stronger cuts can be derived when the details of the underlying power flow equations are known

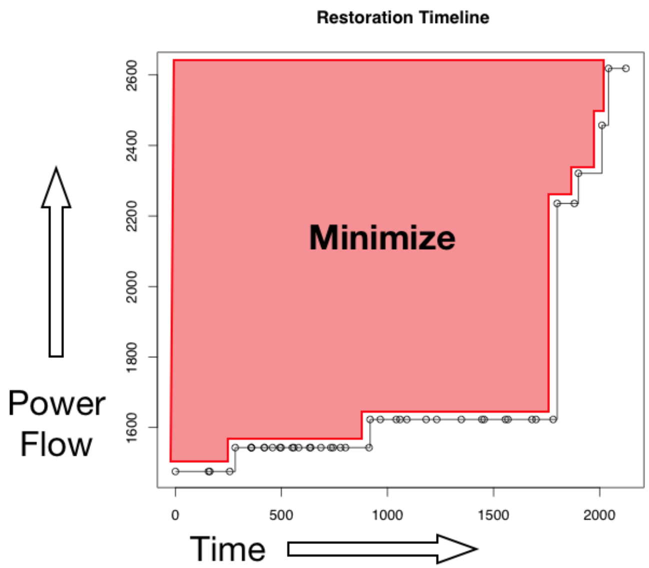
B. Strengthening the power flow model

Long Term: Incorporate restoration



Source: Department of Energy, Office of Electrcity Delivery and Energy Reliability

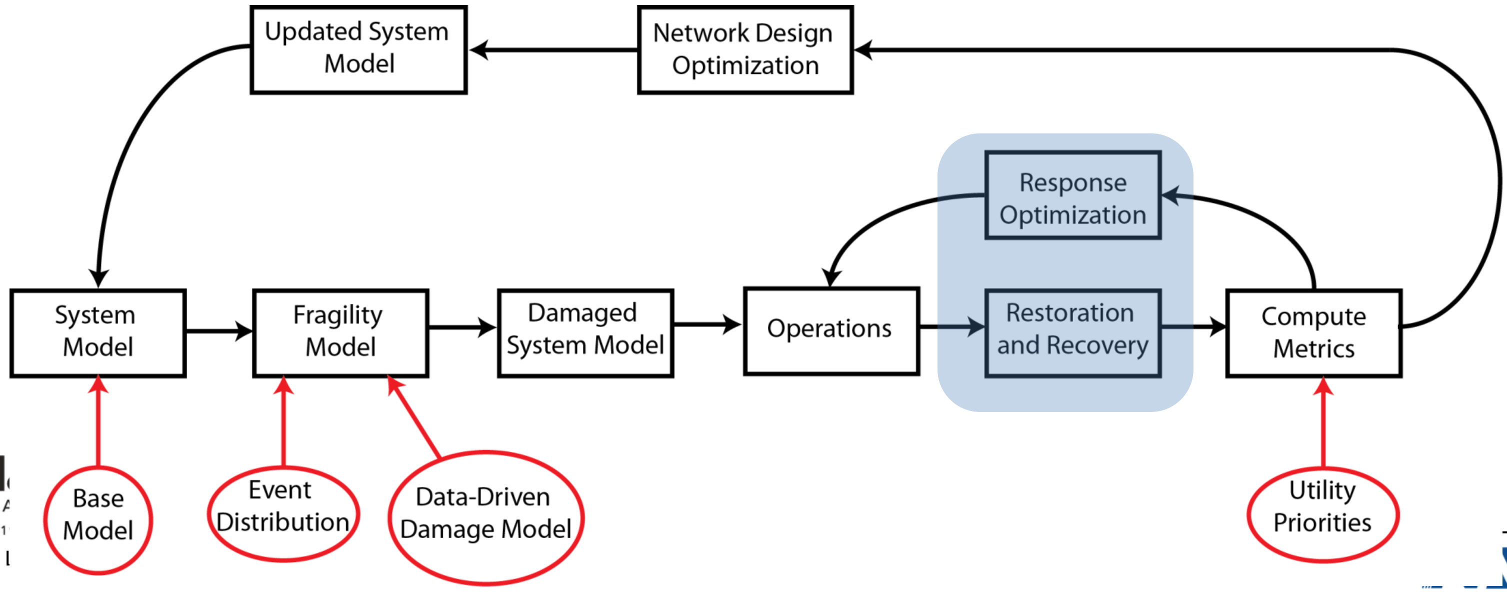
Restoration



Example: Minimize the size and duration of a black out.

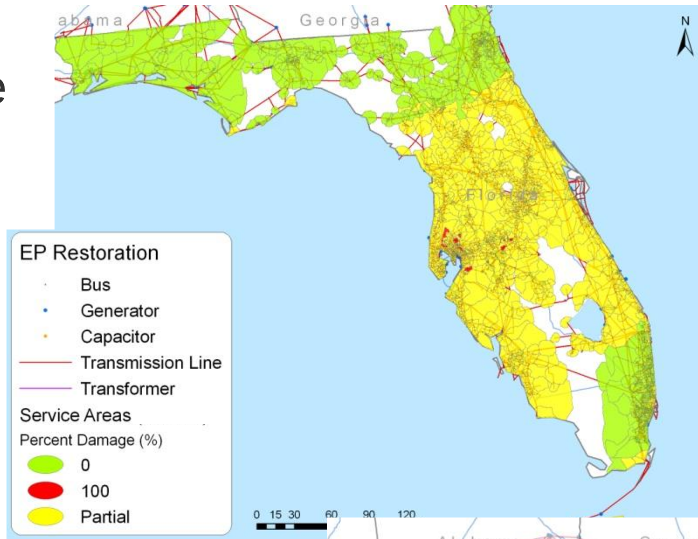
Combine grid operation requirements (restore power as quickly as possible) with transportation requirements (routing crews on a potentially damaged road network)

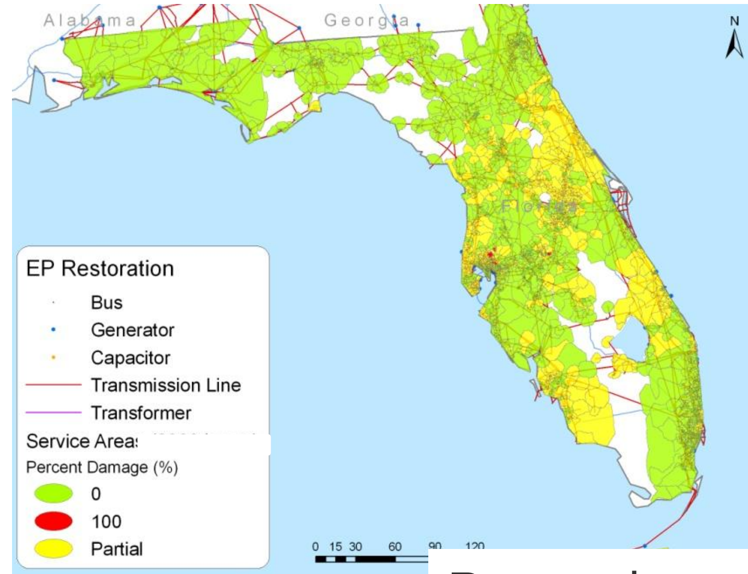
P. van Hentenryck, C. Coffrin, and R. Bent *Vehicle Routing for the Last Mile of Power System Restoration*. 17th Power Systems Computation Conference (PSCC 2011), August 2011, Stockholm, Sweden.



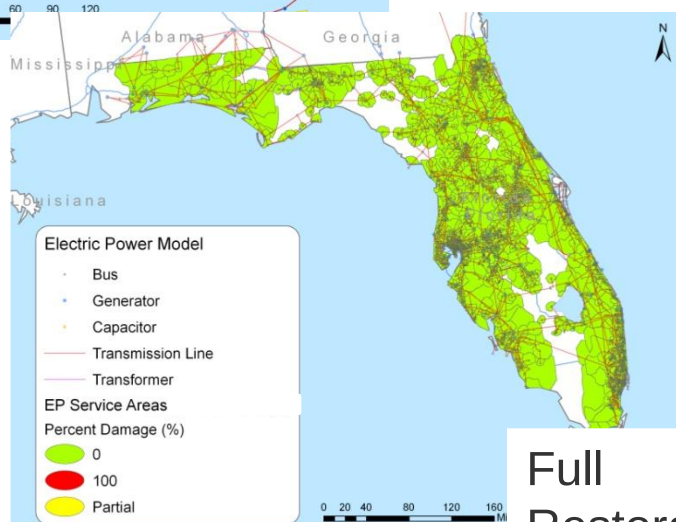
Restoration (only) Example—Applied To Transmission Setting

Initial Outage Area





Restoration After 2 Weeks

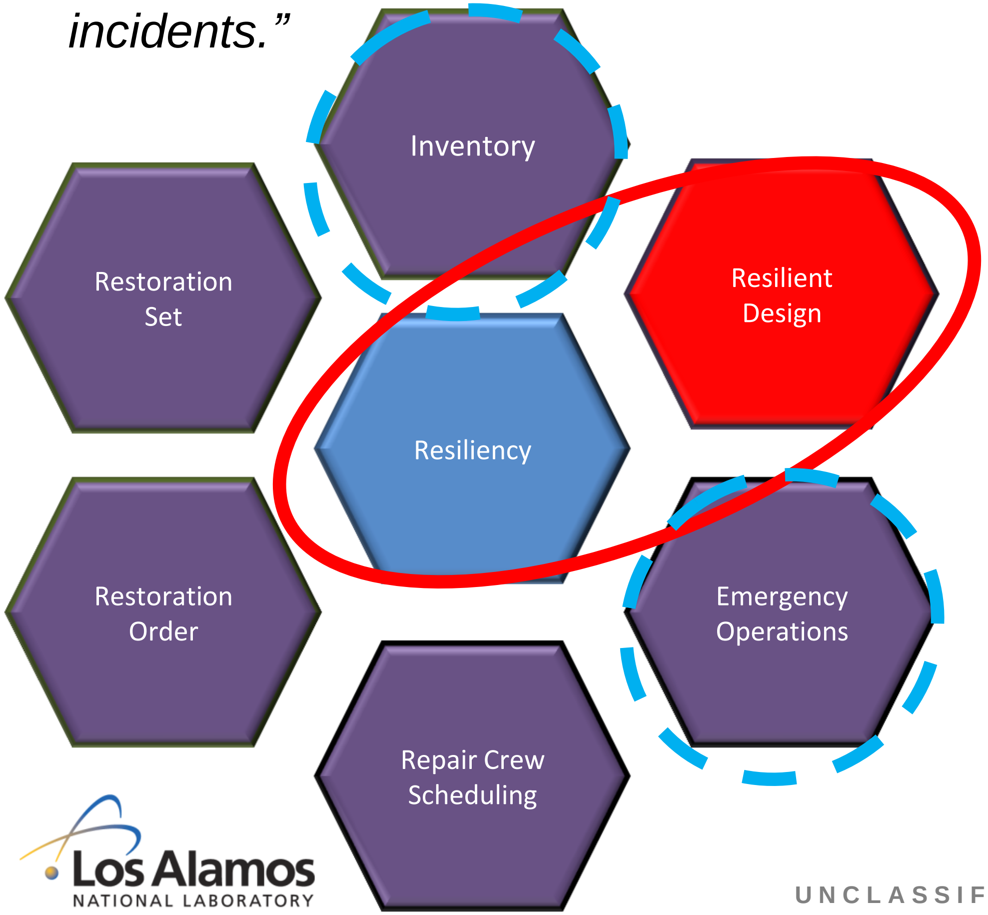


Full Restoration

Beyond the End Goal—Resiliency Tool Suite

Presidential Policy Directive - Critical Infrastructure Security and Resilience

*"The ability to **prepare for** and adapt to changing conditions and **withstand** and **recover** rapidly from disruptions. Resilience includes the ability to **withstand** and **recover** from deliberate attacks, accidents, or naturally occurring threats or incidents."*



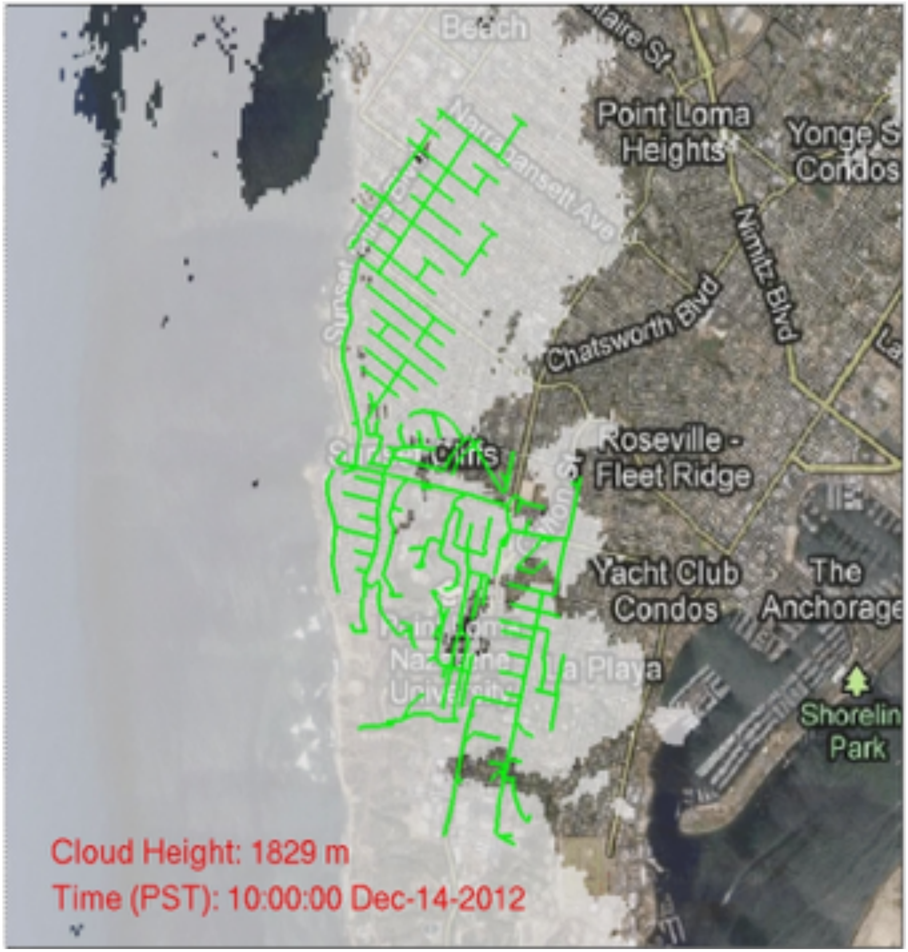
Decision support tool for critical infrastructure disaster planning and response, composed of interconnected modules

Today—Resilient deign to **withstand** initial blow

End Goal— + System restoration to capture **recovery** from initial blow

Beyond the End Goal— + Inventory and Emergency operation to **prepare** for events

Short Term Solar Forecasting Using Sky Imagery and Its Applications in Control and Optimization for a Smart Grid



Projection of clouds using sky imagery on a feeder in San Diego city from our study on impacts of high PV penetration on distribution feeders.

LANL Winter School
Jan 15th, 2015

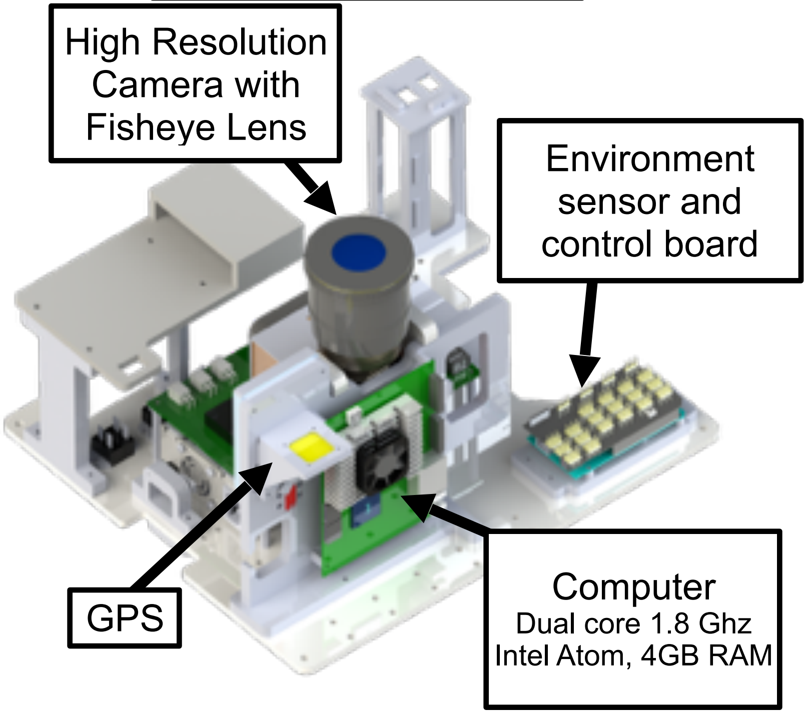
Andu Nguyen
Jan Kleissl

UCSD SRAF
solar resource and forecasting laboratory

UC San Diego
Dept. of Mechanical and Aerospace Engineering
La Jolla, California, USA

Short term solar forecasting using sky imagery

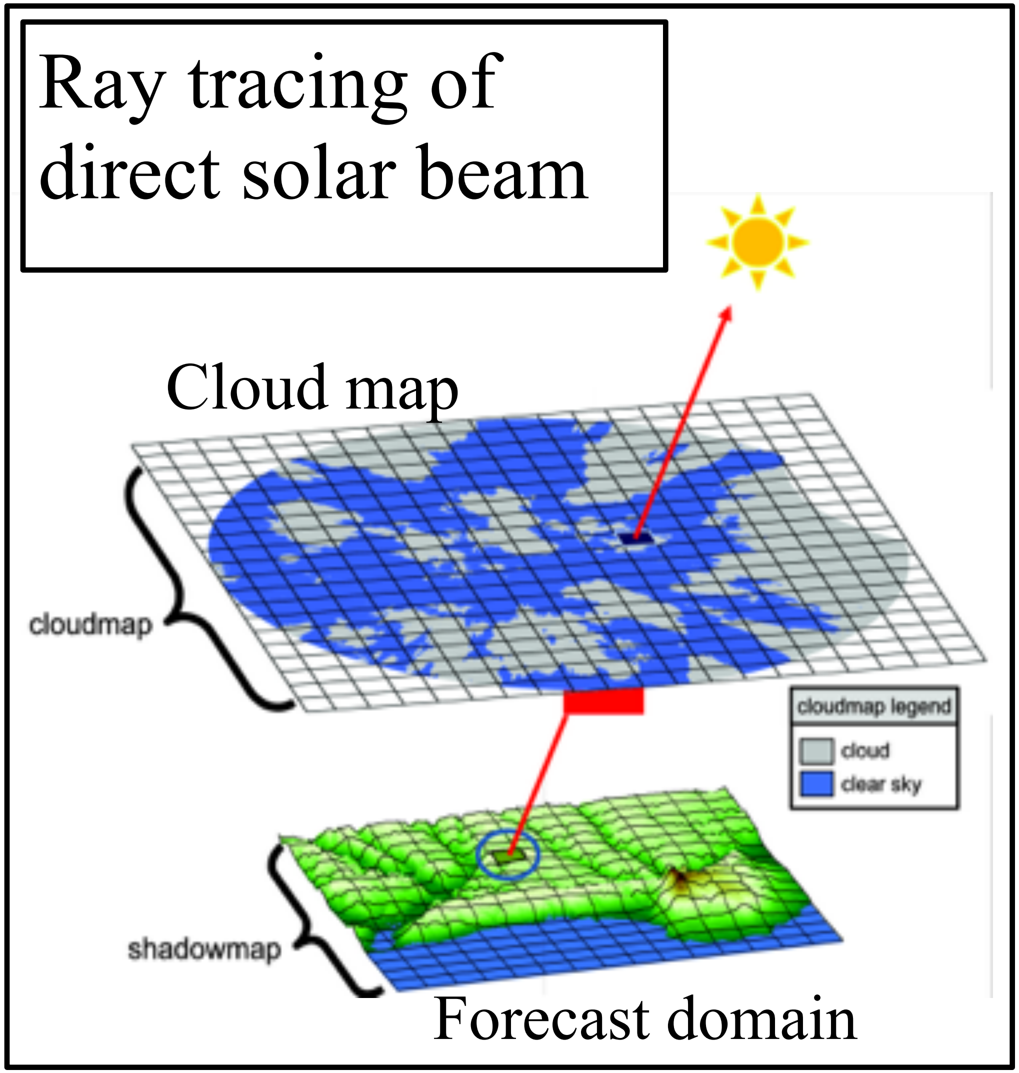
Hardware



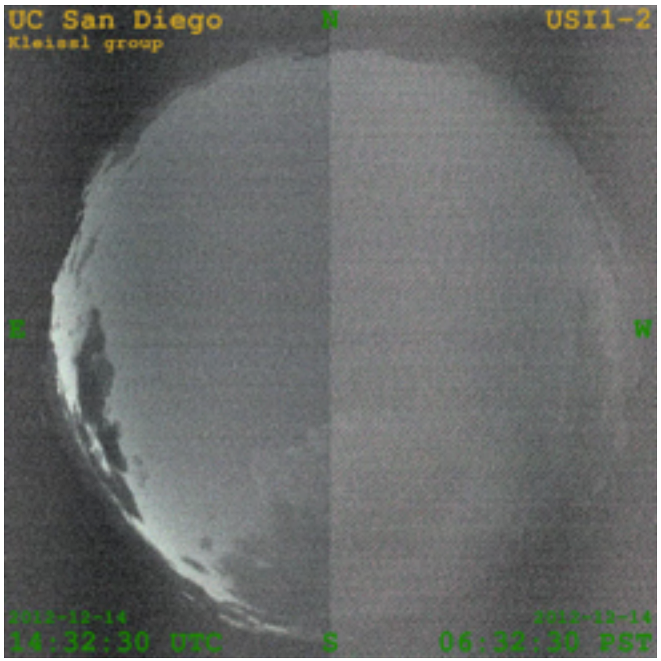
USI Deployed in Redlands, CA

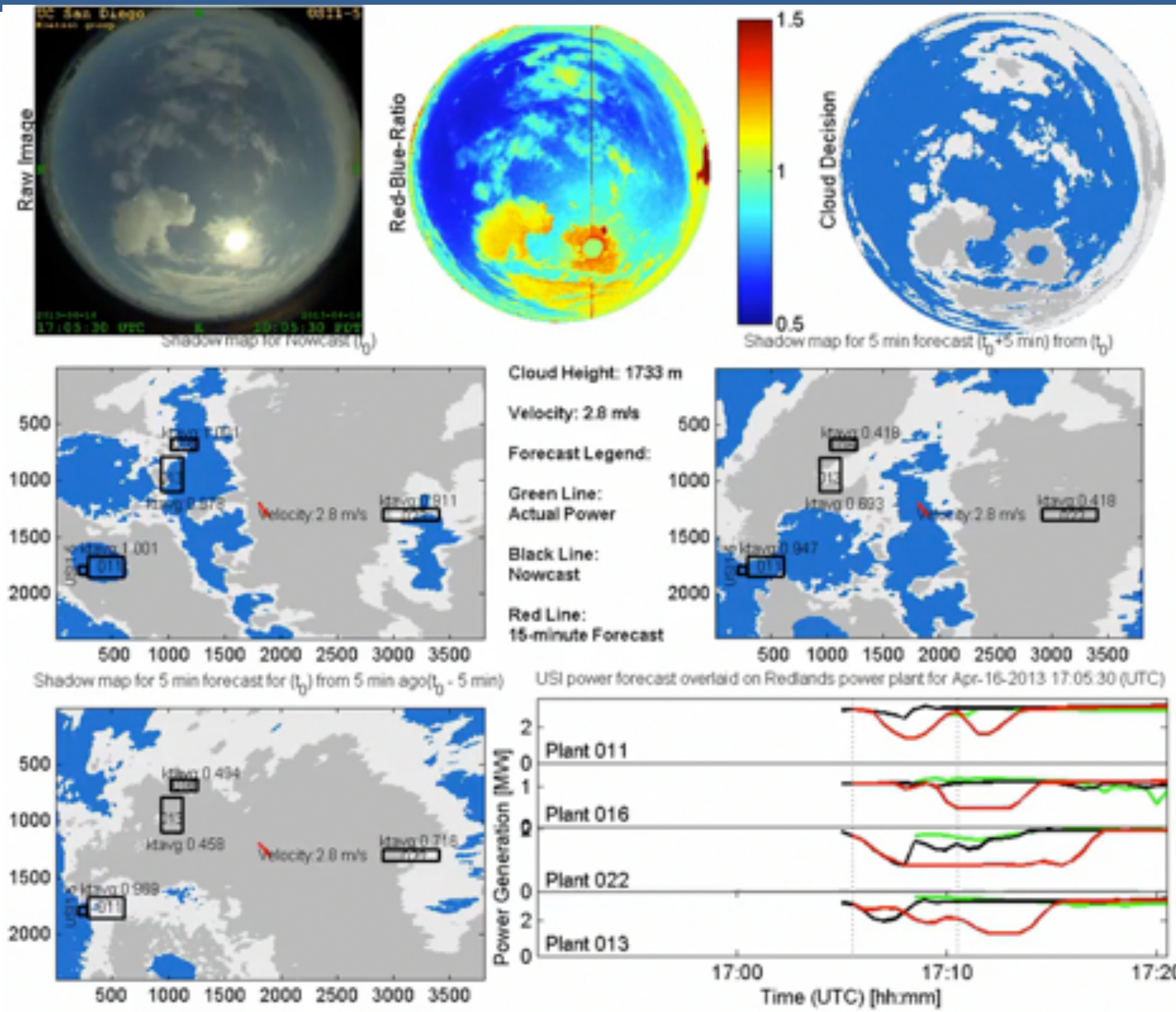
Short term solar forecasting using sky imagery

- Basic steps [1,2]:
  - Cloud detection
  - Cloud height determination
  - Cloud direction and velocity determination
  - Ray tracing/ Projection of cloud to the ground based on the Sun's location for irradiance forecast
  - Convert from irradiance to power forecast
- Provides 15-minute forecast every 30 seconds down to ground resolution of 2m x 2m.



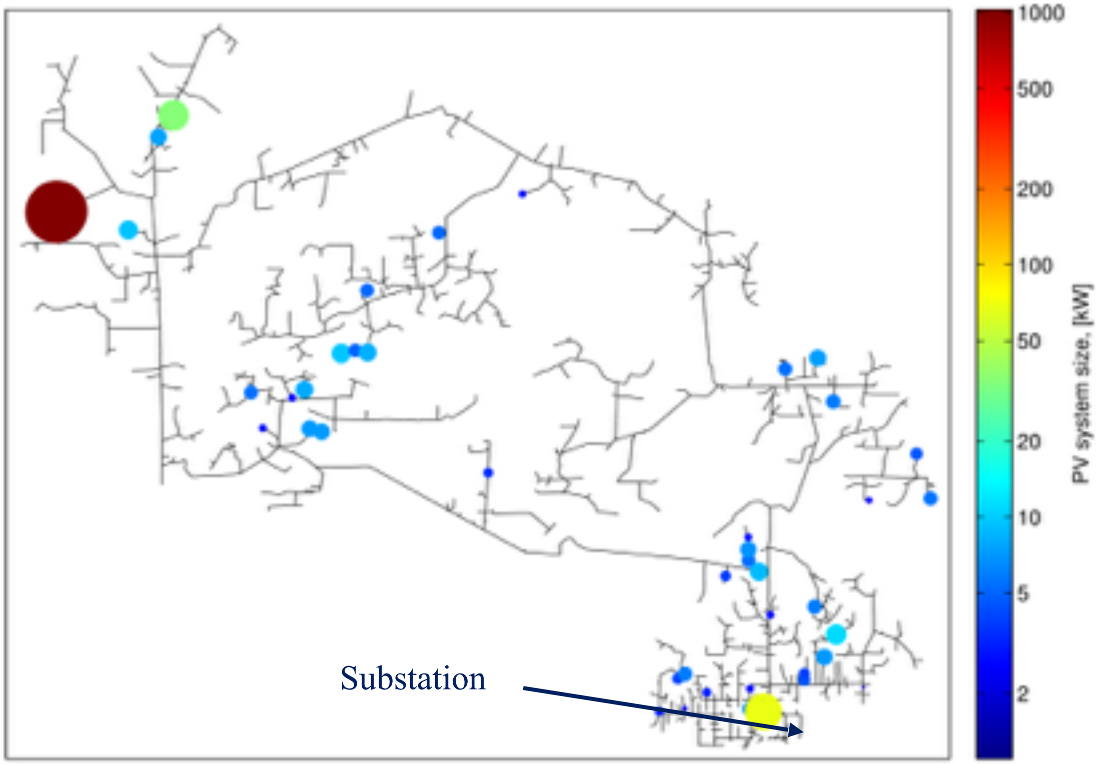
Short term solar forecasting using sky imagery





Model SDG&E feeder

- Large feeder (10 x 10 km^2) with peak load (11.12 MW) in rural area
- 1 large 2MW-PV site at the end of the feeder; Total PV: 2.3 MW peak.
- 1 large 2.5 MW load at the end of the feeder



Feeder A configuration with PV systems in circles

General information Fallbrook Feeder		
General	*Buses*	2463
	Nodes	6125
	Devices	4374
Conductors	*Length of three-phase lines*	311.953 kft / 95.08 km
	Length of two-phase lines	252.695 kft / 77.02 km
	Length of one-phase lines	18.518 kft / 5.64 km
Substation	*Voltage Level*	12 kV
Loads	*Total Active Power*	11.1225 MW
	Total Reactive Power	6.5007 MVar
	Number Of 1-Phase Loads	*556*
	Number Of 3-Phase Loads	*29*
Transformers	*Number Of Transformers*	1 (substation)
	Number Of Voltage Regulators	7
Capacitor Banks	*Total Number Of Capacitor Banks*	5 at 5 different locations
	Rating	4.3 MVar

Summary on characteristics of Feeder A

Model SDG&E feeder

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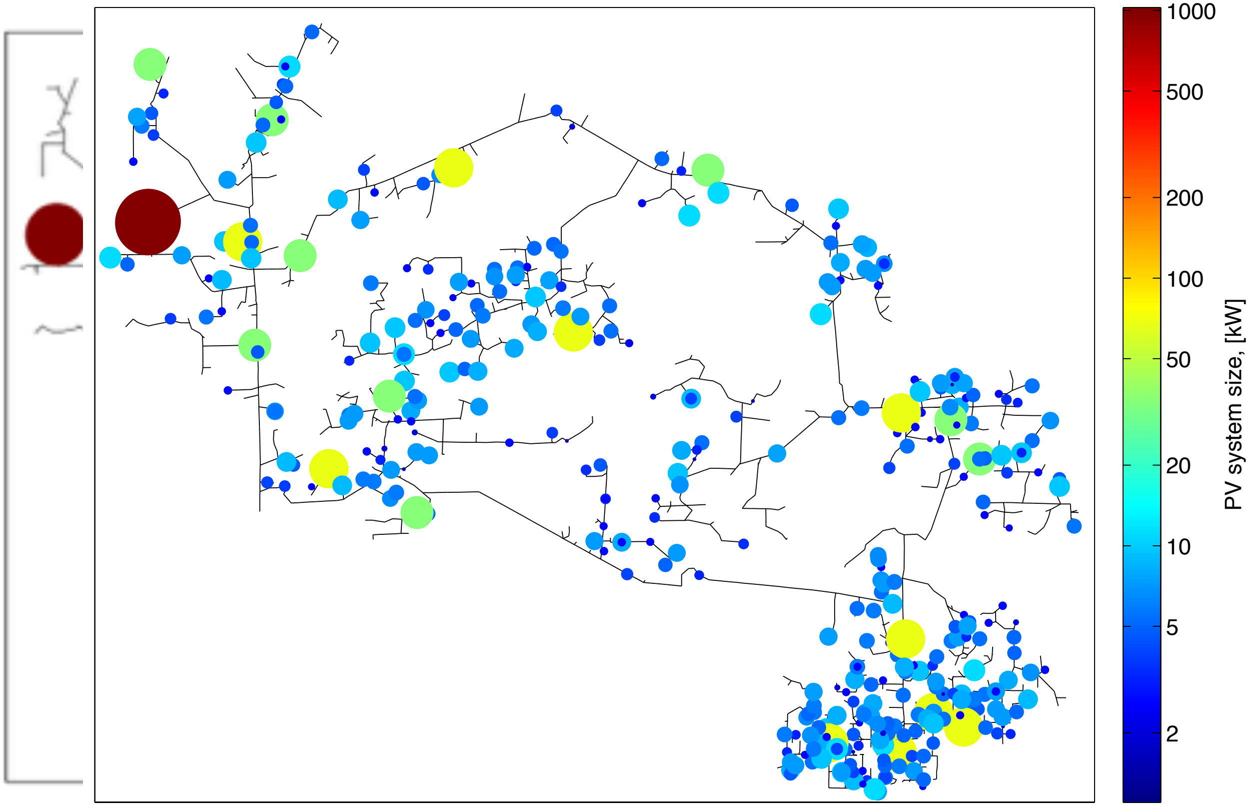

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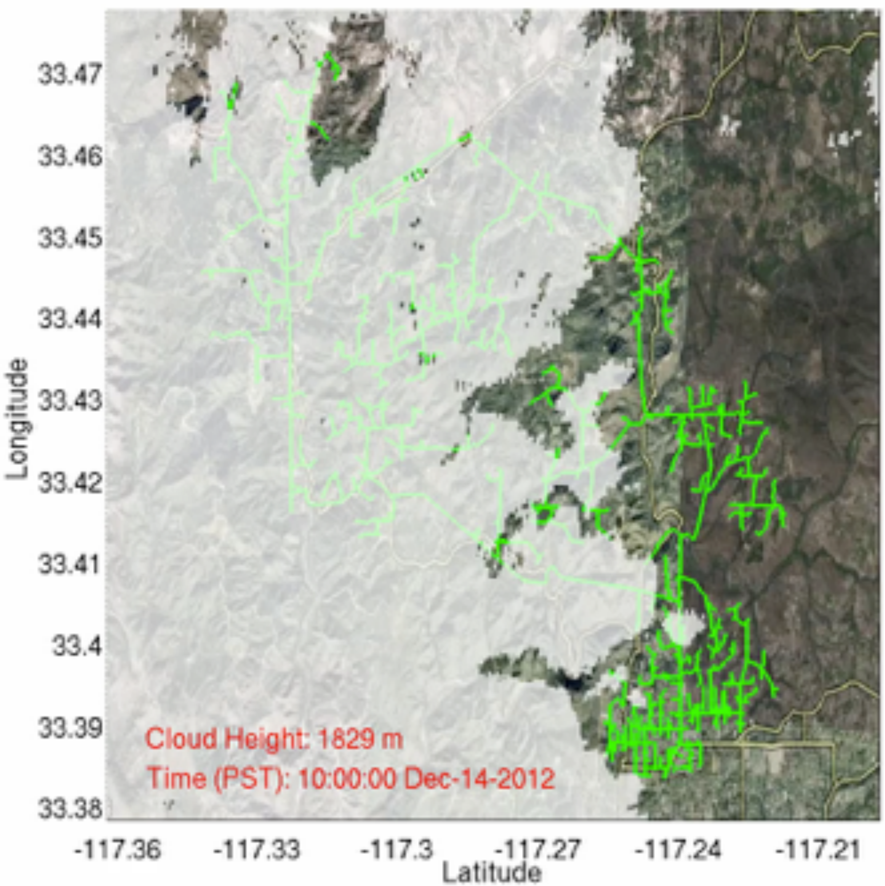
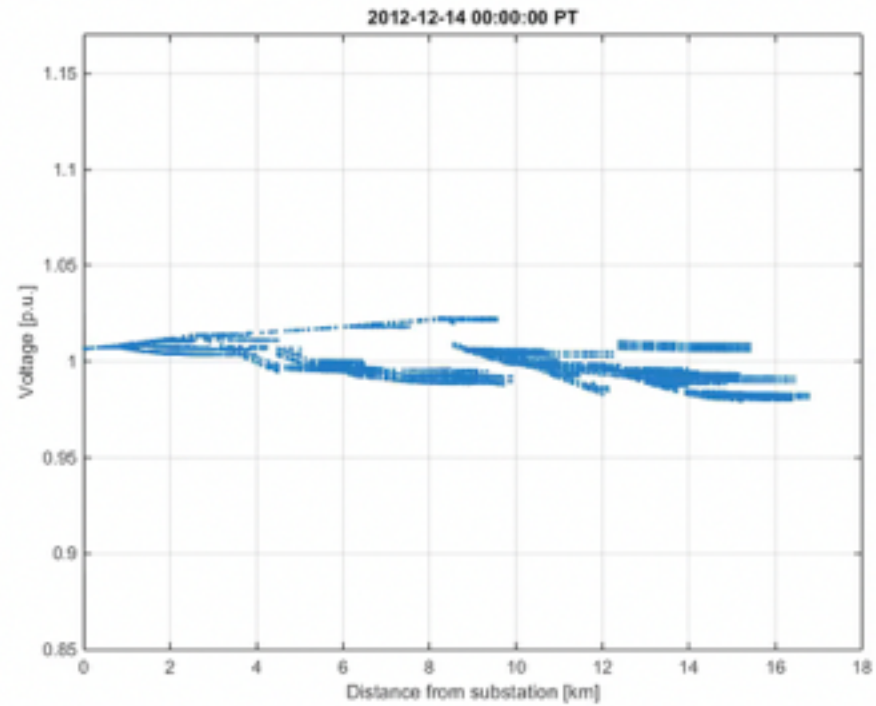
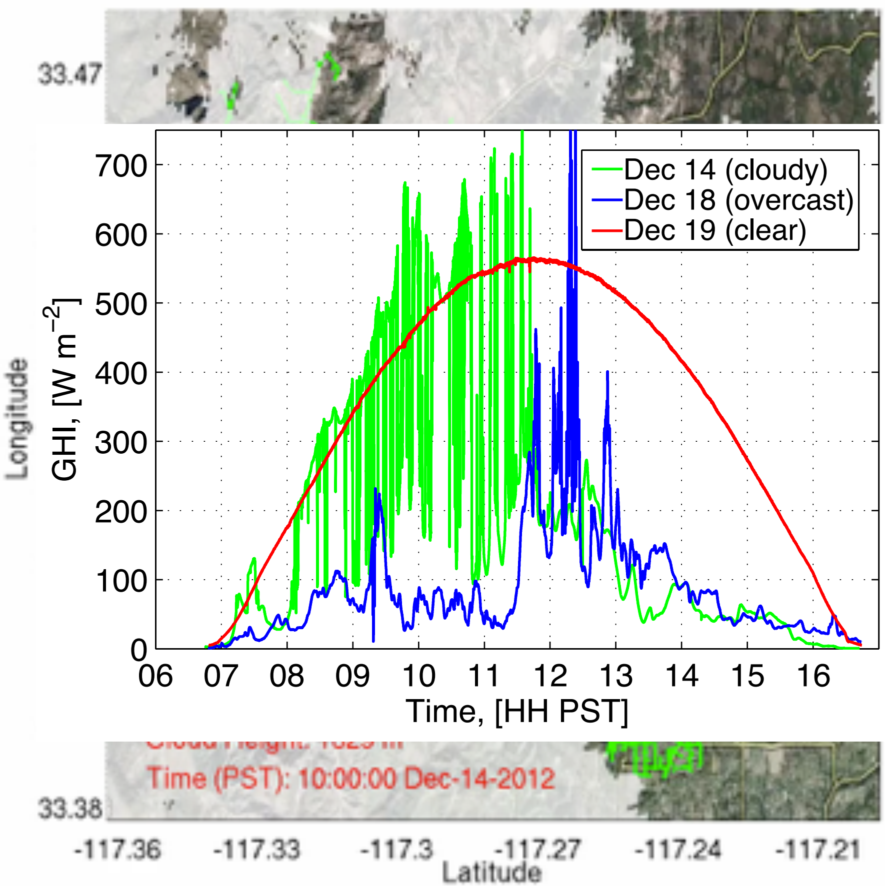
Summary on characteristics of Feeder A

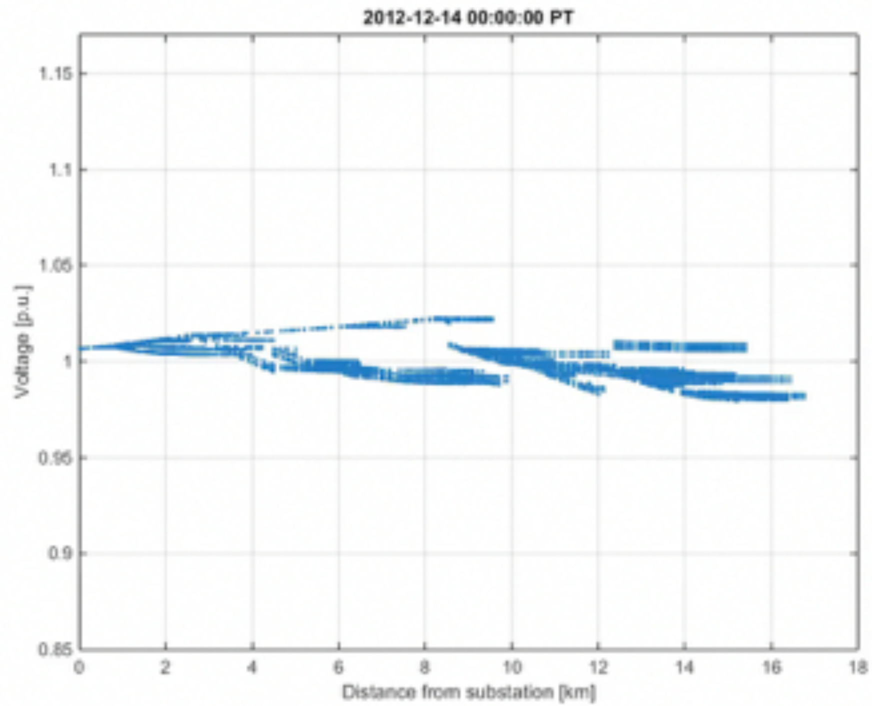
Impacts of high PV penetration on Dist. systems



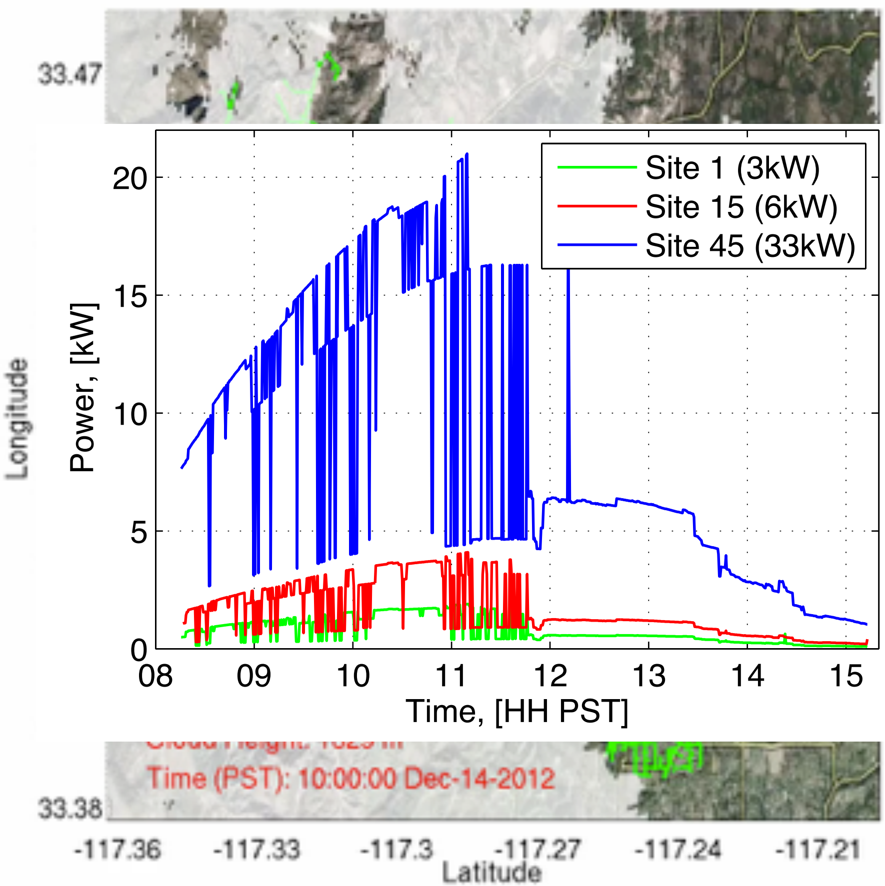


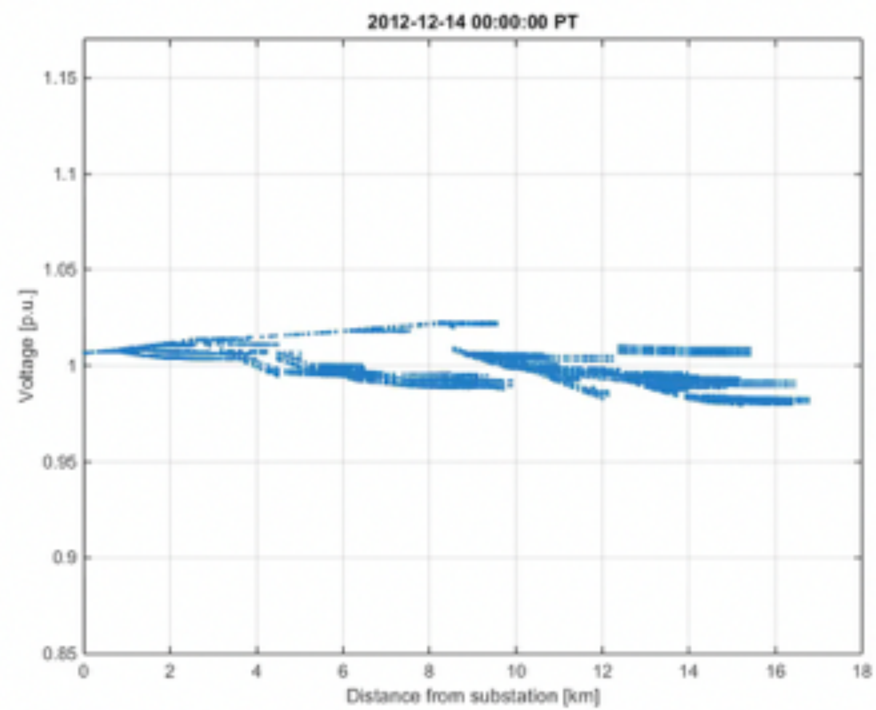
Impacts of high PV penetration on Dist. systems



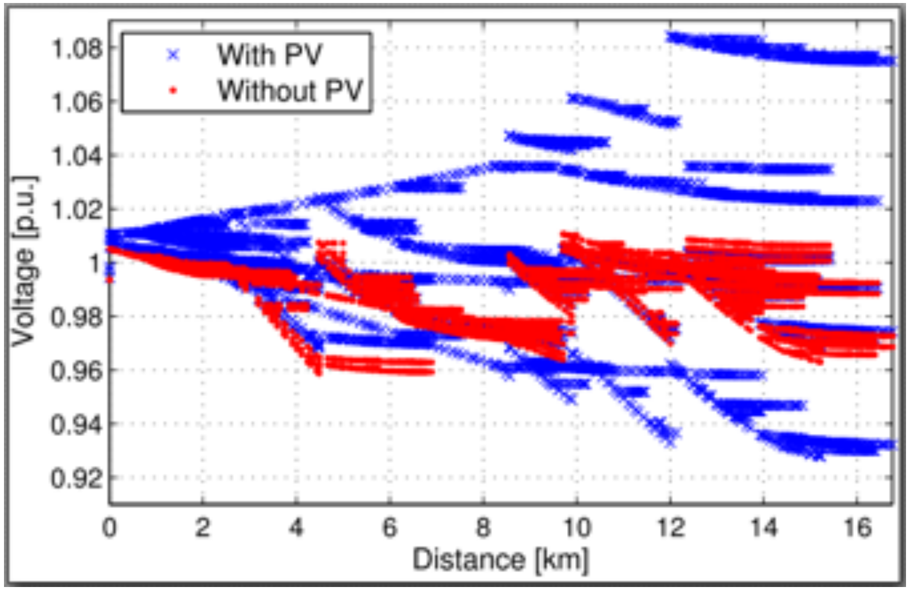


Impacts of high PV penetration on Dist. systems

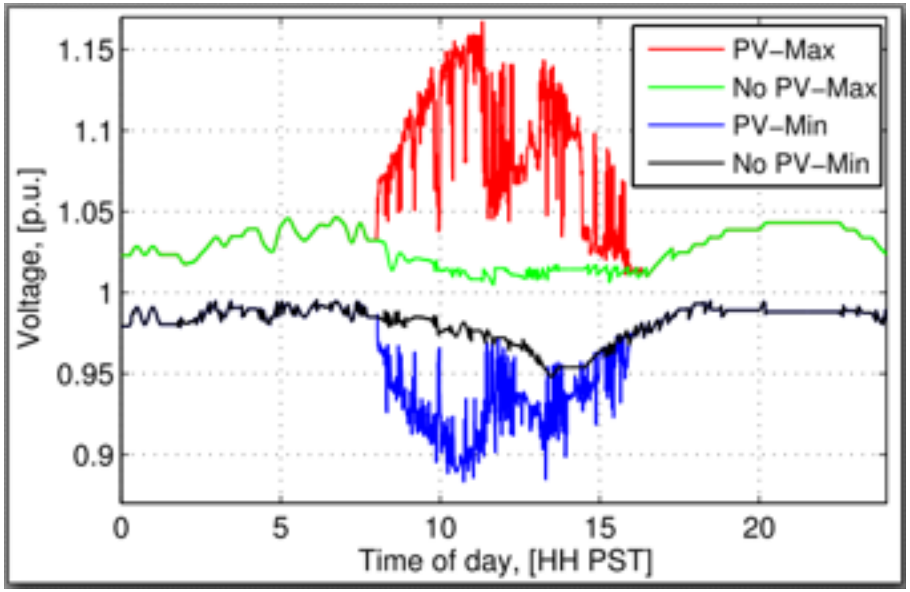




Comparison: With v.s. Without PV

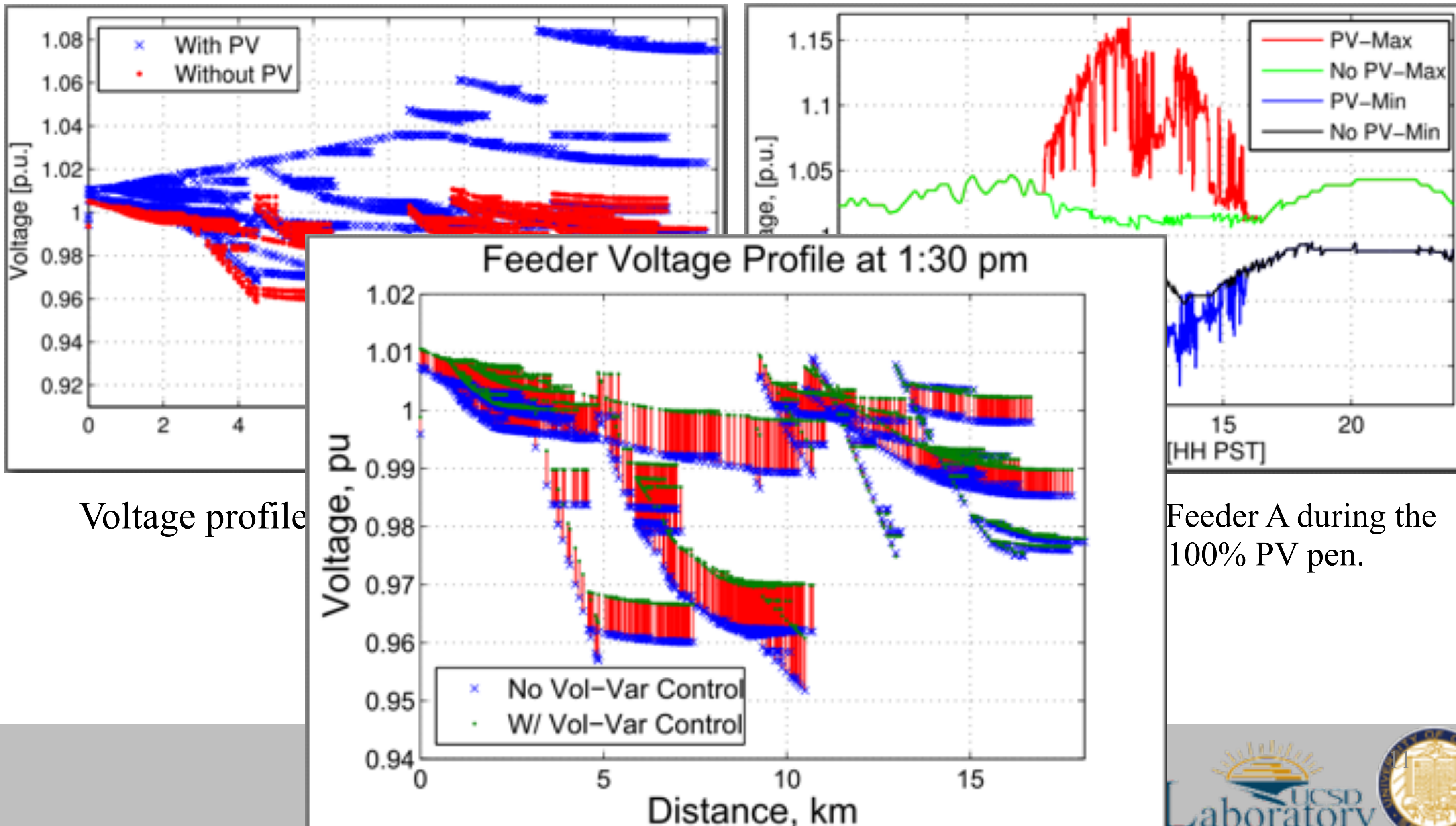


Voltage profile snapshot at 1300 PST



Max-min voltage profile on Feeder A during the partly cloudy day with 100% PV pen.

Comparison: With v.s. Without PV



Voltage profile

Feeder A during the 100% PV pen.

Optimization and control using PV inverters and Energy Storage systems

$$\min_{p_{\mathrm{PV}},q_{\mathrm{PV}},p_{\mathrm{ES}},q_{\mathrm{ES}}} \sum_{t\in\tau} (J_{\mathrm{loss}} + \alpha_1 J_{\mathrm{power}} + \alpha_2 J_{\mathrm{ramp\_viol}} + \alpha_3 J_{\mathrm{ES\_cc}} + \alpha_4 J_{\mathrm{TO}} + \alpha_5 J_{\mathrm{VoltExcursions}})$$

$$\text{s.t.} \quad \forall t \in \tau \begin{cases} (p_k^t)^2 + (q_k^t)^2 \le S_k^{max}, & \forall k \in \mathcal{G} \cup \mathcal{S}, \\ V_{\min} \le v_k(t) \le V_{\max}, & \forall k \in \mathcal{G} \cup \mathcal{S}, \\ D_{\mathrm{dis}}^k \le d_k(t) \le D_{\mathrm{ch}}^k, & \forall k \in \mathcal{S}, \\ 0 \le c_k(0) + \frac{1}{C_k} \sum_{i=1}^{T} d_k(i) \le 1, & \forall k \in \mathcal{S}, \\ \text{Power flow equations hold} & \end{cases}$$

$$J_{\mathrm{ramp\_viol}} = \sum_i^n \left[\left(\frac{dp_{\mathrm{PV}}^i}{dt} \right)^2 - R_R^2 \right]_+ = \sum_i^n \left[\left(\frac{dp_{\mathrm{PV}}^i}{dt} \right)^2 - \left(\frac{P_{\max}^i}{60s} \right)^2 \right]_+$$

Ramp rate <10%/min

$$J_{\mathrm{TO}} = \sum |\Delta s| = \sum_{i=1}^{N_{\mathrm{VR}}} \sum_{t=0}^{T} \left| s_t^i - s_{(t-1)}^i \right|, \quad \frac{v_{t-1}^i - v_{\mathrm{ref}}^i}{v_{\mathrm{bw}}^i} - \frac{1}{2} \le s_t^i \le \frac{v_{t-1}^i - v_{\mathrm{ref}}^i}{v_{\mathrm{bw}}^i} + \frac{1}{2}, i = 1, 2, \ldots, N_{\mathrm{VR}}$$

UCSD Microgrid

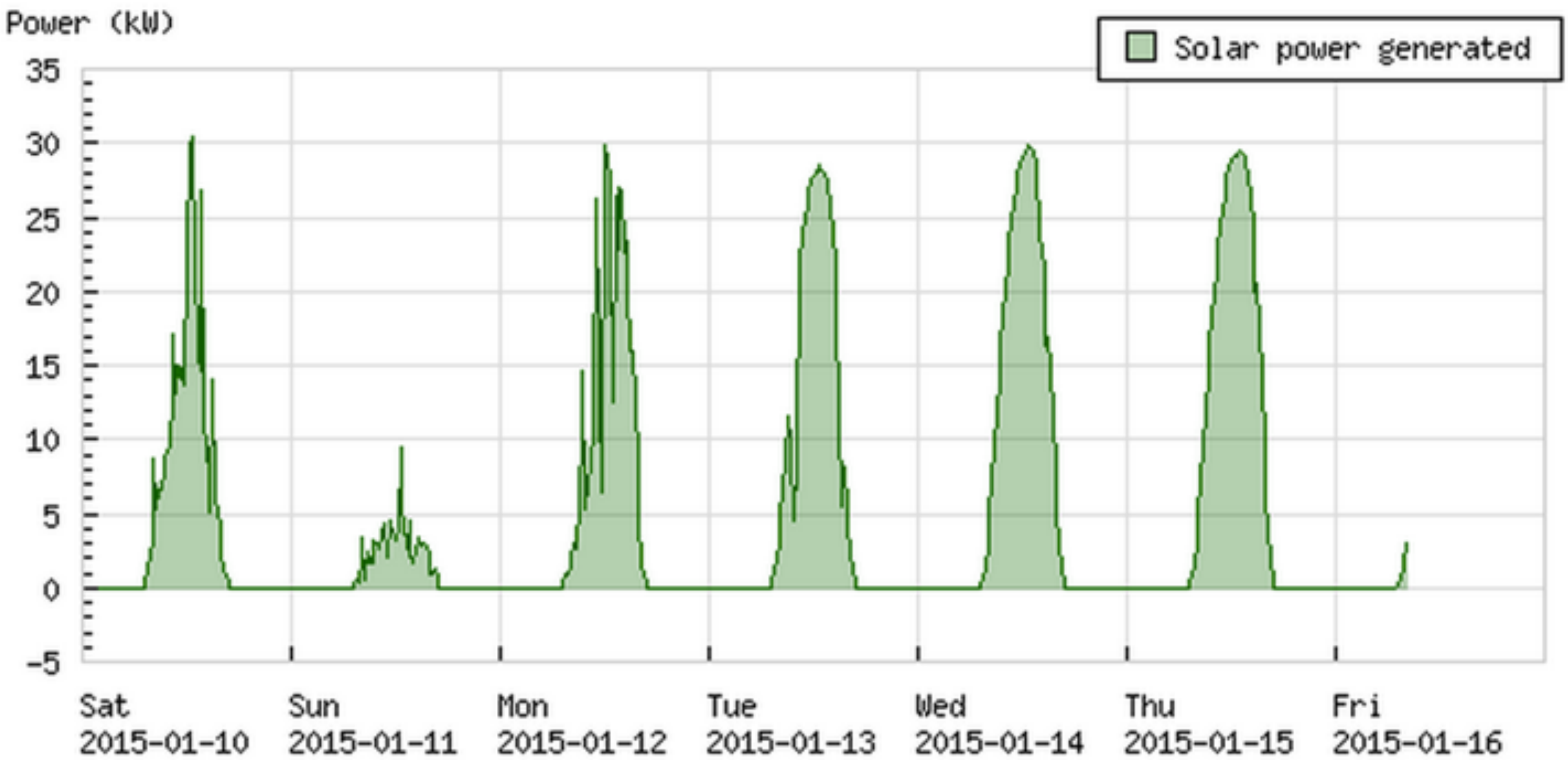
UCSD Microgrid

UCSD Microgrid

UCSD Microgrid

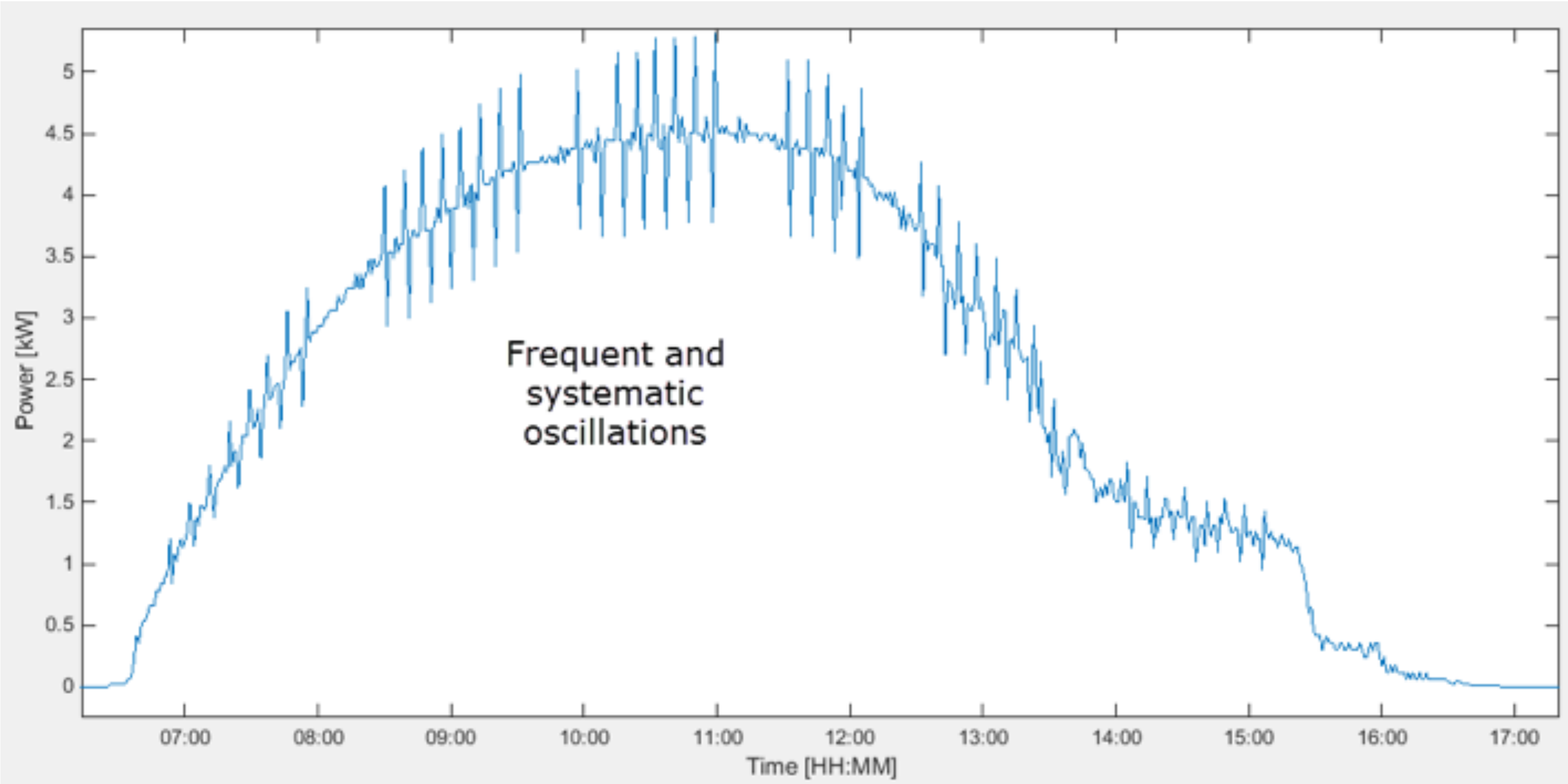
- 42 MW peak load
- 3.1 MW PV
- 2.8 MW Fuel Cell
- 30 MW Natural gas plant generating 80% annual demand
- 1.8 MW / 11.2 MWh electric energy storage
- Meters 50,000 data points for power, voltage, current, temperature, etc.
- 5 PMUs currently, and planning to install 15 more in coming year

UCSD Microgrid



UCSD Microgrid

Thank you! Questions?



- Food for thought: SolarCity's 1-min data

SRAF Laboratory UCSD

Contacts

- If you are interested in the videos, please contact me using my email below and I'll send them separately to you since some of them are quite large in size.
  - Andu Nguyen: andunguyen.ucsd@gmail.com or andunguyen@ucsd.edu
- You can also contact my advisor if you are interested in our work in general. His email is below:
  - Jan Kleissl: jkleissl@ucsd.edu

ETH zürich

Modeling Frameworks for Future Energy Systems

Göran Andersson
Power System Laboratory, ETH Zürich

eeh power systems laboratory

Outline

- Introduction & Motivation
- Energy Hubs
- Power Nodes
- Other Models
- Concluding Remarks

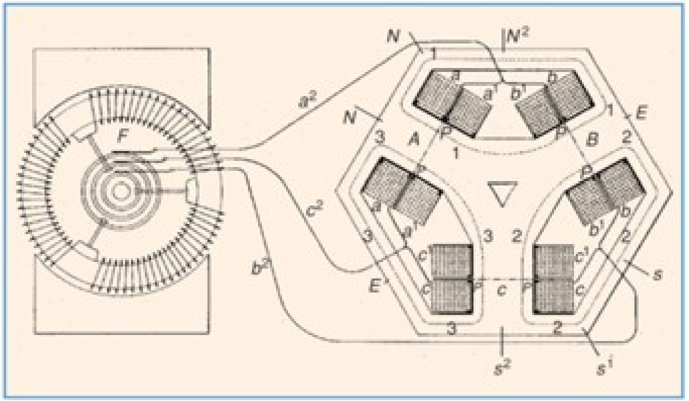
ETH zürich

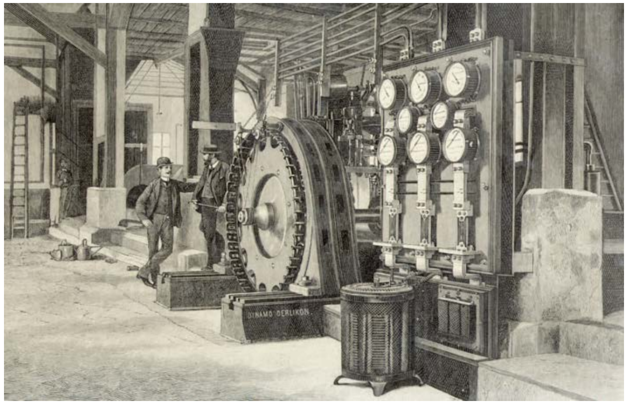
History of Challenges of the Power System

eeh power systems laboratory

The First Challenge of Electric Power Engineering

1880 – 1920: To make it work





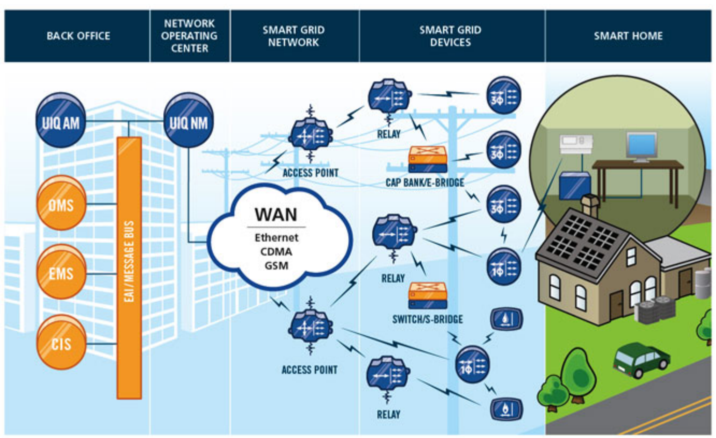
The Second Challenge of Electric Power Engineering

1920 – 1990: To make it big

To make it big (1000 kV, China 2008)

The Third Challenge of Electric Power Engineering

1990 - : To make it sustainable



ETH zürich

About Planning the Future

- *“Plans are useless, but planning is indispensable.”*

  Dwight D. Eisenhower

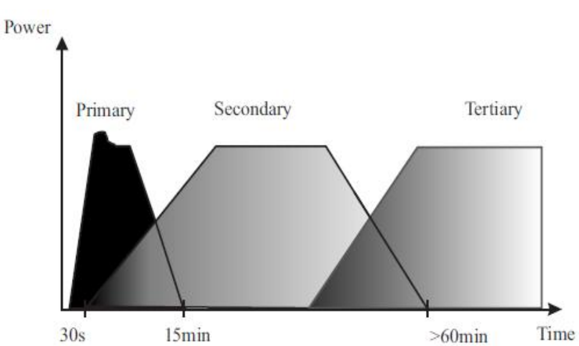
Complexity of Power Systems

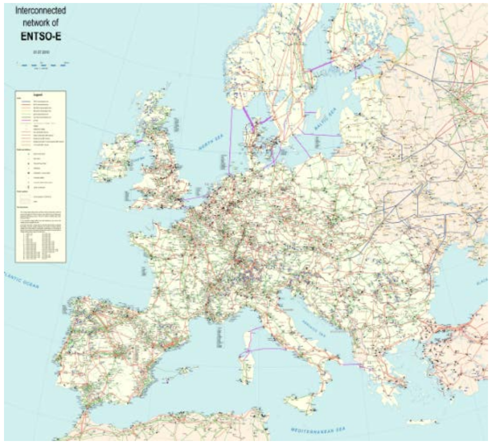
Complexity along several dimensions

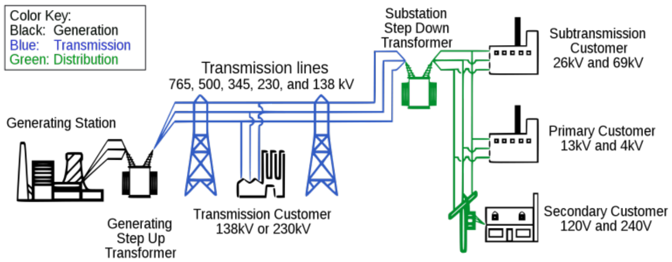
- **Time** — **(milli)seconds** (e.g. frequency inertia, frequency&voltage control),
  minutes (e.g. secondary/tertiary frequency&voltage control),
  hours/days (e.g. spot market-based plant/storage scheduling),
  months/years (e.g. seasonal storage, infrastructure planning).

- **Space** — **1‘000+ km**, e.g. interconnected continental European grid
  (Portugal – Poland: 3‘600 km, Denmark – Sicily: 3‘000 km).

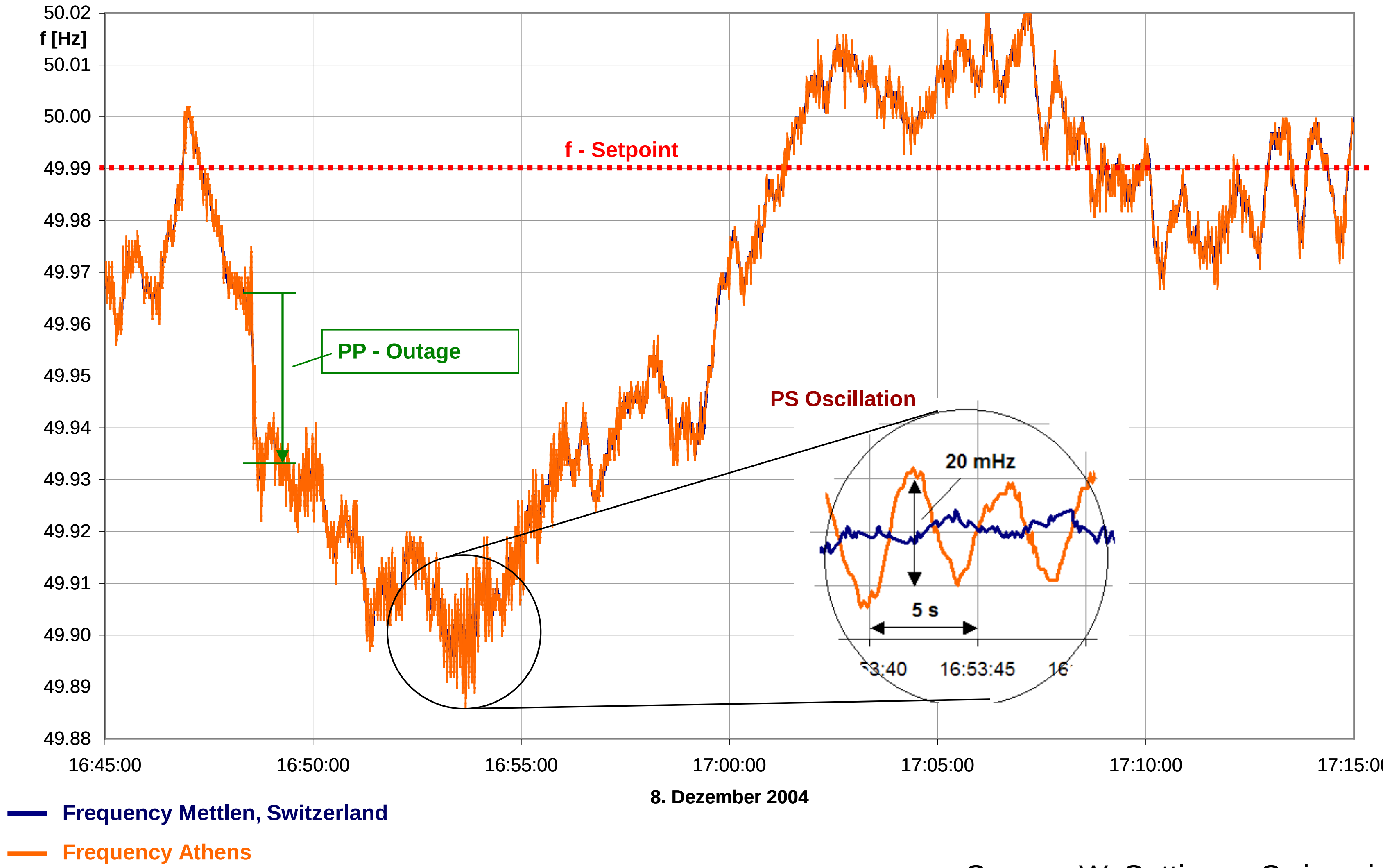
- **Hierarchy** — **from distribution grid** (e.g. 120/240 V, 10 kV) **to**
  high-voltage transmission grid (220/380/500/… kV, AC and DC).







The grid frequency – A key indicator of the state of the system



Source: W. Sattinger, Swissgrid

Spectrum of the system frequency and the AGC signal

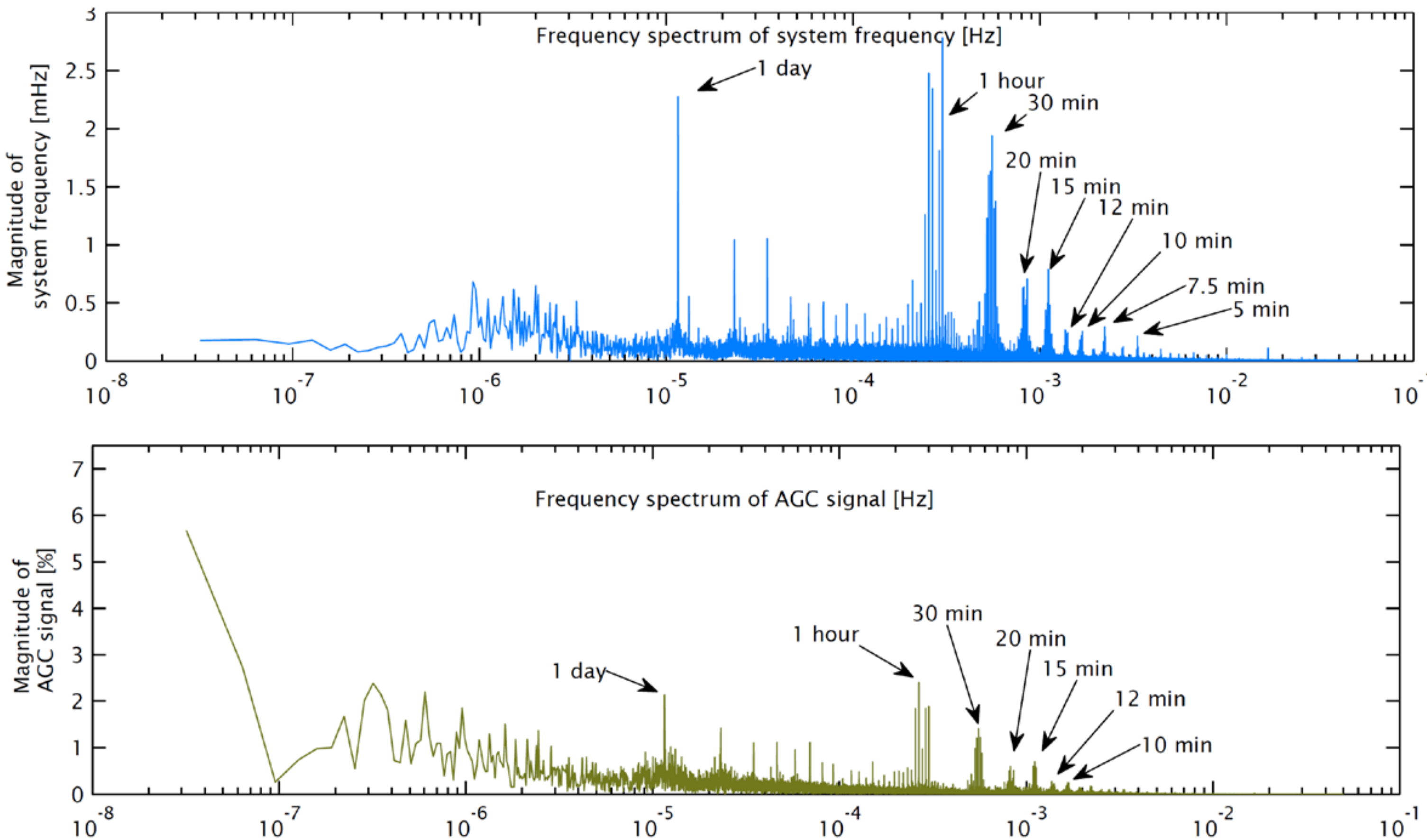


Figure 1. Spectrum of the system frequency and the AGC signal

Source: A new frequency control reserve framework based on energy-constrained units (Borsche, Ulbig, Andersson, PSCC 2014)

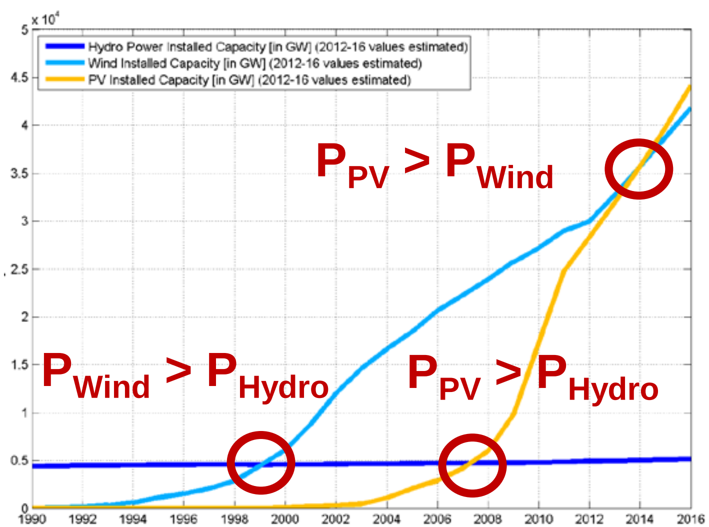
Trends and Challenges

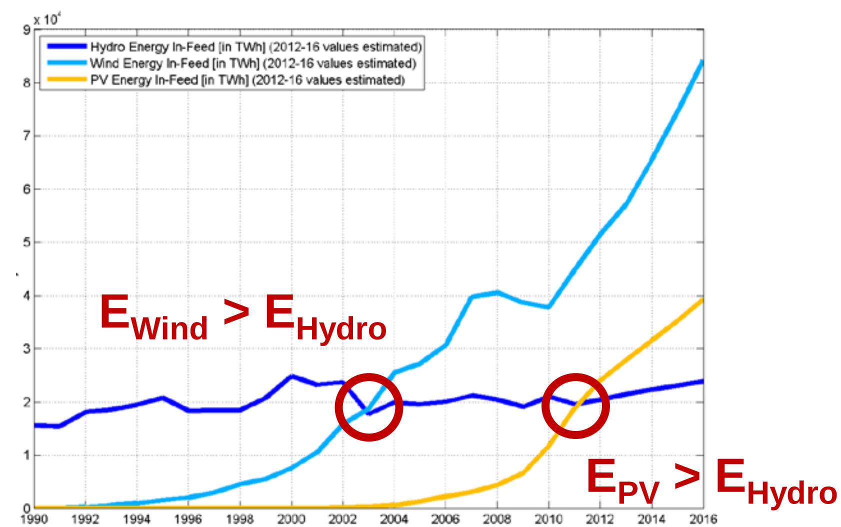
Increasing fluctuating RES deployment = Fluctuating power in-feed

- Germany 2012: (Wind+PV) **63.9 GW power capacity** ≈ 75% of fully dispatchable fossil generation. **77.1 TWh energy produced** ≈ 15.2% of final electricity consumption.
- Wind+PV: *Still* mostly uncontrolled power feed-in – Hydro: «well»-predictable power feed-in.

Mitigation Options

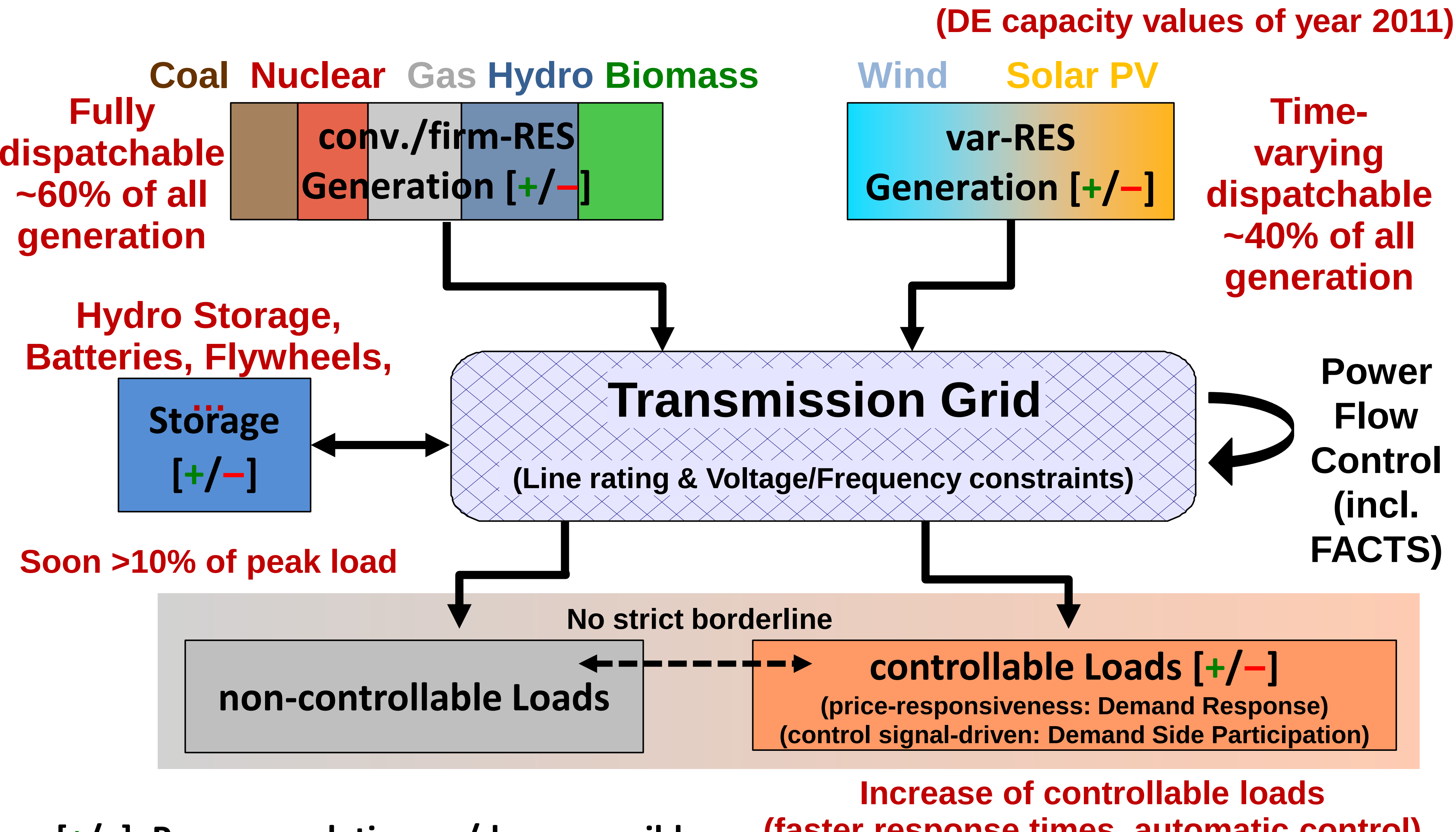
- Improvement of Controllability: Implementation of Wind/PV curtailment in some countries.
- Improvement of Observability: More measurements and better predictions of PV and wind power feed-in (state estimation & prediction).





Sources: BaSt 2012, IEA Electricity Information 2011, BMU AGEE 2013, own calculations

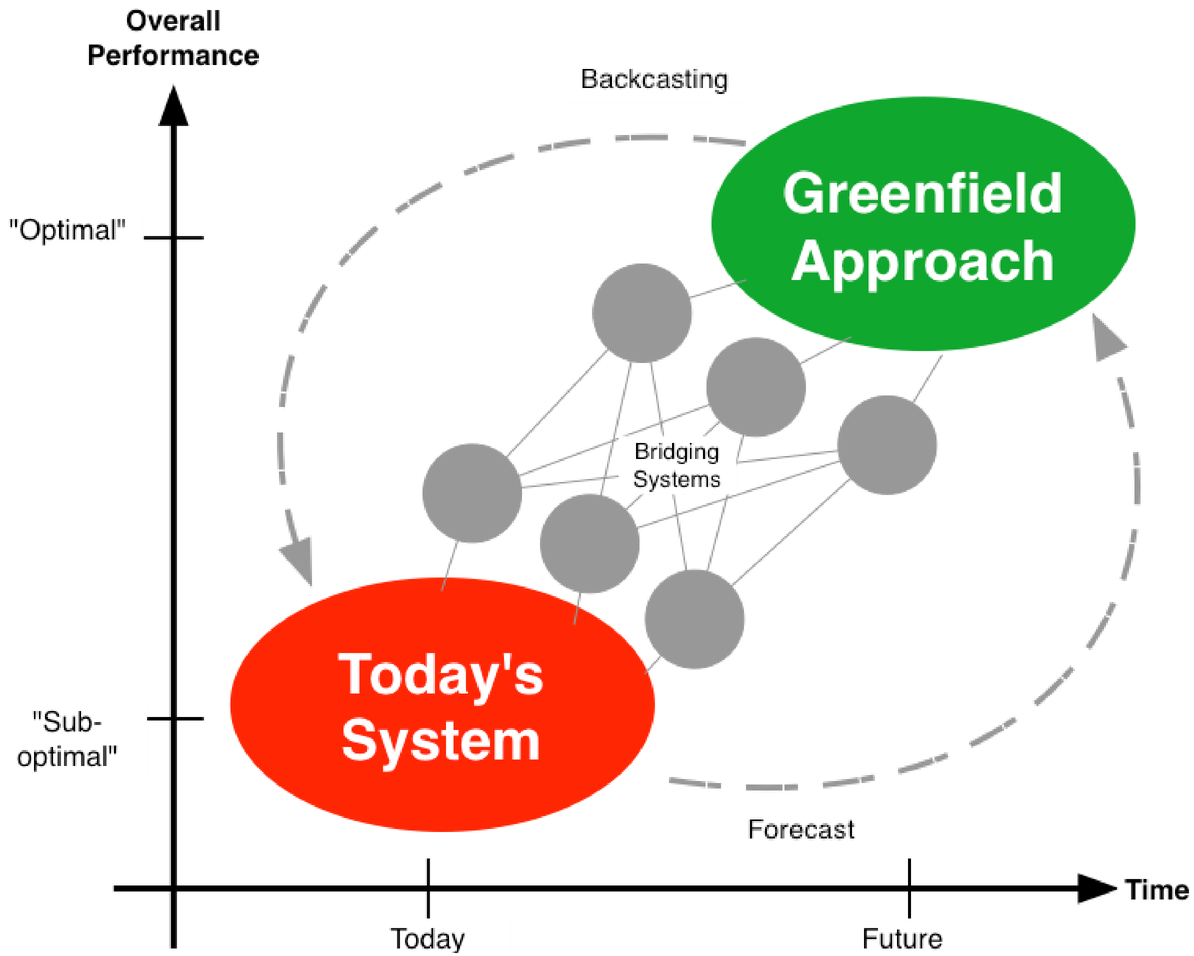
PRESENT & FUTURE – high RES shares & *Smart Grid Vision*



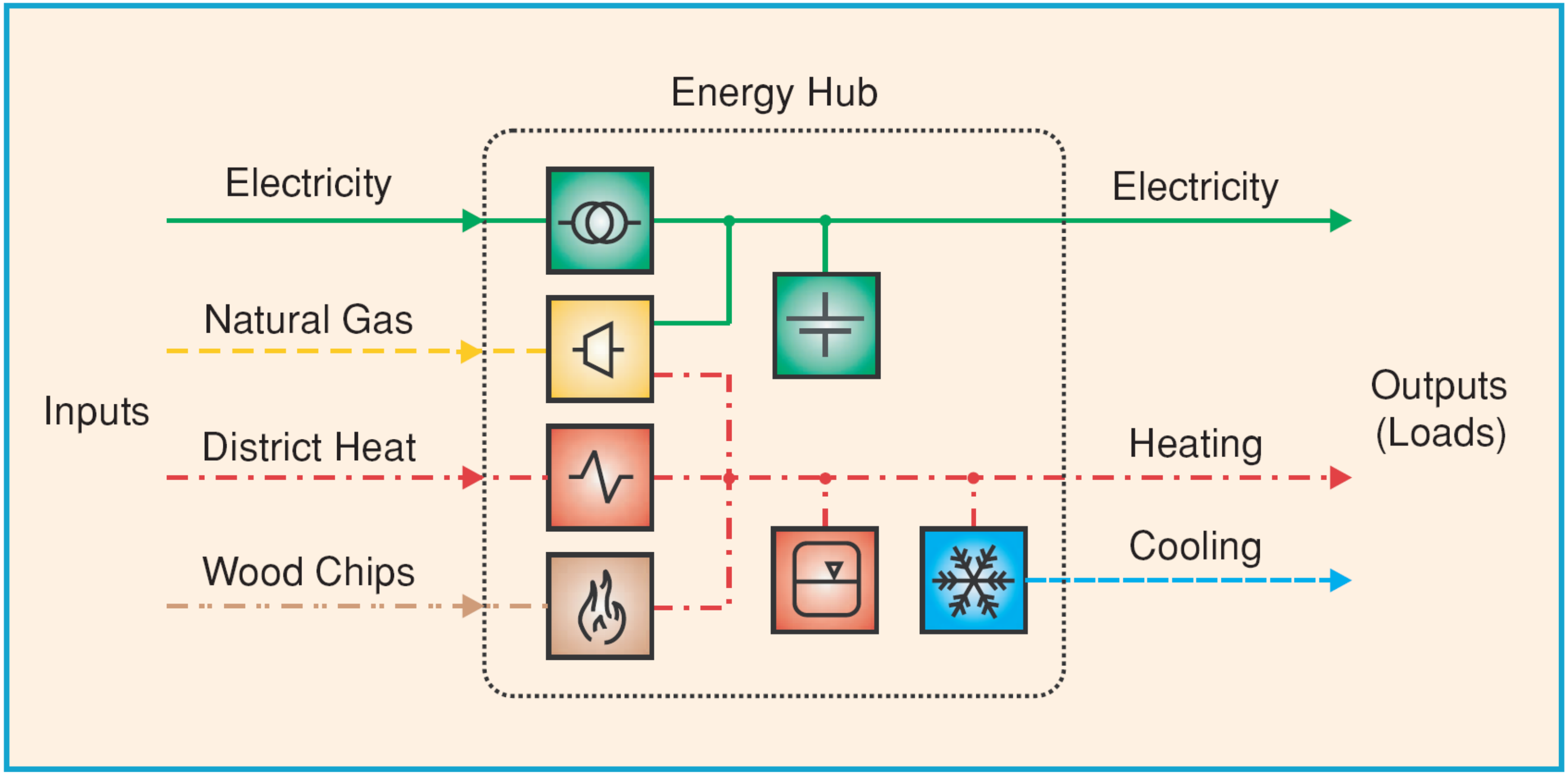
[+/–]: Power regulation up/down possible.

Energy Hubs

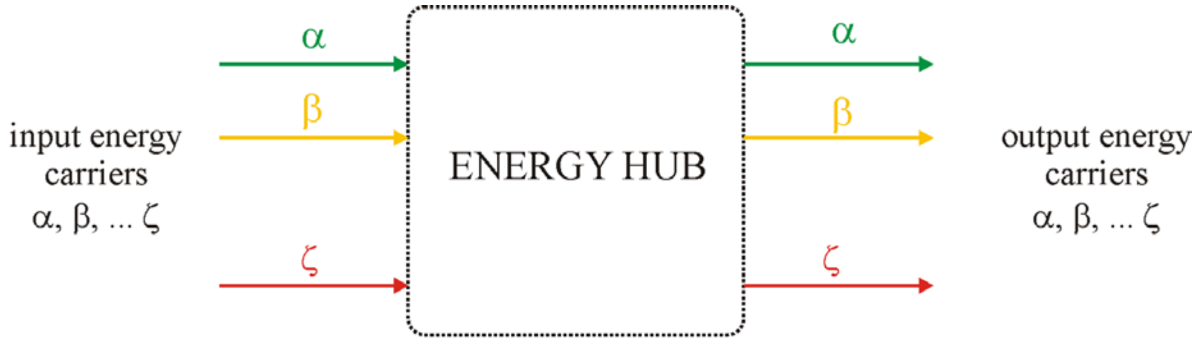
- ETH Zürich: Michèle Arnold, Martin Geidl, Florian Kienzle, Gaudenz Koeppel, Thilo Krause, …
- University of Michigan: Mads Almassalkhi, Ian Hiskens


Overall
Performance
Backcasting
"Optimal"
Greenfield
Approach
Bridging
Systems
Today's
System
"Sub-
optimal"
Forecast
Time
Today
Future

The Energy Hub – A Key Element



Modeling the Energy Hub

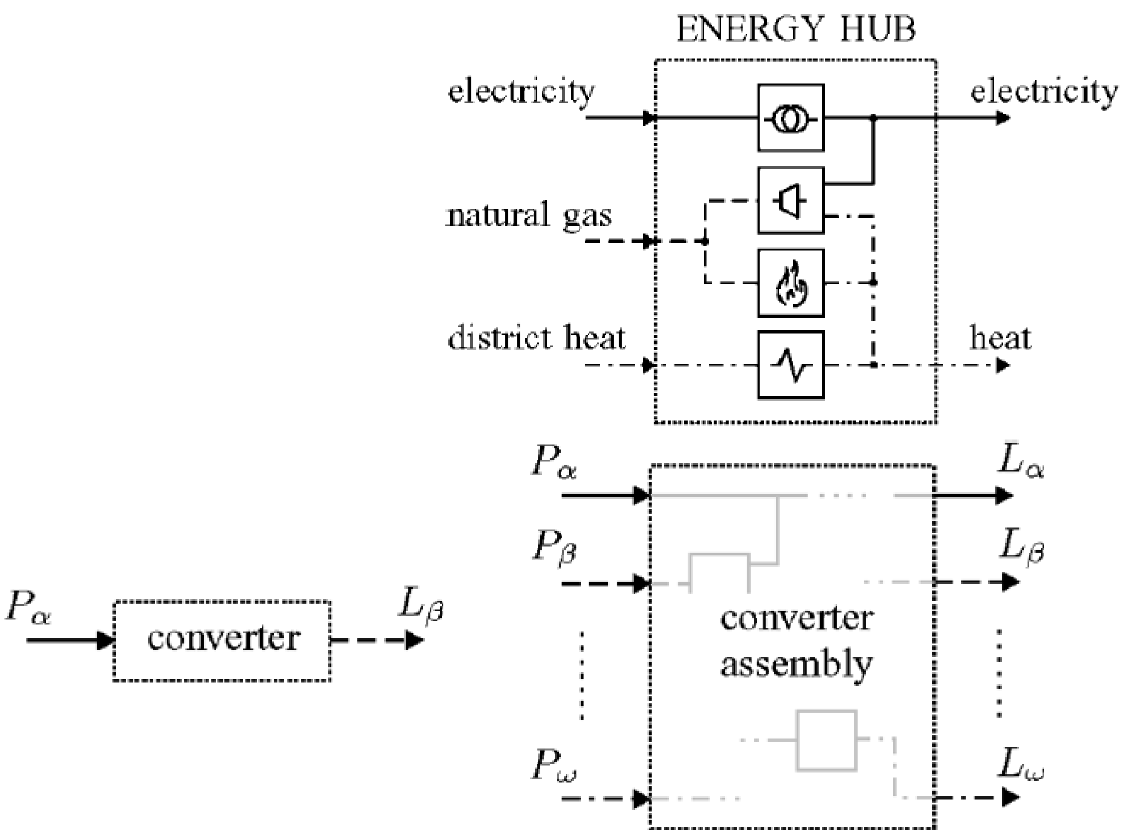


$$\begin{bmatrix} P_{out}^{\alpha} \\ P_{out}^{\beta} \\ \vdots \\ P_{out}^{\zeta} \end{bmatrix} = \begin{bmatrix} c_{\alpha\alpha} & c_{\beta\alpha} & \cdots & c_{\zeta\alpha} \\ c_{\alpha\beta} & c_{\beta\beta} & \cdots & c_{\zeta\beta} \\ \vdots & \vdots & \ddots & \vdots \\ c_{\alpha\zeta} & c_{\beta\zeta} & \cdots & c_{\zeta\zeta} \end{bmatrix} \begin{bmatrix} P_{in}^{\alpha} \\ P_{in}^{\beta} \\ \vdots \\ P_{in}^{\zeta} \end{bmatrix}$$

output power vector coupling matrix input power vector

Motivation for Energy Hub Modelling

- Conversion between different energy carriers, e.g. natural gas into electricity and heat, establishes input-output coupling of power (and energy) flows.



Conversion Matrix C

$$\mathbf{L} + \mathbf{M} = \mathbf{C}\left[\ \mathbf{P} - \mathbf{Q}\ \right]$$

$\mathbf{L}$ = Loads (Output)

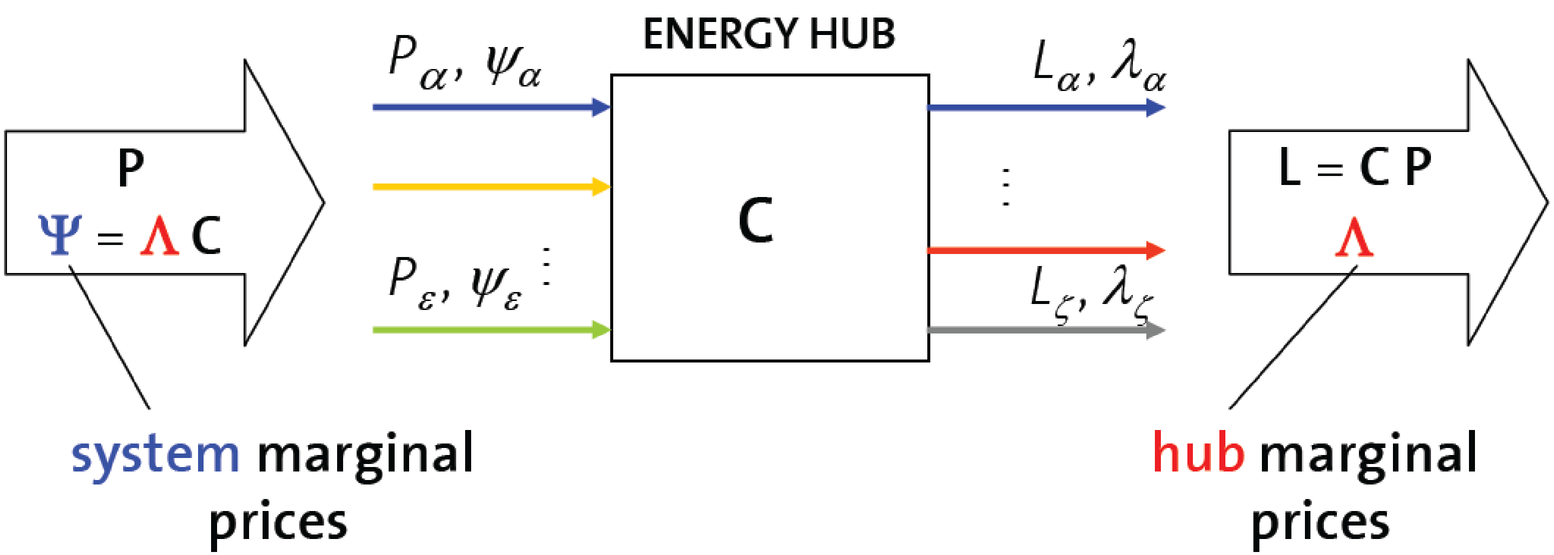
$\mathbf{M}$ = Output side storage flows

$\mathbf{C}$ = Coupling matrix

$\mathbf{P}$ = Input power flows

$\mathbf{Q}$ = Input storage flows

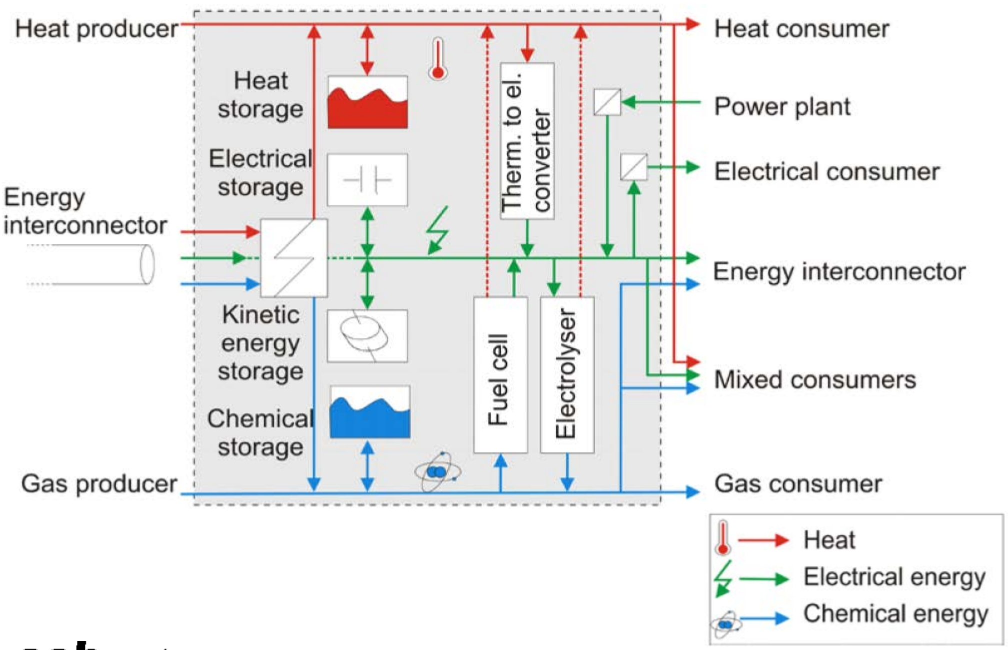
- Power conversion ⇔ price conversion

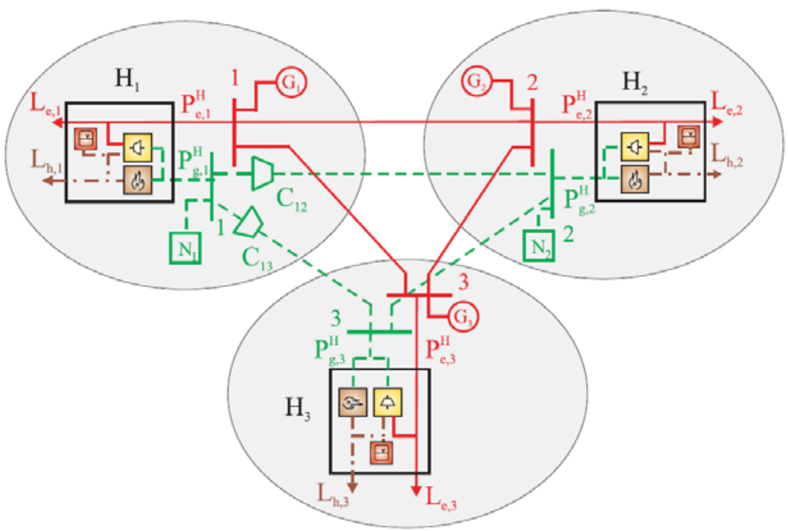


Modeling of Energy Networks – Energy Hubs

- Energy Hub concept allows unified modelling of energy networks and resulting synergies of electricity networks ($\mathbf{P_{el}}$, $\mathbf{E_{el}}$), natural gas networks ($\mathbf{P_{gas}}$, $\mathbf{E_{gas}}$) and district heat networks ($\mathbf{P_{heat}}$, $\mathbf{E_{heat}}$)

- Energy Hub concept allows analysis and optimization of investment optimality, operation efficiency and operation reliability.



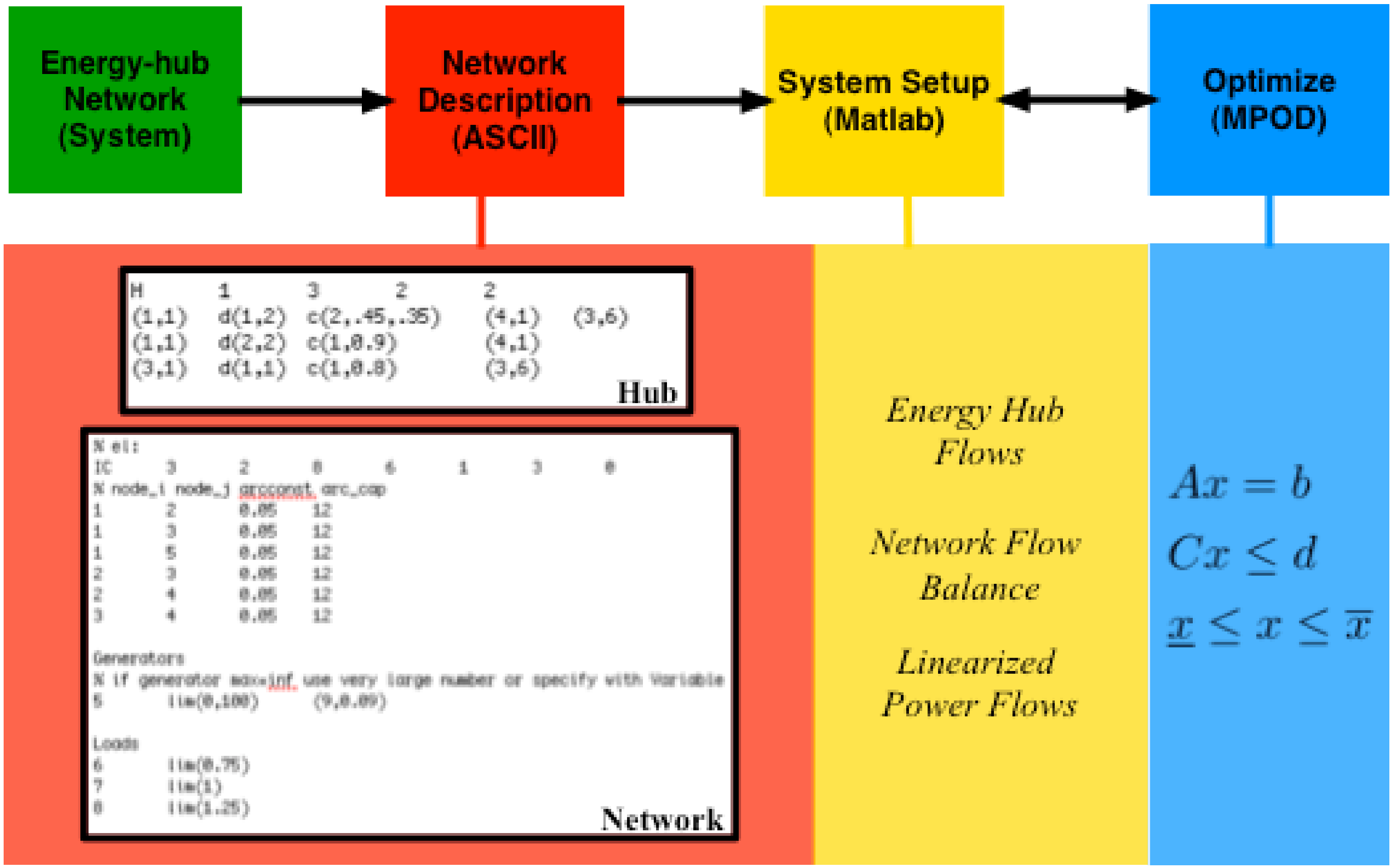


Geidl, Andersson et al., 2007 and 2008

- **Multi-period Optimal Dispatch (MPOD) of hub systems**
  - *Minimize energy costs in system*
    - Also includes penalty on load control and wind curtailment
  - *Subject to*
    - Energy hub flows, limits on hub elements
    - Hub storage integrator dynamics, limits on storage devices
    - Physics of power flow, limits on network elements
    - Forecasted energy demand, fuel costs, and renewable

  - Solution represents <u>**optimal energy schedule**</u> over MP horizon
  - Similar to economic dispatch in electric power systems
    - Energy storage enforces tighter coupling between time-steps

- **<u>HUBERT</u>- automated simulation of arbitrarily large hub systems**

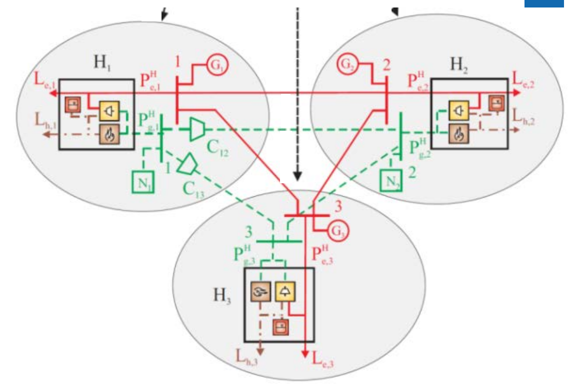


From Mads Almassalkhi

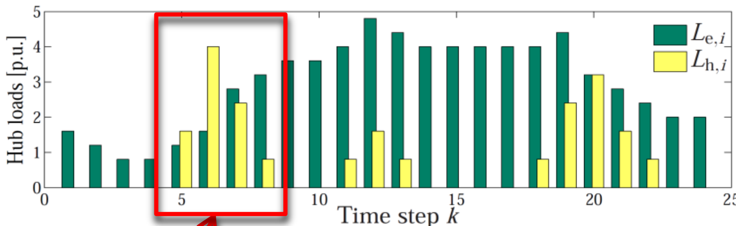
Some Applications

- Long term energy planning of the city of Bern
- Energy planning of several Swiss municipalities
- Analysis of e-mobility
- Energy/Exergy analysis of cities of Zürich and Geneva
- Long term energy network expansion in Europe
- Energy efficiency studies of airports, harbours, etc in Europe (EPICAP)

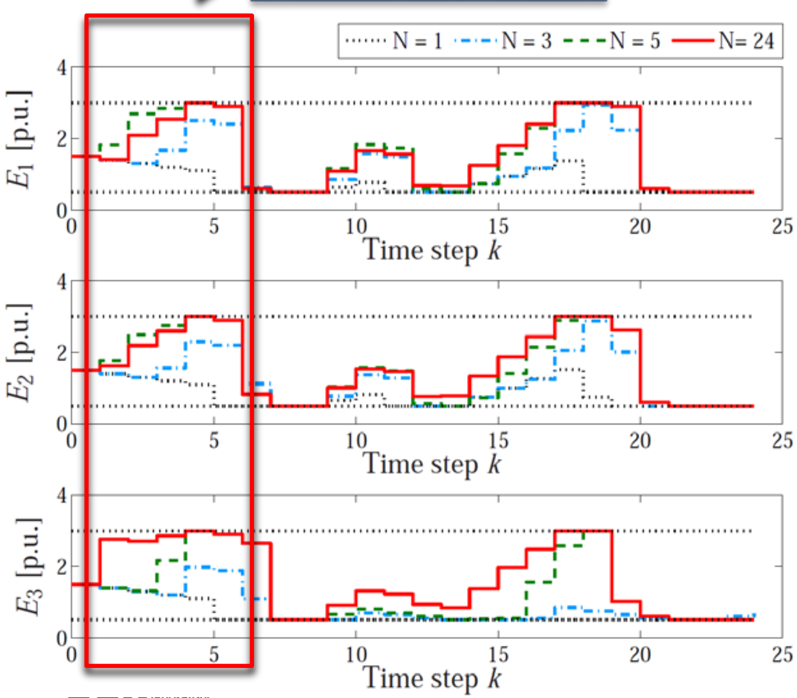
Influence of Prediction Horizon



Load Profiles: Electricity and Heat

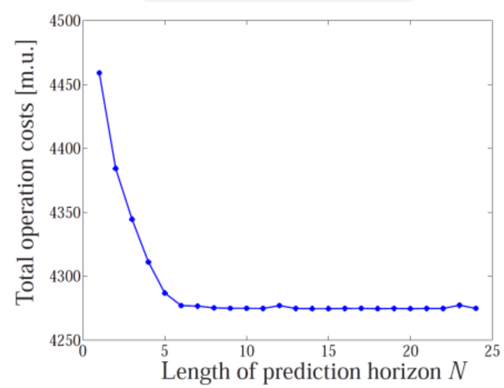


SOC Heat Storage

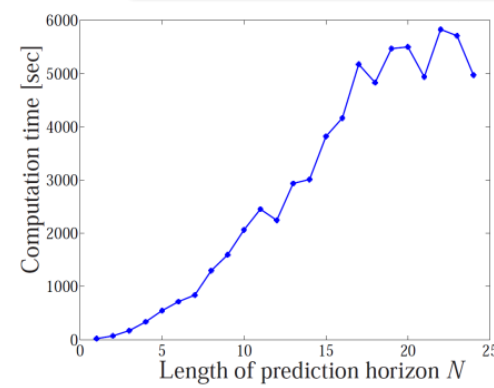


- Operation of heat storage dependent on heat load and CHP operation

Costs



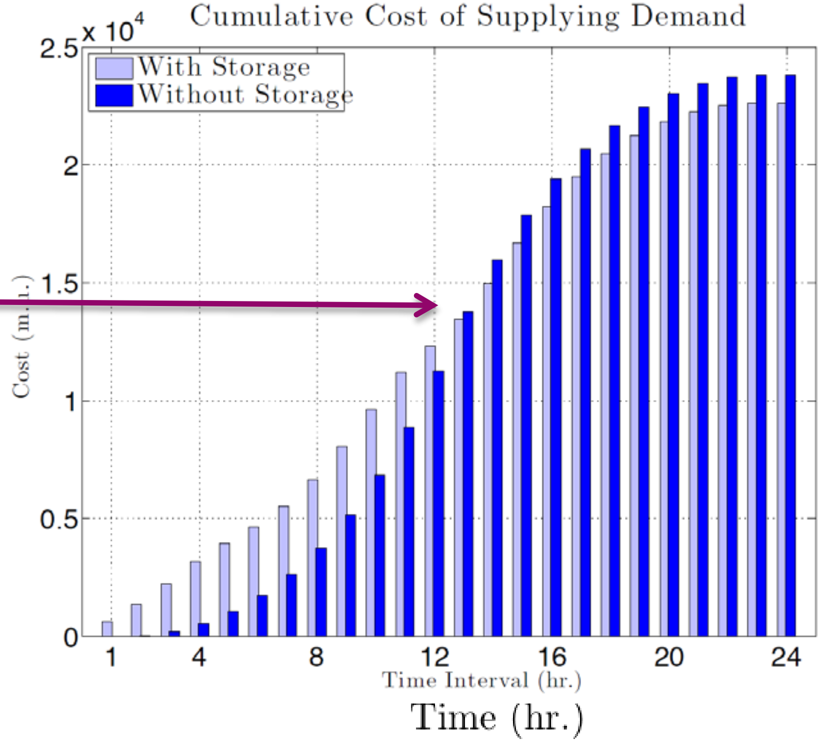
Computational Effort



Energy hub optimization

- **Simulating large multi-energy systems**
  - Example: 102 energy hubs,
    - electric + natural gas networks & wind farms + heating loads

Break-even time



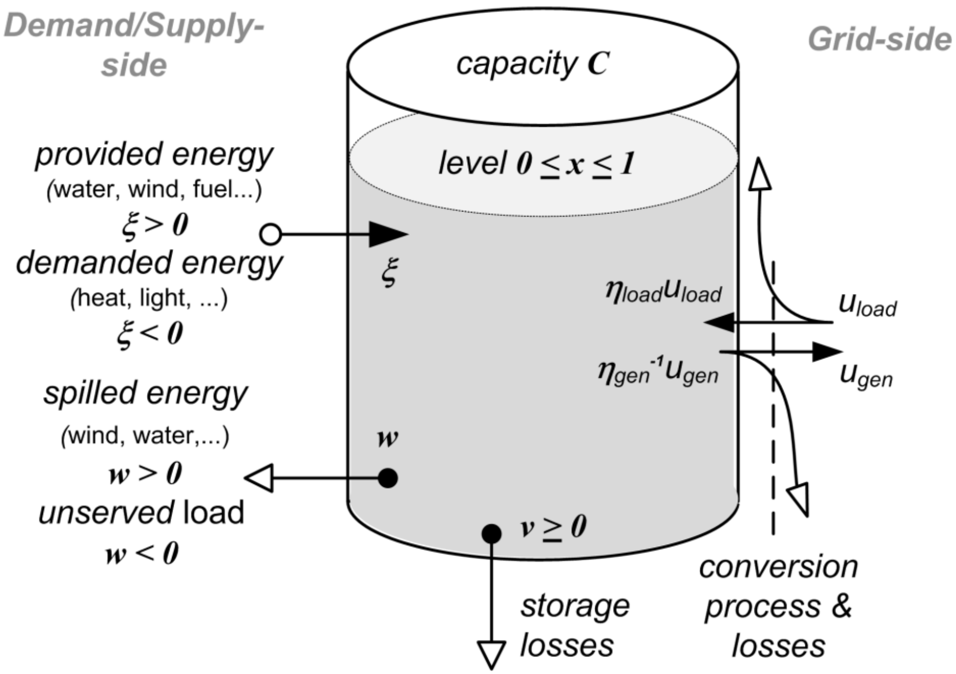
Economic benefits of storage

From Mads Almassalkhi

Power Nodes Framework

Kai Heussen (DTU)
Stephan Koch
Andreas Ulbig
...

Power Node Modeling Approach

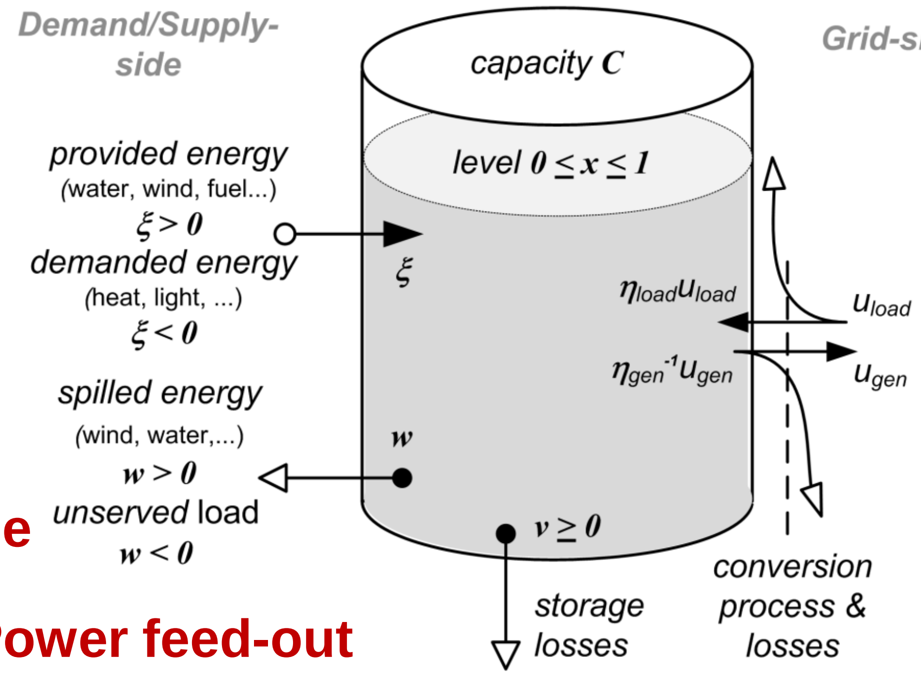


State-Descriptor Form

$$C_{soc,i}\dot{x}_i$$
$$= a_i x_i + b_i{}^T \underline{u}$$

$$C_i \dot{x}_i = \eta_{\mathrm{load}_i} u_{\mathrm{load}_i} - \eta_{\mathrm{gen}_i}^{-1} u_{\mathrm{gen}_i} + \xi_i - w_i - v_i$$

Power Node Modeling Approach



State-Descriptor Form

$$C_{soc,i}\dot{x}_i = a_i x_i + b_i^T \underline{u}$$

Internal storage losses v(x)

Storage capacity × state-of-charge (SOC)

Power feed-out from grid

Power feed-in to grid

Shedding term

$$C_i \dot{x}_i = \eta_{\text{load}_i} u_{\text{load}_i} - \eta_{\text{gen}_i}^{-1} u_{\text{gen}_i} + \xi_i - w_i - v_i$$

Efficiency factors

Provided / demanded power

Examples of Power Node Definitions

General formulation: $$C_i \dot{x}_i = \eta_{\text{load}_i} u_{\text{load}_i} - \eta_{\text{gen}_i}^{-1} u_{\text{gen}_i} + \xi_i - w_i - v_i$$

Combined Heat/ Power Plant(CHP), Berlin-Mitte

- **Fully dispatchable generation**
- **No load, no storage (C)**
- **Fuel: natural gas ($\xi>0$)**

$$\eta_{\text{gen}_i}^{-1} u_{\text{gen}_i} = \xi_i$$

Offshore Wind Farm, Denmark

- **Time-dependent dispatchable generation, if wind blows, $\xi \geq 0$, and if energy waste term $w \geq 0$**
- **No load, no storage (C)**
- **Fuel: wind power ($\xi>0$)**

$$\eta_{\text{gen}_i}^{-1} u_{\text{gen}_i} = \xi_i - w_i$$

Examples of Power Node Definitions

General formulation: $$C_i \dot{x}_i = \eta_{\text{load}_i} u_{\text{load}_i} - \eta_{\text{gen}_i}^{-1} u_{\text{gen}_i} + \xi_i - w_i - v_i$$

Residential electric water heaters

- **Time-dependent dispatchable load (heating element)**
- **Constrained "storage" ($C \approx 10\ kWh$)**
- **Demand: hot water, daily pattern ($\xi < 0$), internal heat loss ($v > 0$)**

$$C_i \dot{x}_i = \eta_{\text{load}_i} u_{\text{load}_i} + \xi_i - v_i$$

Plug-In (Hybrid) Electric Vehicle (PHEV/EV)

- **Dispatchable generation & load**
- **Battery storage ($C \approx 10\text{-}20\ kWh$), very small losses ($v \approx 0$)**
- **Demand: driving profile ($\xi < 0$), EV: ($w = 0$)**
- **PHEV: Substitute electricity by fuel ($w \geq 0$)**

Charging only: $$C_i \dot{x}_i = \eta_{\text{load}_i} u_{\text{load}_i} + \xi_i$$

Full V2G support: $$C_i \dot{x}_i = \eta_{\text{load}_i} u_{\text{load}_i} - \eta_{\text{gen}_i}^{-1} u_{\text{gen}_i} + \xi_i - w_i$$

Examples of Power Node Definitions

General formulation: $$C_i \dot{x}_i = \eta_{\text{load}_i} u_{\text{load}_i} - \eta_{\text{gen}_i}^{-1} u_{\text{gen}_i} + \xi_i - w_i - v_i$$

Goldisthal Hydro Pumped Storage, Germany

- Fully dispatchable generation (turbine) and load (pump)
- Constrained storage ($C \approx 8$ GWh)
- Fuel: almost no water influx ($\xi \approx 0$)

$$C_i \dot{x}_i = \eta_{\text{load}_i} u_{\text{load}_i} - \eta_{\text{gen}_i}^{-1} u_{\text{gen}_i}$$

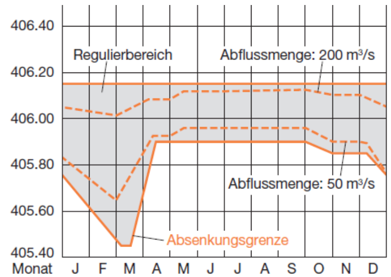
Emossion Storage Lake, Switzerland

- Fully dispatchable generation (turbine), but no load (pump)
- Large storage ($C \approx 1000$ GWh)
- Fuel supply: rain, snow melting ($\xi > 0$)

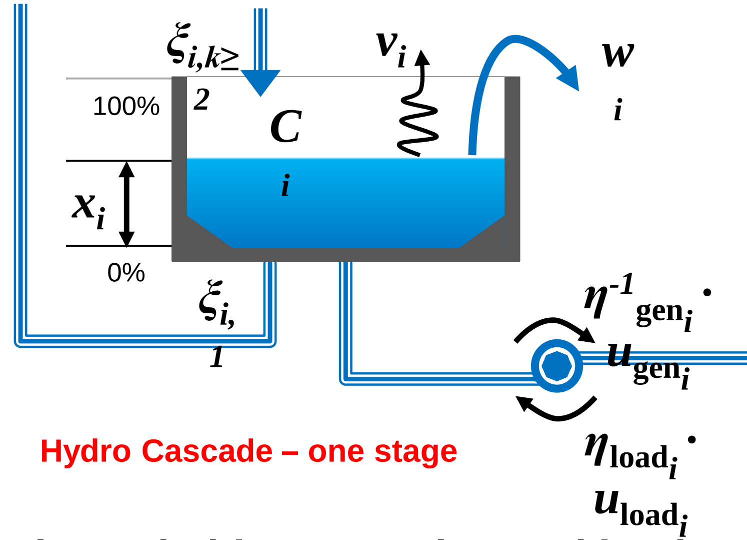
$$C_i \dot{x}_i = -\eta_{\text{gen}_i}^{-1} u_{\text{gen}_i} + \xi_i$$

Examples of Power Node Definitions

General formulation: $$C_i \dot{x}_i = \eta_{\text{load}_i} u_{\text{load}_i} - \eta_{\text{gen}_i}^{-1} u_{\text{gen}_i} + \xi_i - w_i - v_i$$




Run-of-River Plant, Zurich



Hydro Cascade – one stage

- **Dispatchable generation, but no load**
- **Storage function dependent on geography, $C \in [0, \ldots, GWh, TWh]$**
- **Fuel (ξ): water influx from river, ($\xi>0$)**
- **Waste (w): water flow over barrage (high water-level) or intentional water diversion**

$$C_i \dot{x}_i = -\eta_{\text{gen}_i}^{-1} u_{\text{gen}_i} + \xi_i^{\text{water inflow}} - w_i$$

- **Dispatchable generation and load**
- **Constrained storage ($C \approx GWh$ range)**
- **Fuel ($\xi_{i,k}$): water influx from upper basin and other inflows ($\xi_{i,k\geq 2}$)**
- **Waste (w): water discharge into lower basin (or river)**
- **Loss (v): evaporation from bassin**

$$C_i \dot{x}_i = \eta_{\text{load}_i} u_{\text{load}_i} - \eta_{\text{gen}_i}^{-1} u_{\text{gen}_i} + \sum_k \xi_{i,k} - w_i - v_i$$

Power Nodes Simulations – Predictive Power Dispatch

$$
\begin{aligned}
\min J(k) \quad = \quad & \sum_{l=k}^{l=k+N-1} (x(l) - x_{ref})^T \cdot Q_x \cdot (x(l) - x_{ref}) \\
& + u(l)^T \cdot Q_u \cdot u(l) + R_u \cdot u(l) \\
& + \delta u(l)^T \cdot \delta Q_u \cdot \delta u(l) \\
\text{s.t.} \quad (a) \quad & x(l+1) = A \cdot x(l) + B \cdot u(l) \\
(b) \quad & 0 \leq x^{min} \leq x(l) \leq x^{max} \leq 1 \\
(c) \quad & 0 \leq u^{min} \leq u(l) \leq u^{max} \\
(d) \quad & \delta u^{min} \leq \delta u(l) \leq \delta u^{max} \\
(e) \quad & \xi_1(l) = \xi_{drv,1}(l \cdot T) \\
(f) \quad & \xi_2(l) = \xi_{drv,2}(l \cdot T) \\
(g) \quad & \xi_3(l) = \xi_{drv,3}(l \cdot T) \\
(h) \quad & \xi_7(l) = \xi_{drv,7}(l \cdot T) \\
(i) \quad & u_{gen,4}(l) \cdot u_{load,4}(l) = 0 \\
(j) \quad & u_{gen,5}(l) \cdot u_{load,5}(l) = 0 \\
(k) \quad & \sum_{i=\{2,3,4,5,6\}} u_{gen,i}(l) - \sum_{i=\{1,4,5,7\}} u_{load,i}(l) = 0 \\
(a\text{-}k) \quad & \forall l = \{k, \ldots, k+N-1\}
\end{aligned}
$$

- **Unit Commitment (UC) or Optimal Power Flow (OPF) including energy storage units**
- Demand and RES power in-feed forecasts (perfect or imperfect)
- Optimisation based on marginal generation costs (+ ramping costs)
- **UC: Copperplate simplification**
- **OPF: Grid constraints included**
- In addition: Representation of transmission and distribution grid constraints (line capacity, voltage)
- **Implementation: Matlab, Yalmip**

Verification of the Power Node approach, 1

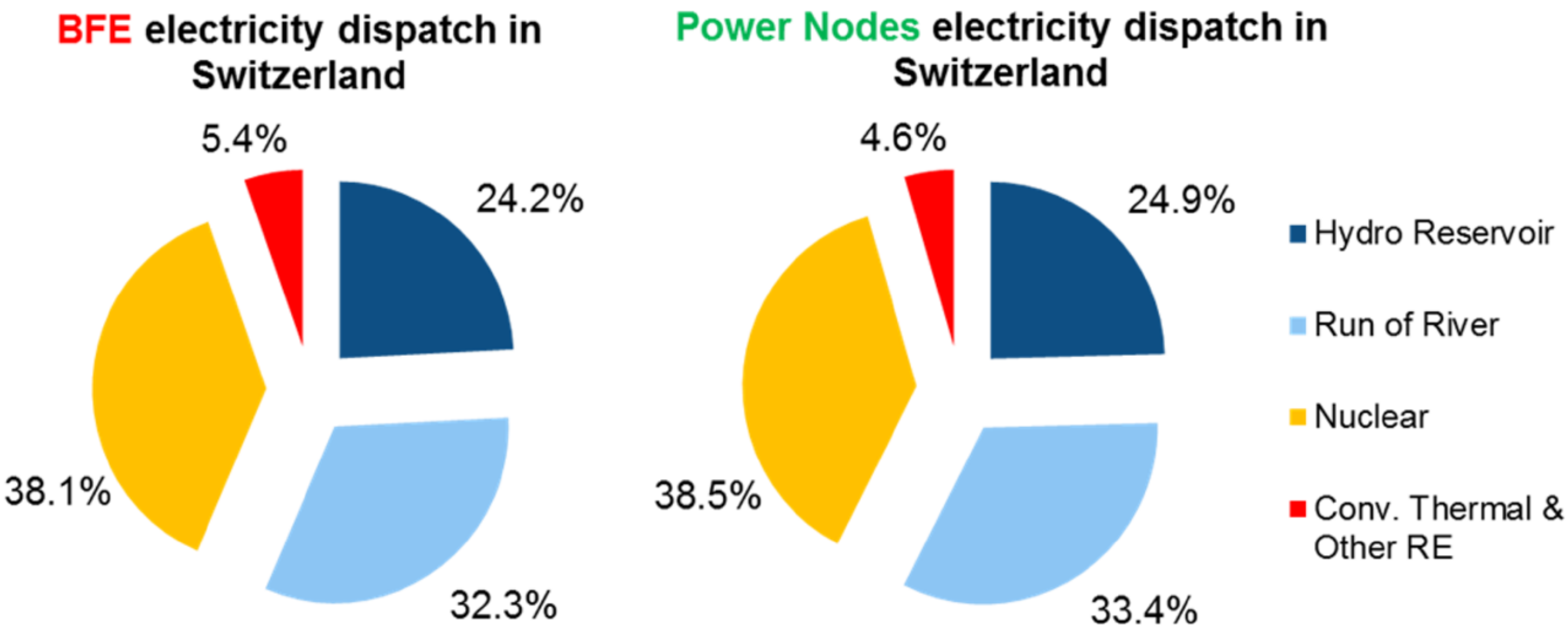


Figure 10 – BFE measurements [18] vs Power node dispatch in Switzerland in 2010.

Source: Swiss energy strategy 2050 and the consequences for electricity grid operation – full report (Comaty, Ulbig, Andersson, ETH 2014)

ETH zürich

Verification of the Power Node approach, 2

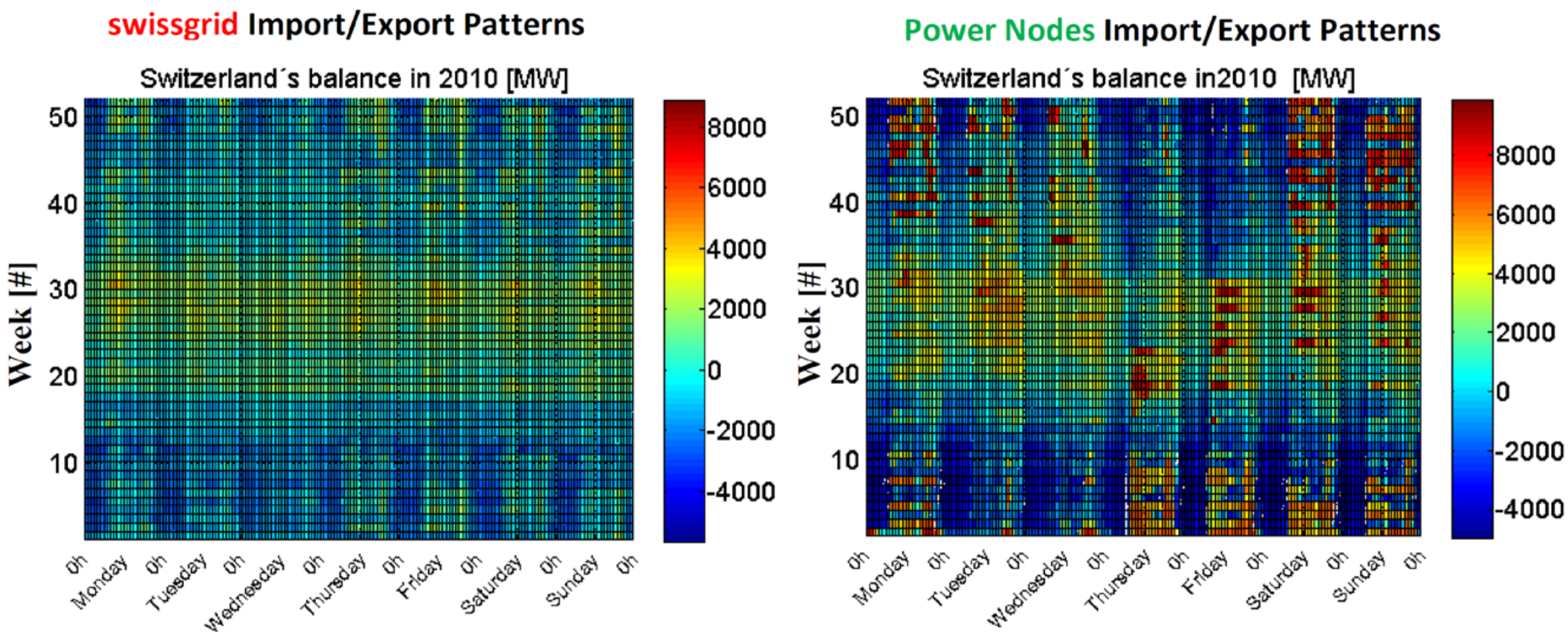


Figure 11 – Power Exchange Comparison between swissgrid Measurements and Power Node Dispatch.

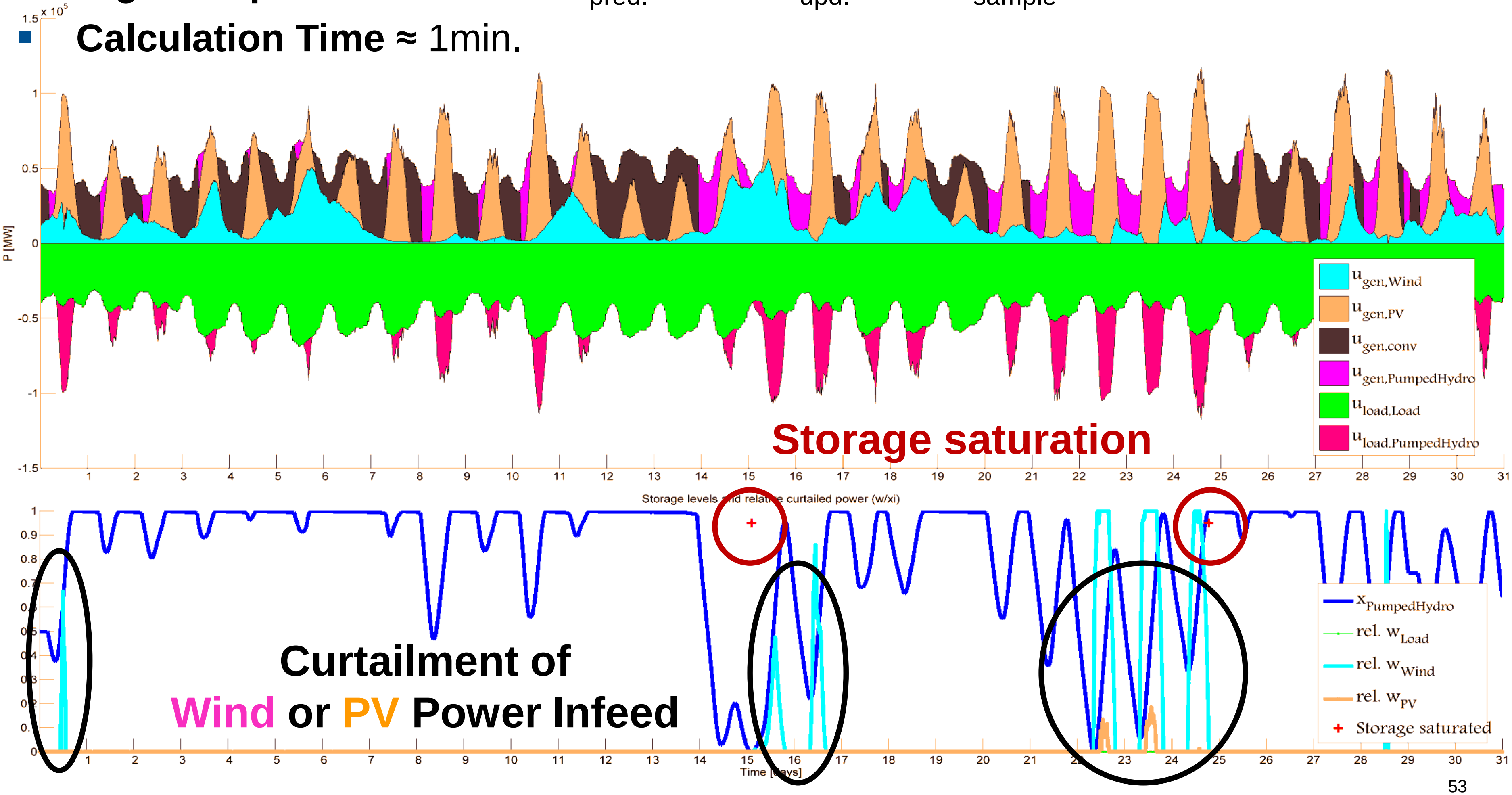
BfE statistics: Import 32.9 TWh_e/a
Export 30.9 TWh_e/a

Power Node approach: Import 30.2 TWh_e/a
Export 36.6 TWh_e/a

Source: Swiss energy strategy 2050 and the consequences for electricity grid operation – full report (Comaty, Ulbig, Andersson, ETH 2014)

Simulation Results – Predictive Power Dispatch (Case Study Germany)

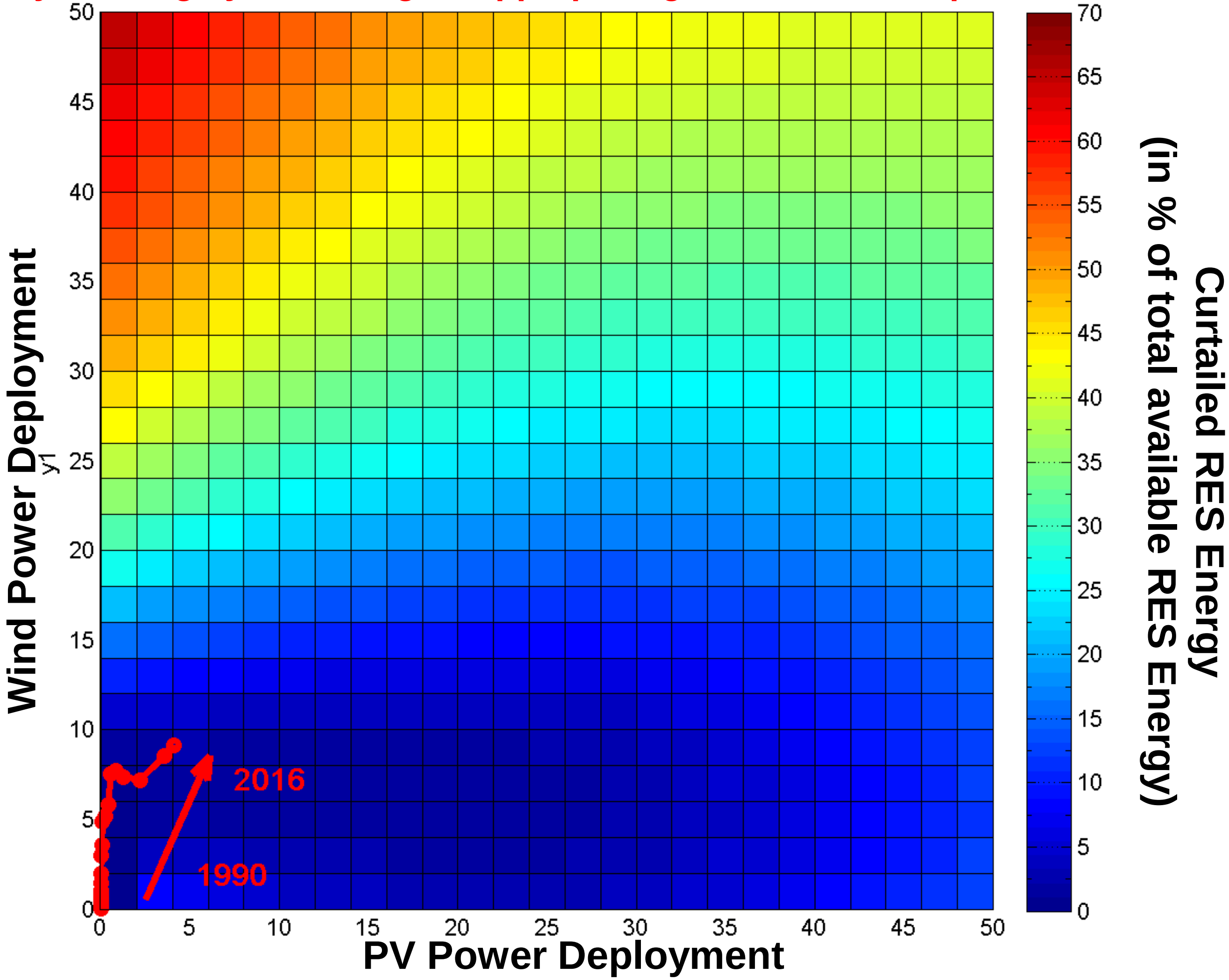
- **Simulation Period** May 2010 (30% Wind, **50% PV, no DSP**)
- **High Temporal Resolution** $T_{pred.}$ = 72h, $T_{upd.}$ = 4h, T_{sample}= 15min.
- **Calculation Time** ≈ 1min.



Assessment of Flexibility – Curtailed Renewable Energy in Germany

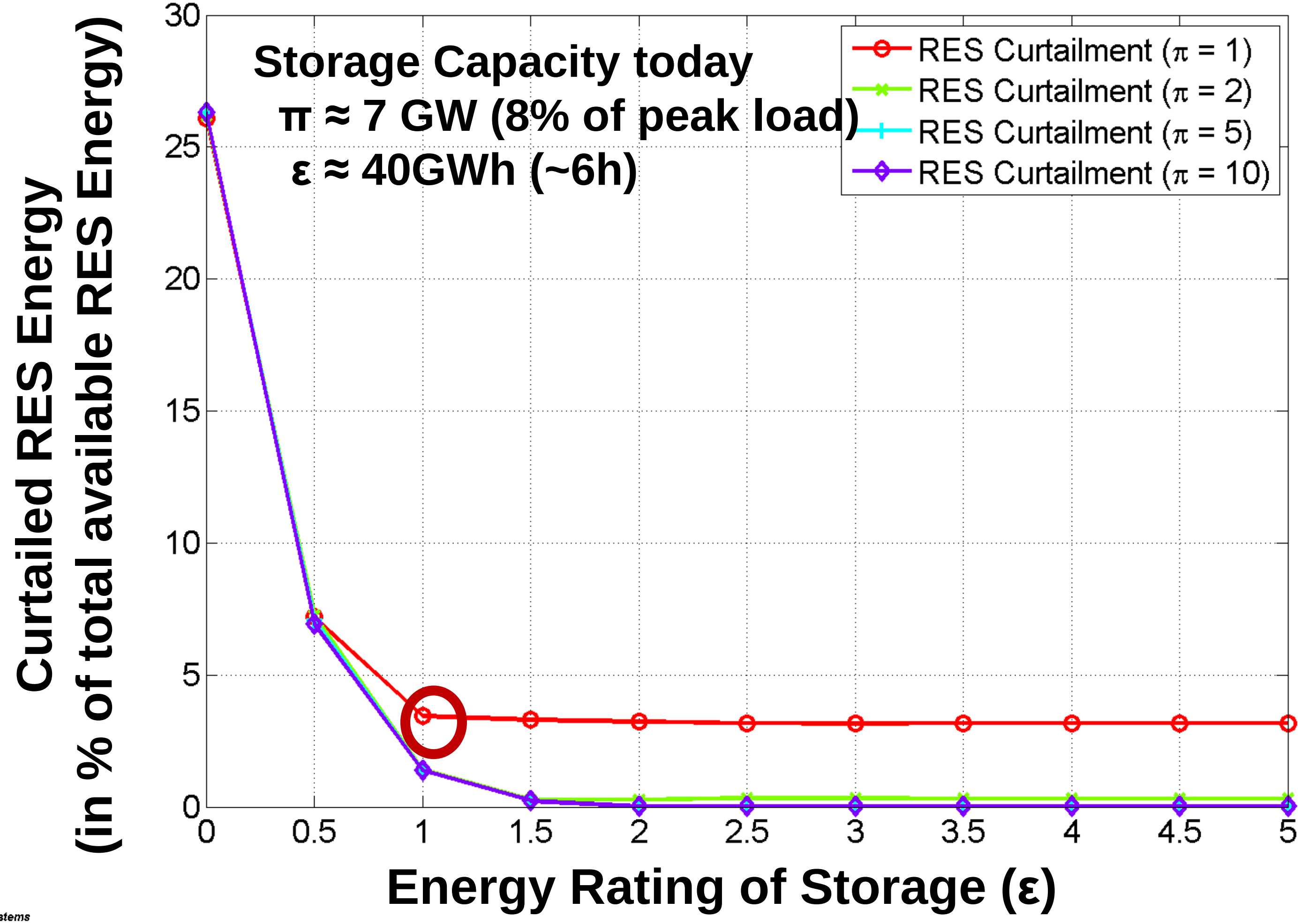
0-50% Wind Energy, 0-50% PV Energy, Full-Year 2011 simulations

only existing hydro storage, copperplate grid model, no export, no DSP



Assessment of Flexibility – Curtailed Renewable Energy in Germany

20% Wind Energy, 10% PV Energy (EU-NREAP Goals), Full-Year 2011 simulations
only existing hydro storage, copperplate grid model, no export, no DSP



Why is a predictive dispatch optimization necessary?

- Strong impact of prediction horizon length (T_p) on dispatch performance visible.
- **Example** German power system (with varying wind/PV energy shares).
- **Simulation parameters** full-year 2010, 15min sampling time, **artificial pumped hydro storage capacity of 50x nominal values** (7GW/42 GWh nominal power/energy)
- Full-year simulations of 25 setups with varying wind/PV share

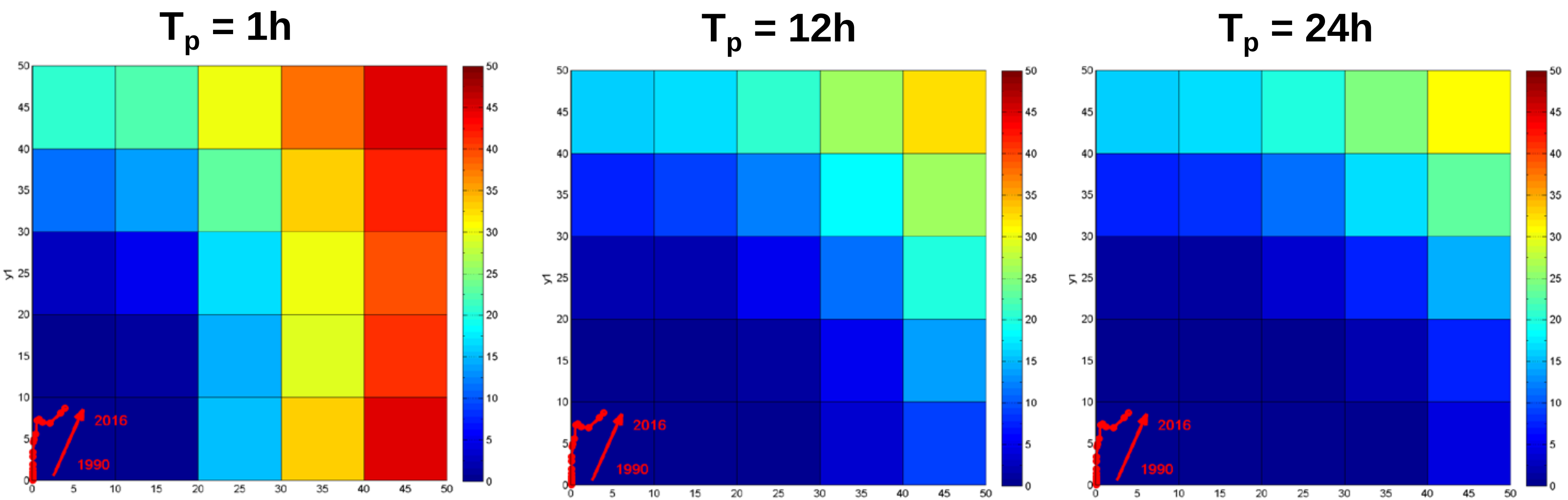


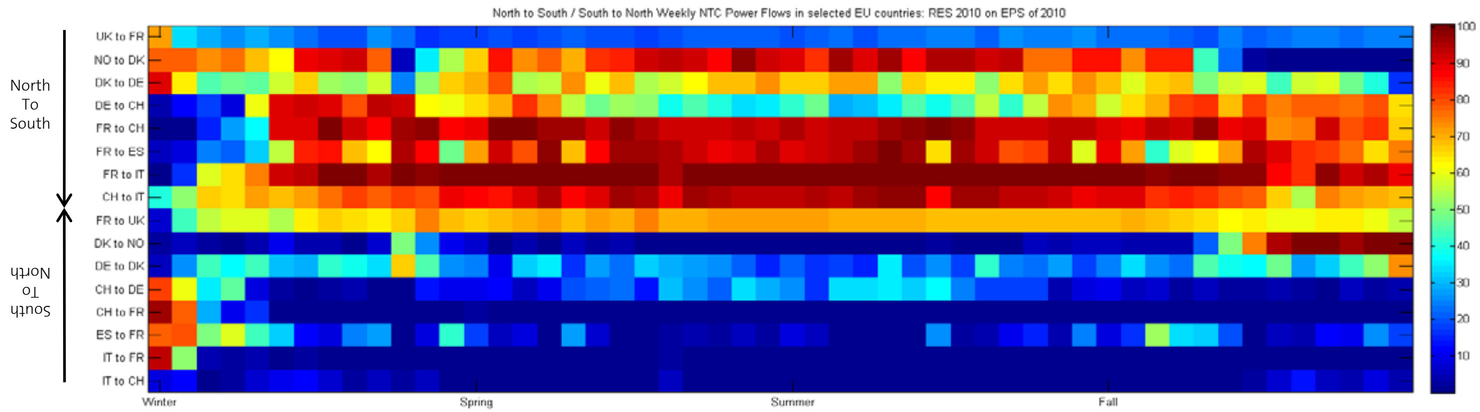
Figure description

– **x-axis: [0, 5, 10, …, 50%] of PV energy share of total yearly load demand.**
– **y-axis: [0, 5, 10, …, 50%] of wind energy share of total yearly load demand.**
– **color coding: Curtailment of Wind&PV energy (dark blue: ≈0%, dark red: ≈50%).**

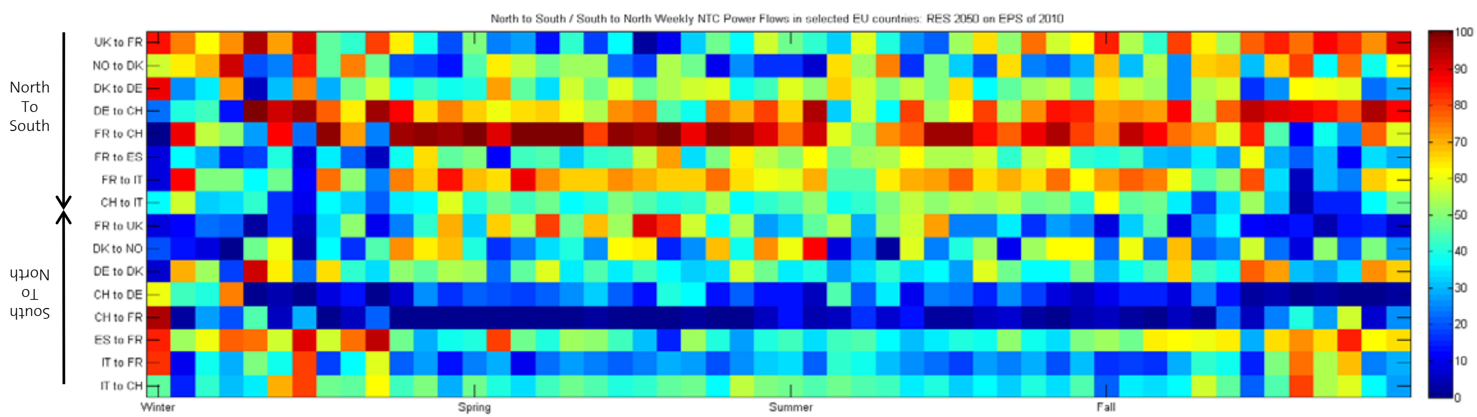
A Comment on Volatility

Change of Load Flow Patterns in European Power System

Year 2010



Year 2010 (50% RES)



Other Models

- Cyber-Physical Models of Power Systems

Daniel Kirschen & François Bouffard,
IEEE Energy & Power Magazine, 2009

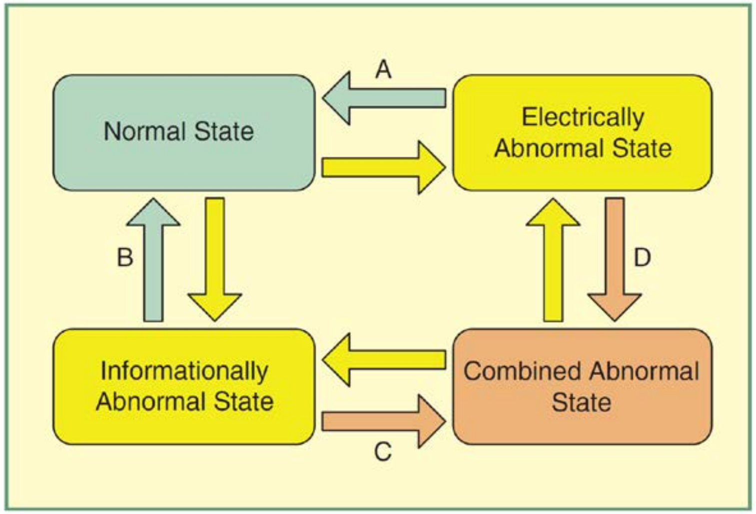


figure 2. Expanded power system security analysis framework.

Some Conclusions (1)

- The challenges of integrating renewables are manifold but – in principal – managable.

- Accurate modeling, simulation and analysis tools necessary for studying power systems and derive **adaptation strategies** from such decision support tools.

  - **Hard Paths** – Solve problems simply by oversizing everything.
    (= oversized, expensive, inefficiently operated power system)

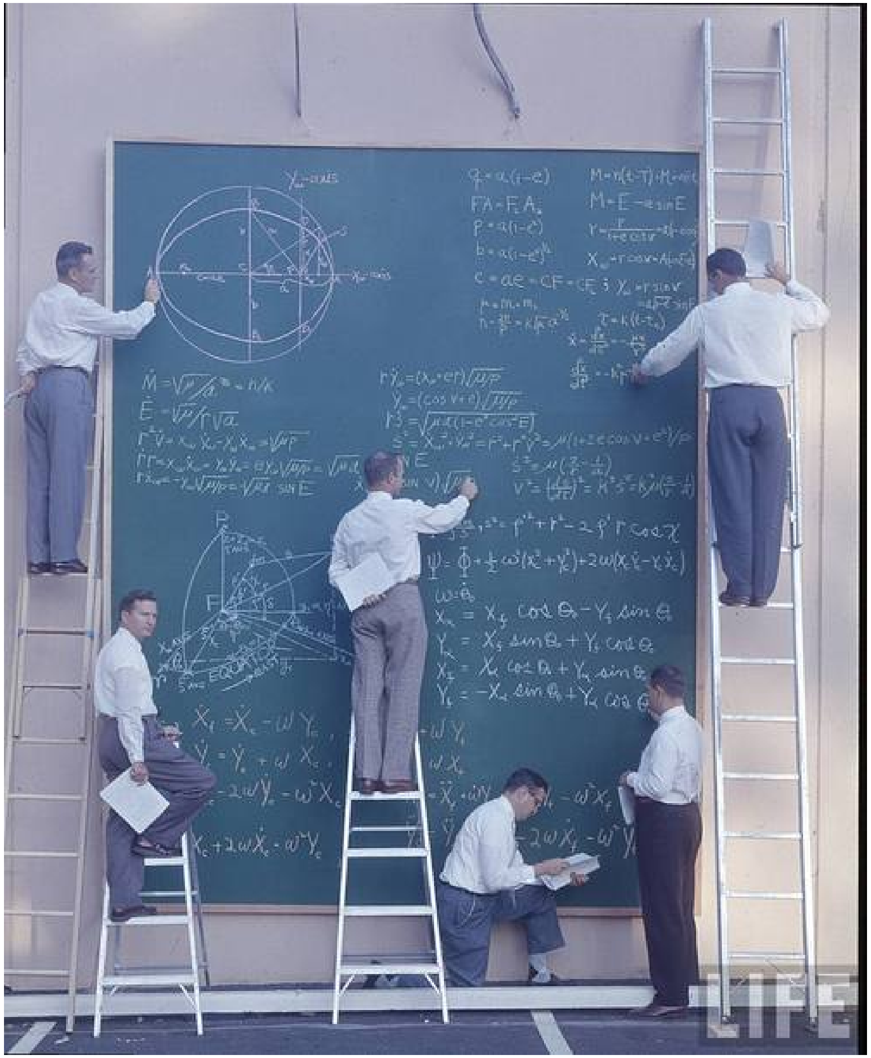
  - **Soft Paths** – Solve problems via more control & optimal operation.
    (= right sized, less expensive, efficiently operated power system)

  Control Based Expansion

- Computation and communication is cheap (and getting cheaper),
  (physical grid investments are expensive)

- Also other challenges (power markets, consumption growth, …)

Building an Energy System is a Team Work



A general reflection on research

Tomas Tranströmer

Nobel Prize Laureate in Literature 2011

Det finns i skogen en oväntad glänta
som bara kan hittas av den som gått vilse.

In the middle of the forest there is an unexpected glade
that can only be found by someone who is lost.